Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/115-4.5.0-a-sec-^m-b-trg-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [299]. This is test number [115].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (299)	0.00 (0)
Mathematica	100.00 (299)	0.00 (0)
Maple	75.59 (226)	24.41 (73)
Fricas	66.56 (199)	33.44 (100)
Maxima	31.10 (93)	68.90 (206)
Mupad	26.09 (78)	73.91 (221)
Giac	12.04 (36)	87.96 (263)
Sympy	8.36 (25)	91.64 (274)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

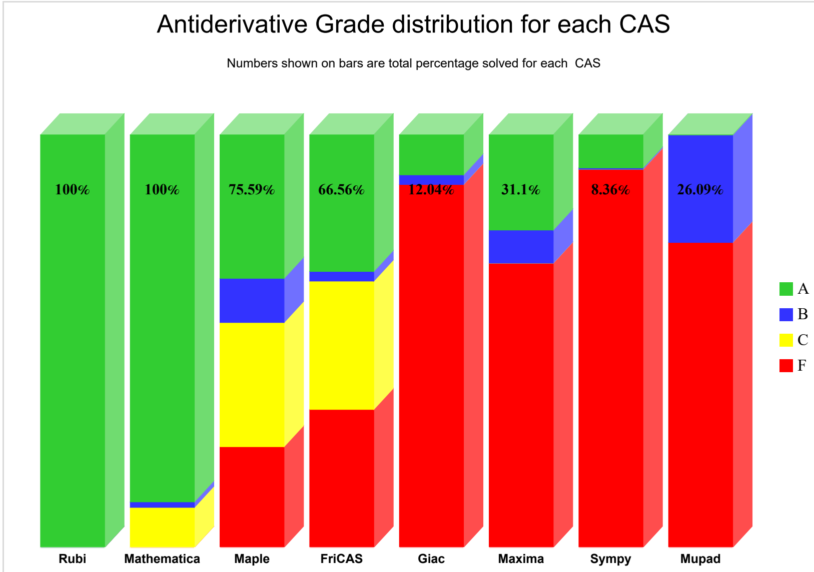
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

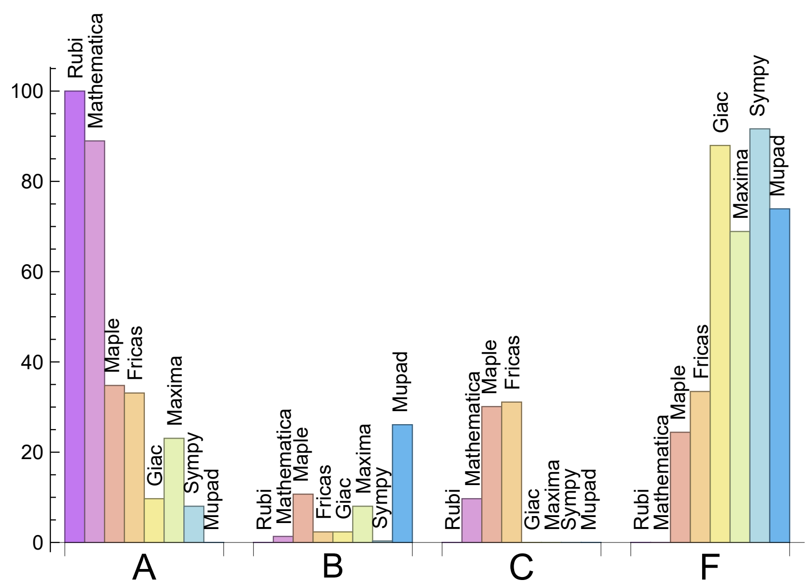
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	88.96	1.34	9.70	0.00
Maple	34.78	10.70	30.10	24.41
Fricas	33.11	2.34	31.10	33.44
Maxima	23.08	8.03	0.00	68.90
Giac	9.70	2.34	0.00	87.96
Sympy	8.03	0.33	0.00	91.64
Mupad	N/A	26.09	0.00	73.91

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	73	100.00 %	0.00 %	0.00 %
Fricas	100	81.00 %	13.00 %	6.00 %
Giac	263	99.24 %	0.00 %	0.76 %
Maxima	206	100.00 %	0.00 %	0.00 %
Sympy	274	62.41 %	23.72 %	13.87 %
Mupad	221	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

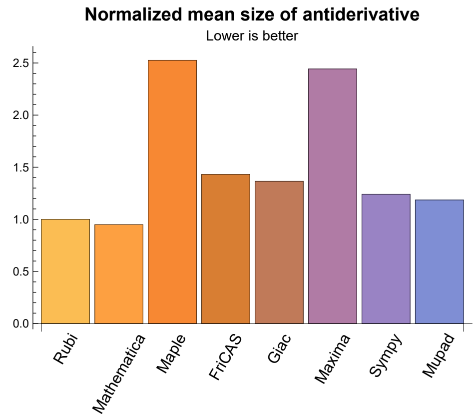
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

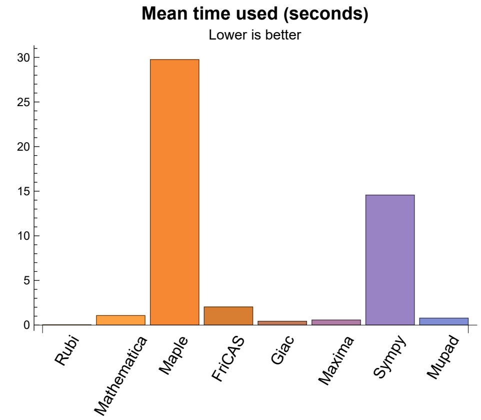
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	79.00	1.00	70.00	1.00
Mathematica	1.07	63.82	0.95	57.00	0.87
Maple	29.75	203.73	2.53	124.00	1.58
Maxima	0.57	168.66	2.44	42.00	0.96
Fricas	2.04	85.23	1.43	84.00	1.25
Sympy	14.57	53.08	1.24	46.00	1.08
Giac	0.44	38.17	1.37	35.50	0.98
Mupad	0.78	61.22	1.19	46.00	1.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
30
25
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{280, 281, 282, 283, 298\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

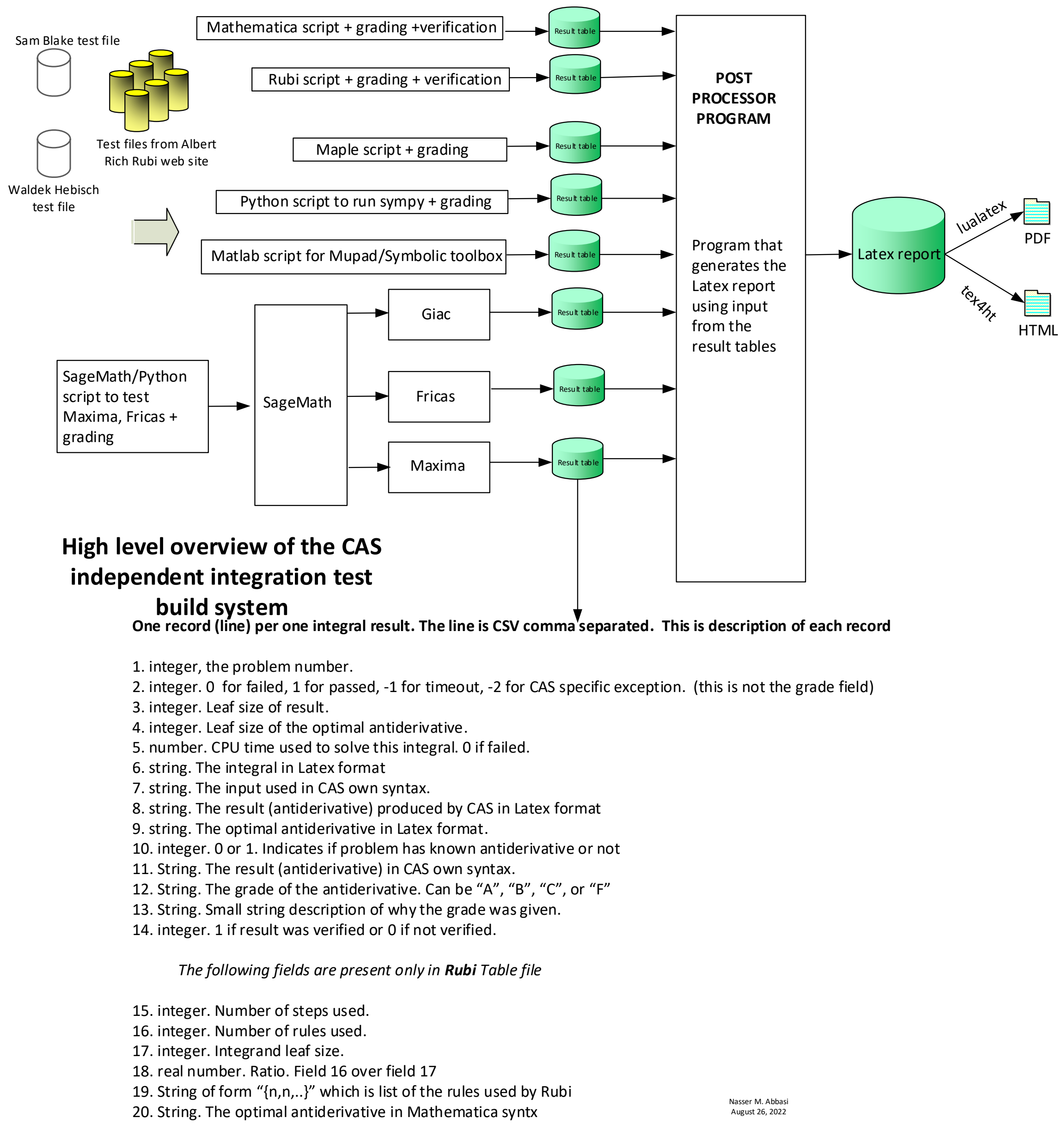
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 233, 235, 237, 238, 240, 242, 244, 247, 249, 251, 253, 254, 256, 258, 260, 262, 264, 266, 268, 270, 271, 273, 275, 277, 279, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 296, 297, 299 }

B grade: { 41, 42, 294, 295 }

C grade: { 230, 232, 234, 236, 239, 241, 243, 245, 246, 248, 250, 252, 255, 257, 259, 261, 263, 265, 267, 269, 272, 274, 276, 278, 280, 281, 282, 283, 298 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 229, 231, 233, 236, 238, 240, 242, 247, 249, 250, 251, 252, 254, 256, 262, 264, 267, 269, 271 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 16, 40, 41, 42, 228, 230, 232, 234, 239, 241, 243, 245, 246, 248, 255, 257, 259, 261, 263, 265, 272, 274, 276, 278, 287 }

C grade: { 17, 18, 19, 20, 21, 22, 23, 24, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 235, 237, 244, 253, 258, 260, 266, 268, 270, 273, 275, 277, 279, 288 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 68, 69, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 135, 137, 138, 139, 140, 141, 145, 147, 148, 149, 150, 151, 155, 157, 158, 159, 160, 164, 165, 166, 167, 171, 172, 173, 174, 175, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 287, 288, 289 }

B grade: { 47, 48, 49, 132, 133, 134, 136, 142, 143, 144, 146, 152, 153, 154, 156, 161, 162, 163, 168, 169, 170, 176, 177, 178 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.5 FriCAS

A grade: { 2, 4, 5, 6, 7, 8, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 231, 233, 238, 240, 242, 247, 249, 254, 256, 262, 287, 288, 289 }

B grade: { 1, 3, 41, 42, 251, 264, 271 }

C grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 230, 232, 234, 246, 248, 250, 252, 263, 265 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 68, 69, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 235, 236, 237, 239, 241, 243, 244, 245, 253, 255, 257, 258, 259, 260, 261, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.6 Sympy

A grade: { 43, 44, 45, 46, 51, 52, 53, 54, 137, 138, 139, 147, 148, 164, 165, 166, 167, 171, 172, 173, 174, 180, 181, 182 }

B grade: { 1 }

C grade: { }

F grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 175, 176, 177, 178, 179, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 225, 226, 228, 229 }

B grade: { 1, 40, 41, 42, 223, 224, 227 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 55, 56, 57, 58, 59, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 12, 20, 43, 51, 61, 62, 63, 64, 74, 133, 135, 137, 138, 139, 140, 141, 143, 145, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 159, 160, 162, 164, 165, 166, 167, 169, 171, 172, 173, 174, 175, 177, 179, 180, 181, 182, 183, 223, 224, 225, 226, 227, 228, 229, 231, 233, 238, 240, 242, 247, 249, 251, 254, 256, 262, 264, 271, 287, 288, 289 }

C grade: { }

F grade: { 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 136, 142, 144, 146, 154, 156, 161, 163, 168, 170, 176, 178, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 230, 232, 234, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 255, 257, 258, 259, 260, 261, 263, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	18	28	36	44	11
N.S.	1	1.00	1.00	1.73	1.64	2.55	3.27	4.00	1.00
time (sec)	N/A	0.003	0.004	0.040	0.303	2.381	0.991	0.440	0.397

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	18	0	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.80	0.00	1.00	1.00
time (sec)	N/A	0.007	0.004	0.187	0.293	2.026	0.000	0.438	0.098

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	46	61	0	48	36
N.S.	1	1.00	1.00	1.06	1.35	1.79	0.00	1.41	1.06
time (sec)	N/A	0.011	0.011	0.203	0.290	2.065	0.000	0.474	0.111

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	24	22	31	0	22	21
N.S.	1	1.00	0.88	0.92	0.85	1.19	0.00	0.85	0.81
time (sec)	N/A	0.009	0.045	0.154	0.294	2.472	0.000	0.441	0.077

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	49	71	74	0	63	58
N.S.	1	1.00	0.76	0.89	1.29	1.35	0.00	1.15	1.05
time (sec)	N/A	0.021	0.080	0.247	0.300	2.598	0.000	0.412	0.124

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	34	34	41	0	34	31
N.S.	1	1.00	0.85	0.83	0.83	1.00	0.00	0.83	0.76
time (sec)	N/A	0.012	0.120	0.152	0.294	3.005	0.000	0.425	0.089

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	59	91	84	0	73	79
N.S.	1	1.00	0.68	0.78	1.20	1.11	0.00	0.96	1.04
time (sec)	N/A	0.029	0.137	0.237	0.271	4.068	0.000	0.427	0.158

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	44	44	51	0	44	39
N.S.	1	1.00	0.81	0.83	0.83	0.96	0.00	0.83	0.74
time (sec)	N/A	0.014	0.196	0.216	0.276	4.605	0.000	0.421	0.082

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	59	358	0	110	0	0	-1
N.S.	1	1.00	0.69	4.21	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.169	4.839	0.000	0.894	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	213	0	88	0	0	-1
N.S.	1	1.00	0.74	3.44	0.00	1.42	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.065	2.125	0.000	1.474	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	182	0	73	0	0	-1
N.S.	1	1.00	0.78	3.14	0.00	1.26	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.043	2.848	0.000	0.409	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	51	0	0	33
N.S.	1	1.00	1.00	3.69	0.00	1.42	0.00	0.00	0.92
time (sec)	N/A	0.014	0.023	1.582	0.000	0.815	0.000	0.000	0.126

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	133	0	57	0	0	-1
N.S.	1	1.00	1.00	3.69	0.00	1.58	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.030	1.624	0.000	1.108	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	49	179	0	68	0	0	-1
N.S.	1	1.00	0.79	2.89	0.00	1.10	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.051	2.007	0.000	0.736	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	202	0	74	0	0	-1
N.S.	1	1.00	0.89	3.26	0.00	1.19	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.067	2.112	0.000	0.867	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	61	199	0	87	0	0	-1
N.S.	1	1.00	0.72	2.34	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.105	2.579	0.000	0.689	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	354	0	125	0	0	-1
N.S.	1	1.00	0.63	3.61	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.181	42.482	0.000	0.475	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	101	0	0	-1
N.S.	1	1.00	0.73	1.83	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.079	41.752	0.000	0.654	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	320	0	84	0	0	-1
N.S.	1	1.00	0.73	4.85	0.00	1.27	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.046	42.370	0.000	0.827	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	57	0	0	35
N.S.	1	1.00	1.00	2.58	0.00	1.50	0.00	0.00	0.92
time (sec)	N/A	0.015	0.027	27.224	0.000	0.597	0.000	0.000	0.196

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	66	0	0	-1
N.S.	1	1.00	1.00	8.05	0.00	1.74	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.034	29.639	0.000	0.912	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	87	0	0	-1
N.S.	1	1.00	0.82	1.82	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.068	27.165	0.000	0.957	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	323	0	95	0	0	-1
N.S.	1	1.00	0.83	4.49	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.081	28.654	0.000	1.019	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	100	0	0	-1
N.S.	1	1.00	0.66	1.53	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.107	27.337	0.000	1.027	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.057	0.220	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	55	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.040	0.208	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.040	0.212	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.089	0.181	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.088	0.206	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.066	0.195	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.045	0.199	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.044	0.138	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.044	0.214	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.058	0.109	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.057	0.142	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.074	0.138	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.058	0.193	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.047	0.145	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	72	42	49	0	59	-1
N.S.	1	1.00	1.48	1.44	0.84	0.98	0.00	1.18	-0.02
time (sec)	N/A	0.012	0.315	0.401	0.501	3.290	0.000	0.435	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	68	64	30	43	0	53	-1
N.S.	1	1.00	1.89	1.78	0.83	1.19	0.00	1.47	-0.03
time (sec)	N/A	0.009	0.164	0.290	0.522	2.701	0.000	0.438	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	52	55	18	34	0	44	-1
N.S.	1	1.00	2.36	2.50	0.82	1.55	0.00	2.00	-0.05
time (sec)	N/A	0.006	0.069	0.203	0.506	2.737	0.000	0.427	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	44	21	3	17	0	35	-1
N.S.	1	1.00	14.67	7.00	1.00	5.67	0.00	11.67	-0.33
time (sec)	N/A	0.005	0.011	0.194	0.536	2.818	0.000	0.452	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	4	10	6	12
N.S.	1	1.00	1.00	1.27	1.00	0.36	0.91	0.55	1.09
time (sec)	N/A	0.005	0.009	0.224	0.301	2.892	0.185	0.430	0.159

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	25	10	27	16	-1
N.S.	1	1.00	0.79	0.72	0.86	0.34	0.93	0.55	-0.03
time (sec)	N/A	0.008	0.018	0.197	0.306	2.959	0.315	0.437	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	29	37	18	44	25	-1
N.S.	1	1.00	0.72	0.67	0.86	0.42	1.02	0.58	-0.02
time (sec)	N/A	0.010	0.029	0.216	0.283	3.540	1.525	0.416	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	37	35	49	24	60	34	-1
N.S.	1	1.00	0.65	0.61	0.86	0.42	1.05	0.60	-0.02
time (sec)	N/A	0.014	0.045	0.233	0.301	2.458	12.283	0.440	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	74	2175	65	0	79	-1
N.S.	1	1.00	0.93	0.88	25.89	0.77	0.00	0.94	-0.01
time (sec)	N/A	0.030	0.154	0.362	4.491	3.555	0.000	0.449	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	66	1111	56	0	67	-1
N.S.	1	1.00	1.11	1.02	17.09	0.86	0.00	1.03	-0.02
time (sec)	N/A	0.021	0.092	0.271	0.851	2.957	0.000	0.425	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	55	324	39	0	42	-1
N.S.	1	1.00	1.20	1.20	7.04	0.85	0.00	0.91	-0.02
time (sec)	N/A	0.017	0.059	0.220	0.541	3.228	0.000	0.428	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	46	23	38	55	0	31	-1
N.S.	1	1.00	1.84	0.92	1.52	2.20	0.00	1.24	-0.04
time (sec)	N/A	0.011	0.011	0.217	0.540	3.306	0.000	0.415	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	6	16	12	11	15
N.S.	1	1.00	1.00	1.23	0.46	1.23	0.92	0.85	1.15
time (sec)	N/A	0.020	0.008	0.248	0.558	2.641	0.204	0.440	0.211

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	23	14	24	31	26	-1
N.S.	1	1.00	0.75	0.64	0.39	0.67	0.86	0.72	-0.03
time (sec)	N/A	0.014	0.023	0.179	0.605	3.061	0.365	0.436	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	31	22	32	49	36	-1
N.S.	1	1.00	0.65	0.56	0.40	0.58	0.89	0.65	-0.02
time (sec)	N/A	0.019	0.029	0.246	0.538	2.566	1.592	0.438	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	42	37	28	38	66	45	-1
N.S.	1	1.00	0.57	0.50	0.38	0.51	0.89	0.61	-0.01
time (sec)	N/A	0.027	0.041	0.257	0.540	3.378	12.430	0.431	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	59	223	0	102	0	0	-1
N.S.	1	1.00	0.50	1.91	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.110	0.902	0.000	0.838	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	87	0	74	0	0	-1
N.S.	1	1.00	0.66	1.34	0.00	1.14	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.045	0.572	0.000	1.185	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	32	189	0	57	0	0	-1
N.S.	1	1.00	0.76	4.50	0.00	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.023	0.505	0.000	0.742	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	76	0	58	0	0	-1
N.S.	1	1.00	0.70	1.73	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.046	0.498	0.000	0.790	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	43	198	0	73	0	0	-1
N.S.	1	1.00	0.59	2.71	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.106	0.549	0.000	1.022	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	59	114	0	79	0	0	-1
N.S.	1	1.00	0.50	0.97	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.110	0.494	0.000	1.330	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	54	53	61	76	0	67	589
N.S.	1	1.00	0.33	0.33	0.37	0.47	0.00	0.41	3.61
time (sec)	N/A	0.027	0.194	0.450	0.505	3.246	0.000	0.446	4.684

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	42	41	43	58	0	49	119
N.S.	1	1.00	0.36	0.35	0.37	0.50	0.00	0.42	1.02
time (sec)	N/A	0.022	0.109	0.290	0.536	2.522	0.000	0.441	2.355

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	30	29	25	34	0	22	36
N.S.	1	1.00	0.49	0.48	0.41	0.56	0.00	0.36	0.59
time (sec)	N/A	0.017	0.066	0.210	0.535	2.539	0.000	0.454	0.562

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	6	13	0	6	6
N.S.	1	1.00	1.00	0.93	0.40	0.87	0.00	0.40	0.40
time (sec)	N/A	0.011	0.008	0.204	0.506	2.903	0.000	0.424	0.111

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	22	25	27	0	39	-1
N.S.	1	1.00	0.64	0.61	0.69	0.75	0.00	1.08	-0.03
time (sec)	N/A	0.011	0.027	0.280	0.519	2.841	0.000	0.424	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	38	41	58	43	0	0	-1
N.S.	1	1.00	0.44	0.48	0.67	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.048	0.236	0.510	2.797	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	55	57	88	55	0	0	-1
N.S.	1	1.00	0.42	0.43	0.67	0.42	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.099	0.453	0.507	3.229	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	69	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.111	0.322	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.086	0.177	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	69	152	0	114	0	0	-1
N.S.	1	1.00	0.71	1.57	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.225	42.838	0.000	0.958	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	69	356	0	120	0	0	-1
N.S.	1	1.00	0.73	3.75	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.188	39.435	0.000	0.865	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	130	0	98	0	0	-1
N.S.	1	1.00	0.74	1.88	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.096	41.897	0.000	0.726	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	314	0	83	0	0	-1
N.S.	1	1.00	0.75	4.98	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.061	46.708	0.000	0.666	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	57	0	0	35
N.S.	1	1.00	1.00	2.58	0.00	1.50	0.00	0.00	0.92
time (sec)	N/A	0.015	0.026	31.422	0.000	0.644	0.000	0.000	0.232

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	303	0	63	0	0	-1
N.S.	1	1.00	1.00	7.77	0.00	1.62	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.042	27.283	0.000	0.561	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	123	0	84	0	0	-1
N.S.	1	1.00	0.76	1.84	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.062	31.474	0.000	0.850	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	57	315	0	92	0	0	-1
N.S.	1	1.00	0.81	4.50	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.071	29.362	0.000	0.560	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	145	0	97	0	0	-1
N.S.	1	1.00	0.66	1.53	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.100	28.644	0.000	0.627	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	71	325	0	105	0	0	-1
N.S.	1	1.00	0.72	3.32	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.262	28.067	0.000	0.685	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	64	152	0	117	0	0	-1
N.S.	1	1.00	0.67	1.60	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.204	45.441	0.000	0.699	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	64	356	0	121	0	0	-1
N.S.	1	1.00	0.65	3.63	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.191	42.894	0.000	0.661	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	122	0	99	0	0	-1
N.S.	1	1.00	0.73	1.82	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.089	42.546	0.000	0.917	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	322	0	84	0	0	-1
N.S.	1	1.00	0.73	4.88	0.00	1.27	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.045	43.447	0.000	0.632	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	98	0	57	0	0	-1
N.S.	1	1.00	1.00	2.51	0.00	1.46	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.025	31.973	0.000	0.843	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	309	0	63	0	0	-1
N.S.	1	1.00	1.00	7.54	0.00	1.54	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.030	28.625	0.000	1.278	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	129	0	85	0	0	-1
N.S.	1	1.00	0.74	1.84	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.058	29.224	0.000	0.586	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	319	0	93	0	0	-1
N.S.	1	1.00	0.81	4.43	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.058	30.388	0.000	0.976	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	64	151	0	99	0	0	-1
N.S.	1	1.00	0.65	1.54	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.087	30.078	0.000	0.624	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	72	331	0	107	0	0	-1
N.S.	1	1.00	0.72	3.31	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.164	32.345	0.000	0.842	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	61	152	0	121	0	0	-1
N.S.	1	1.00	0.62	1.55	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.203	41.405	0.000	0.526	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	348	0	125	0	0	-1
N.S.	1	1.00	0.63	3.59	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.184	44.937	0.000	0.792	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	101	0	0	-1
N.S.	1	1.00	0.73	1.83	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.023	43.882	0.000	0.778	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	322	0	86	0	0	-1
N.S.	1	1.00	0.74	4.74	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.041	49.264	0.000	0.638	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	57	0	0	-1
N.S.	1	1.00	1.00	2.39	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.021	29.452	0.000	0.375	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	311	0	63	0	0	-1
N.S.	1	1.00	0.93	7.59	0.00	1.54	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.051	30.359	0.000	0.922	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	131	0	87	0	0	-1
N.S.	1	1.00	0.75	1.82	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.059	29.697	0.000	1.103	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	95	0	0	-1
N.S.	1	1.00	0.83	4.46	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.060	30.701	0.000	1.299	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	103	0	0	-1
N.S.	1	1.00	0.66	1.53	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.083	28.704	0.000	0.752	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	74	333	0	111	0	0	-1
N.S.	1	1.00	0.74	3.33	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.185	30.566	0.000	0.859	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	354	0	125	0	0	-1
N.S.	1	1.00	0.63	3.61	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.024	49.855	0.000	0.680	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	152	0	117	0	0	-1
N.S.	1	1.00	0.69	1.52	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.100	47.637	0.000	0.544	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	356	0	123	0	0	-1
N.S.	1	1.00	0.63	3.67	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.251	47.756	0.000	0.805	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	130	0	101	0	0	-1
N.S.	1	1.00	0.71	1.81	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.085	47.069	0.000	0.805	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	319	0	86	0	0	-1
N.S.	1	1.00	0.74	4.91	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.093	47.328	0.000	0.595	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	60	0	0	-1
N.S.	1	1.00	1.00	2.39	0.00	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.021	31.972	0.000	0.581	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	66	0	0	-1
N.S.	1	1.00	1.00	8.05	0.00	1.74	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.020	29.859	0.000	0.470	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	126	0	87	0	0	-1
N.S.	1	1.00	0.87	1.83	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.083	32.525	0.000	0.878	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	60	316	0	95	0	0	-1
N.S.	1	1.00	0.90	4.72	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.098	32.358	0.000	0.788	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	148	0	100	0	0	-1
N.S.	1	1.00	0.68	1.53	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.094	28.477	0.000	1.119	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	328	0	108	0	0	-1
N.S.	1	1.00	0.74	3.45	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.203	30.836	0.000	0.961	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	152	0	117	0	0	-1
N.S.	1	1.00	0.69	1.52	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.180	48.153	0.000	0.769	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	356	0	123	0	0	-1
N.S.	1	1.00	0.64	3.56	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.061	48.590	0.000	0.915	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	125	0	101	0	0	-1
N.S.	1	1.00	0.78	1.74	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.115	48.372	0.000	0.442	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	322	0	86	0	0	-1
N.S.	1	1.00	0.75	4.74	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.064	51.001	0.000	0.909	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	60	0	0	-1
N.S.	1	1.00	1.00	2.39	0.00	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.021	35.836	0.000	0.741	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	311	0	66	0	0	-1
N.S.	1	1.00	1.00	7.59	0.00	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.038	34.569	0.000	1.148	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	87	0	0	-1
N.S.	1	1.00	0.82	1.82	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.057	35.145	0.000	0.785	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	323	0	95	0	0	-1
N.S.	1	1.00	0.87	4.68	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.037	35.948	0.000	0.733	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	66	153	0	100	0	0	-1
N.S.	1	1.00	0.67	1.56	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.104	33.449	0.000	0.611	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	333	0	108	0	0	-1
N.S.	1	1.00	0.75	3.43	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.088	36.142	0.000	0.906	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	152	0	117	0	0	-1
N.S.	1	1.00	0.64	1.52	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.219	52.506	0.000	0.646	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	351	0	123	0	0	-1
N.S.	1	1.00	0.64	3.51	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.150	51.204	0.000	0.603	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	130	0	101	0	0	-1
N.S.	1	1.00	0.71	1.81	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.064	52.883	0.000	1.213	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	322	0	86	0	0	-1
N.S.	1	1.00	0.75	4.74	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.060	53.596	0.000	0.735	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	98	0	60	0	0	-1
N.S.	1	1.00	1.00	2.39	0.00	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.020	33.806	0.000	0.762	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	311	0	66	0	0	-1
N.S.	1	1.00	0.93	7.59	0.00	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.042	33.748	0.000	0.673	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	131	0	87	0	0	-1
N.S.	1	1.00	0.86	1.82	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.037	34.959	0.000	0.552	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	95	0	0	-1
N.S.	1	1.00	0.83	4.46	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.023	33.386	0.000	0.562	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	153	0	100	0	0	-1
N.S.	1	1.00	0.68	1.58	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.046	35.129	0.000	1.158	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	333	0	108	0	0	-1
N.S.	1	1.00	0.74	3.40	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.135	33.619	0.000	1.093	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	66	153	0	100	0	0	-1
N.S.	1	1.00	0.66	1.53	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.026	33.977	0.000	0.674	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	64	131	1656	229	0	0	-1
N.S.	1	1.00	0.60	1.22	15.48	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.160	32.078	0.716	2.821	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	294	43	0	0	126
N.S.	1	1.00	0.64	0.74	4.20	0.61	0.00	0.00	1.80
time (sec)	N/A	0.014	0.095	33.391	0.599	2.758	0.000	0.000	2.548

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	50	112	661	199	0	0	-1
N.S.	1	1.00	0.69	1.56	9.18	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.059	33.328	0.609	3.688	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	39	54	30	0	0	46
N.S.	1	1.00	1.00	1.22	1.69	0.94	0.00	0.00	1.44
time (sec)	N/A	0.009	0.020	32.120	0.561	3.628	0.000	0.000	0.288

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	52	65	111	0	0	-1
N.S.	1	1.00	1.00	1.58	1.97	3.36	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.015	33.717	0.619	2.890	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	26	98	22	0	24
N.S.	1	1.00	1.00	1.33	1.08	4.08	0.92	0.00	1.00
time (sec)	N/A	0.002	0.016	34.552	0.577	2.137	4.767	0.000	0.204

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	13	30	46	0	32
N.S.	1	1.00	1.00	1.28	0.41	0.94	1.44	0.00	1.00
time (sec)	N/A	0.005	0.041	34.858	0.689	2.105	9.634	0.000	0.225

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	54	25	158	107	0	41
N.S.	1	1.00	0.71	0.86	0.40	2.51	1.70	0.00	0.65
time (sec)	N/A	0.010	0.074	33.358	0.577	3.144	17.604	0.000	0.439

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	42	48	0	0	45
N.S.	1	1.00	0.64	0.74	0.60	0.69	0.00	0.00	0.64
time (sec)	N/A	0.012	0.122	32.299	0.645	3.670	0.000	0.000	0.522

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	55	74	49	202	0	0	52
N.S.	1	1.00	0.56	0.76	0.50	2.06	0.00	0.00	0.53
time (sec)	N/A	0.018	0.123	36.389	0.633	4.053	0.000	0.000	0.706

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	64	131	1742	236	0	0	-1
N.S.	1	1.00	0.58	1.19	15.84	2.15	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.121	35.186	0.686	3.304	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	52	299	44	0	0	127
N.S.	1	1.00	0.62	0.72	4.15	0.61	0.00	0.00	1.76
time (sec)	N/A	0.014	0.101	34.254	0.658	2.852	0.000	0.000	1.361

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	112	691	202	0	0	-1
N.S.	1	1.00	0.68	1.51	9.34	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.069	32.749	0.609	2.513	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	39	54	31	0	0	47
N.S.	1	1.00	0.97	1.18	1.64	0.94	0.00	0.00	1.42
time (sec)	N/A	0.009	0.023	33.757	0.590	3.145	0.000	0.000	0.217

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	52	68	112	0	0	-1
N.S.	1	1.00	0.97	1.53	2.00	3.29	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.021	33.523	0.621	3.015	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	32	26	99	22	0	25
N.S.	1	1.00	0.96	1.28	1.04	3.96	0.88	0.00	1.00
time (sec)	N/A	0.002	0.028	33.197	0.556	3.035	7.138	0.000	0.196

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	41	13	31	46	0	33
N.S.	1	1.00	0.97	1.24	0.39	0.94	1.39	0.00	1.00
time (sec)	N/A	0.005	0.050	34.118	0.640	3.169	29.162	0.000	0.245

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	54	28	161	0	0	42
N.S.	1	1.00	0.69	0.83	0.43	2.48	0.00	0.00	0.65
time (sec)	N/A	0.010	0.087	36.632	0.647	3.592	0.000	0.000	0.431

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	52	45	51	0	0	46
N.S.	1	1.00	0.62	0.72	0.62	0.71	0.00	0.00	0.64
time (sec)	N/A	0.013	0.151	36.280	0.598	3.853	0.000	0.000	0.435

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	74	53	207	0	0	53
N.S.	1	1.00	0.54	0.73	0.52	2.05	0.00	0.00	0.52
time (sec)	N/A	0.019	0.168	37.973	0.619	3.688	0.000	0.000	0.590

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	57	62	705	63	0	0	205
N.S.	1	1.00	0.49	0.53	6.08	0.54	0.00	0.00	1.77
time (sec)	N/A	0.019	0.232	36.385	0.733	3.166	0.000	0.000	4.700

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	311	48	0	0	129
N.S.	1	1.00	0.59	0.68	4.09	0.63	0.00	0.00	1.70
time (sec)	N/A	0.014	0.093	32.866	0.595	3.344	0.000	0.000	1.367

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	112	747	208	0	0	-1
N.S.	1	1.00	0.64	1.44	9.58	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.084	36.988	0.634	3.682	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	54	33	0	0	66
N.S.	1	1.00	0.91	1.11	1.54	0.94	0.00	0.00	1.89
time (sec)	N/A	0.010	0.038	35.526	0.625	3.302	0.000	0.000	0.725

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	72	114	0	0	-1
N.S.	1	1.00	0.92	1.44	2.00	3.17	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.034	35.341	0.625	3.537	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	26	101	0	0	27
N.S.	1	1.00	0.89	1.19	0.96	3.74	0.00	0.00	1.00
time (sec)	N/A	0.002	0.022	36.029	0.602	3.354	0.000	0.000	0.116

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	41	13	33	0	0	35
N.S.	1	1.00	0.91	1.17	0.37	0.94	0.00	0.00	1.00
time (sec)	N/A	0.005	0.072	35.502	0.590	3.036	0.000	0.000	0.328

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	32	167	0	0	44
N.S.	1	1.00	0.65	0.78	0.46	2.42	0.00	0.00	0.64
time (sec)	N/A	0.010	0.115	35.363	0.621	3.304	0.000	0.000	0.368

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	49	55	0	0	48
N.S.	1	1.00	0.59	0.68	0.64	0.72	0.00	0.00	0.63
time (sec)	N/A	0.012	0.175	38.956	0.705	3.094	0.000	0.000	0.499

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	50	114	661	205	0	0	-1
N.S.	1	1.00	0.69	1.58	9.18	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.059	37.198	0.632	2.980	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	39	59	33	0	0	51
N.S.	1	1.00	1.00	1.22	1.84	1.03	0.00	0.00	1.59
time (sec)	N/A	0.009	0.024	35.918	0.586	3.167	0.000	0.000	0.269

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	52	65	114	0	0	-1
N.S.	1	1.00	1.00	1.58	1.97	3.45	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.015	34.892	0.573	3.420	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	26	101	22	0	27
N.S.	1	1.00	1.00	1.33	1.08	4.21	0.92	0.00	1.12
time (sec)	N/A	0.002	0.014	34.763	0.568	3.412	11.347	0.000	0.296

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	41	13	33	46	0	35
N.S.	1	1.00	1.00	1.28	0.41	1.03	1.44	0.00	1.09
time (sec)	N/A	0.005	0.034	35.234	0.589	3.347	20.366	0.000	0.313

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	54	25	165	107	0	44
N.S.	1	1.00	0.71	0.86	0.40	2.62	1.70	0.00	0.70
time (sec)	N/A	0.009	0.072	37.024	0.595	4.138	21.847	0.000	0.428

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	52	42	51	82	0	48
N.S.	1	1.00	0.64	0.74	0.60	0.73	1.17	0.00	0.69
time (sec)	N/A	0.011	0.090	35.175	0.584	3.891	43.458	0.000	0.516

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	114	670	205	0	0	-1
N.S.	1	1.00	0.64	1.46	8.59	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.067	34.297	0.636	3.612	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	67	33	0	0	51
N.S.	1	1.00	0.91	1.11	1.91	0.94	0.00	0.00	1.46
time (sec)	N/A	0.009	0.038	34.023	0.583	3.381	0.000	0.000	0.266

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	65	114	0	0	-1
N.S.	1	1.00	0.92	1.44	1.81	3.17	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.030	37.467	0.615	3.324	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	26	101	22	0	27
N.S.	1	1.00	0.89	1.19	0.96	3.74	0.81	0.00	1.00
time (sec)	N/A	0.002	0.024	34.400	0.533	4.263	13.445	0.000	0.252

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	41	13	33	46	0	39
N.S.	1	1.00	0.91	1.17	0.37	0.94	1.31	0.00	1.11
time (sec)	N/A	0.005	0.045	35.286	0.606	4.142	11.701	0.000	0.422

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	25	165	107	0	44
N.S.	1	1.00	0.65	0.78	0.36	2.39	1.55	0.00	0.64
time (sec)	N/A	0.011	0.076	35.499	0.596	4.206	21.923	0.000	0.369

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	52	42	51	82	0	48
N.S.	1	1.00	0.59	0.68	0.55	0.67	1.08	0.00	0.63
time (sec)	N/A	0.012	0.103	35.542	0.589	3.772	30.243	0.000	0.327

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	55	74	49	208	0	0	55
N.S.	1	1.00	0.51	0.69	0.46	1.94	0.00	0.00	0.51
time (sec)	N/A	0.020	0.125	32.792	0.653	3.761	0.000	0.000	0.615

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	114	688	205	0	0	-1
N.S.	1	1.00	0.68	1.46	8.82	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.051	36.757	0.599	2.692	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	67	33	0	0	51
N.S.	1	1.00	0.91	1.11	1.91	0.94	0.00	0.00	1.46
time (sec)	N/A	0.010	0.036	36.856	0.609	2.867	0.000	0.000	0.263

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	52	65	114	0	0	-1
N.S.	1	1.00	0.92	1.44	1.81	3.17	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.025	35.564	0.625	3.051	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	32	26	101	0	0	27
N.S.	1	1.00	0.89	1.19	0.96	3.74	0.00	0.00	1.00
time (sec)	N/A	0.002	0.022	35.235	0.591	4.256	0.000	0.000	0.323

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	41	13	33	46	0	39
N.S.	1	1.00	1.00	1.17	0.37	0.94	1.31	0.00	1.11
time (sec)	N/A	0.006	0.027	36.148	0.651	2.445	30.437	0.000	0.392

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	48	54	25	165	107	0	64
N.S.	1	1.00	0.70	0.78	0.36	2.39	1.55	0.00	0.93
time (sec)	N/A	0.011	0.059	32.279	0.583	3.568	19.408	0.000	0.628

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	48	52	42	51	82	0	48
N.S.	1	1.00	0.63	0.68	0.55	0.67	1.08	0.00	0.63
time (sec)	N/A	0.013	0.072	34.748	0.608	3.181	41.973	0.000	0.422

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	58	74	49	208	0	0	55
N.S.	1	1.00	0.54	0.69	0.46	1.94	0.00	0.00	0.51
time (sec)	N/A	0.019	0.063	35.690	0.656	3.625	0.000	0.000	0.459

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.090	0.201	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.053	0.204	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.045	0.200	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.068	0.256	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.161	0.431	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.083	0.207	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.053	0.180	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	57	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.005	0.220	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.009	0.302	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.013	0.595	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.065	0.271	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	60	0	0	0	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.049	0.269	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.057	0.266	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.087	0.408	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.078	0.734	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	60	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.009	0.276	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	58	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.016	0.268	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.006	0.275	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.020	0.400	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.111	0.657	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.102	0.533	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.148	0.473	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.144	0.472	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.139	0.421	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.140	0.441	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	83	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.180	0.462	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.074	0.483	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.053	0.291	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	65	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.041	0.307	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.048	0.160	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.103	0.242	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.084	0.431	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.112	0.547	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.104	0.372	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.095	0.396	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.102	0.353	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.119	0.367	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.137	0.602	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.179	0.575	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	77
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	3.85
time (sec)	N/A	0.025	0.060	0.408	0.285	2.192	0.000	0.449	1.591

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	39
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.95
time (sec)	N/A	0.023	0.049	0.395	0.316	2.072	0.000	0.427	0.252

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	18	0	25	18
N.S.	1	1.00	1.00	0.94	1.28	1.00	0.00	1.39	1.00
time (sec)	N/A	0.025	0.038	0.398	0.299	3.354	0.000	0.431	0.096

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	23	0	22	23
N.S.	1	1.00	1.00	0.94	1.28	1.28	0.00	1.22	1.28
time (sec)	N/A	0.020	0.042	0.457	0.273	2.218	0.000	0.418	0.235

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.40
time (sec)	N/A	0.022	0.059	0.438	0.288	2.397	0.000	0.439	0.243

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	32	357	36	42	0	49	50
N.S.	1	1.00	0.78	8.71	0.88	1.02	0.00	1.20	1.22
time (sec)	N/A	0.033	0.231	88.986	0.292	2.235	0.000	0.442	0.588

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	38	44	0	49	95
N.S.	1	1.00	0.98	0.84	0.88	1.02	0.00	1.14	2.21
time (sec)	N/A	0.034	0.115	82.168	0.282	2.098	0.000	0.470	4.337

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	122	542	0	175	0	0	-1
N.S.	1	1.00	0.95	4.23	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.136	17.595	43.714	0.000	0.710	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	56	54	0	67	0	0	85
N.S.	1	1.00	0.81	0.78	0.00	0.97	0.00	0.00	1.23
time (sec)	N/A	0.068	0.127	29.167	0.000	2.612	0.000	0.000	1.338

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	109	284	0	117	0	0	-1
N.S.	1	1.00	1.17	3.05	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.103	10.905	33.082	0.000	0.837	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	0	36	0	0	36
N.S.	1	1.00	1.00	1.35	0.00	1.16	0.00	0.00	1.16
time (sec)	N/A	0.034	0.074	31.083	0.000	1.777	0.000	0.000	0.366

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	68	155	0	56	0	0	-1
N.S.	1	1.00	1.28	2.92	0.00	1.06	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.695	31.768	0.000	0.607	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	121	275	0	0	0	0	-1
N.S.	1	1.00	0.45	1.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	1.265	32.040	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	188	0	0	0	0	-1
N.S.	1	1.00	0.86	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	10.728	31.638	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	157	514	0	0	0	0	-1
N.S.	1	1.00	0.49	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	1.154	34.271	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	57	64	0	85	0	0	110
N.S.	1	1.00	0.55	0.62	0.00	0.82	0.00	0.00	1.06
time (sec)	N/A	0.109	0.307	60.658	0.000	3.386	0.000	0.000	2.175

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	114	996	0	0	0	0	-1
N.S.	1	1.00	0.69	6.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	11.163	59.421	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	58	0	0	61
N.S.	1	1.00	0.65	0.78	0.00	0.84	0.00	0.00	0.88
time (sec)	N/A	0.070	0.150	58.415	0.000	2.267	0.000	0.000	0.759

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	99	498	0	0	0	0	-1
N.S.	1	1.00	0.79	3.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	10.552	58.206	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	0	36	0	0	36
N.S.	1	1.00	1.00	1.35	0.00	1.16	0.00	0.00	1.16
time (sec)	N/A	0.031	0.069	59.816	0.000	2.550	0.000	0.000	0.304

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	66	497	0	0	0	0	-1
N.S.	1	1.00	0.74	5.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	10.414	60.534	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	142	646	0	0	0	0	-1
N.S.	1	1.00	0.43	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	1.426	56.835	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	69	512	0	0	0	0	-1
N.S.	1	1.00	0.73	5.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	10.468	57.904	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	92	563	0	223	0	0	-1
N.S.	1	1.00	0.55	3.39	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.509	55.387	0.000	0.879	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	57	64	0	89	0	0	112
N.S.	1	1.00	0.54	0.60	0.00	0.84	0.00	0.00	1.06
time (sec)	N/A	0.112	0.197	55.843	0.000	3.167	0.000	0.000	2.319

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	87	306	0	158	0	0	-1
N.S.	1	1.00	0.66	2.34	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.705	56.534	0.000	0.616	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	58	0	0	64
N.S.	1	1.00	0.65	0.78	0.00	0.84	0.00	0.00	0.93
time (sec)	N/A	0.071	0.221	55.291	0.000	3.107	0.000	0.000	0.835

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	68	188	0	117	0	0	-1
N.S.	1	1.00	0.73	2.02	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.100	10.646	55.875	0.000	1.010	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	58	0	0	66
N.S.	1	1.00	1.00	1.27	0.00	1.76	0.00	0.00	2.00
time (sec)	N/A	0.034	0.124	58.515	0.000	2.506	0.000	0.000	0.859

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	70	192	0	120	0	0	-1
N.S.	1	1.00	0.71	1.96	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.107	10.582	56.983	0.000	1.089	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	143	544	0	0	0	0	-1
N.S.	1	1.00	0.43	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	1.307	58.862	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	54	0	79	0	0	99
N.S.	1	1.00	0.65	0.78	0.00	1.14	0.00	0.00	1.43
time (sec)	N/A	0.070	0.278	36.536	0.000	3.301	0.000	0.000	1.829

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	104	992	0	0	0	0	-1
N.S.	1	1.00	0.81	7.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	11.134	34.546	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	0	51	0	0	49
N.S.	1	1.00	1.00	1.27	0.00	1.55	0.00	0.00	1.48
time (sec)	N/A	0.033	0.127	34.086	0.000	3.142	0.000	0.000	0.799

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	502	0	0	0	0	-1
N.S.	1	1.00	0.90	5.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.503	34.231	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	123	316	0	0	0	0	-1
N.S.	1	1.00	0.46	1.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.093	0.844	33.557	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	509	0	0	0	0	-1
N.S.	1	1.00	1.25	9.60	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	10.251	32.532	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	156	658	0	0	0	0	-1
N.S.	1	1.00	0.48	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	1.563	29.888	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	84	523	0	0	0	0	-1
N.S.	1	1.00	0.88	5.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	10.399	33.323	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	57	54	0	88	0	0	125
N.S.	1	1.00	0.52	0.49	0.00	0.80	0.00	0.00	1.14
time (sec)	N/A	0.107	0.303	34.361	0.000	4.240	0.000	0.000	3.695

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	119	550	0	181	0	0	-1
N.S.	1	1.00	0.88	4.07	0.00	1.34	0.00	0.00	-0.01
time (sec)	N/A	0.139	11.518	34.140	0.000	0.498	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	42	0	59	0	0	70
N.S.	1	1.00	1.36	1.27	0.00	1.79	0.00	0.00	2.12
time (sec)	N/A	0.037	0.147	34.289	0.000	2.558	0.000	0.000	1.294

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	105	290	0	120	0	0	-1
N.S.	1	1.00	1.07	2.96	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.103	10.885	32.898	0.000	0.824	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	160	975	0	0	0	0	-1
N.S.	1	1.00	0.49	2.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.825	32.564	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	195	0	0	0	0	-1
N.S.	1	1.00	0.91	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	10.695	35.591	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	157	526	0	0	0	0	-1
N.S.	1	1.00	0.49	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.148	1.268	34.421	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	89	220	0	0	0	0	-1
N.S.	1	1.00	0.66	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.570	35.338	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	168	548	0	0	0	0	-1
N.S.	1	1.00	0.45	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	1.633	32.352	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	42	0	67	0	0	93
N.S.	1	1.00	1.36	1.27	0.00	2.03	0.00	0.00	2.82
time (sec)	N/A	0.035	0.165	34.993	0.000	5.008	0.000	0.000	1.762

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	101	993	0	0	0	0	-1
N.S.	1	1.00	0.75	7.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	11.863	33.462	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	154	1259	0	0	0	0	-1
N.S.	1	1.00	0.47	3.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	1.667	37.474	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	80	515	0	0	0	0	-1
N.S.	1	1.00	0.85	5.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.692	32.235	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	152	668	0	0	0	0	-1
N.S.	1	1.00	0.47	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.144	1.549	34.885	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	530	0	0	0	0	-1
N.S.	1	1.00	0.83	5.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.388	35.932	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	165	696	0	0	0	0	-1
N.S.	1	1.00	0.44	1.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	1.470	34.922	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	90	536	0	0	0	0	-1
N.S.	1	1.00	0.67	3.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	10.683	36.207	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	175	722	0	0	0	0	-1
N.S.	1	1.00	0.43	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.271	1.973	35.195	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	278	0	0	0	0	0	-1
N.S.	1	1.00	3.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	2.320	0.427	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	280	0	0	0	0	0	-1
N.S.	1	1.00	3.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.944	0.373	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	281	0	0	0	0	0	-1
N.S.	1	1.00	3.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.756	0.398	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	283	0	0	0	0	0	-1
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.778	0.319	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.068	0.299	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.045	0.259	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.037	0.245	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	66	31	31	0	0	28
N.S.	1	1.00	0.96	2.75	1.29	1.29	0.00	0.00	1.17
time (sec)	N/A	0.022	0.027	2.548	0.290	3.272	0.000	0.000	0.325

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	2446	62	52	0	0	66
N.S.	1	1.00	0.87	47.04	1.19	1.00	0.00	0.00	1.27
time (sec)	N/A	0.037	0.147	4.345	0.295	2.920	0.000	0.000	0.632

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	81	0	92	77	0	0	134
N.S.	1	1.00	1.04	0.00	1.18	0.99	0.00	0.00	1.72
time (sec)	N/A	0.047	0.592	0.507	0.301	3.358	0.000	0.000	1.411

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.653	0.331	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	77	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.557	0.227	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.494	0.195	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.113	0.149	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	165	0	0	0	0	0	-1
N.S.	1	1.00	2.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.497	0.364	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	246	0	0	0	0	0	-1
N.S.	1	1.00	3.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.629	0.533	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	92	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	11.927	0.447	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	90	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	11.714	0.456	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	326	0	0	0	0	0	-1
N.S.	1	1.00	4.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	32.711	0.434	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	21.134	0.328	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [230] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	4	2	1.00	8	0.250
8	A	2	1	1.00	8	0.125
9	A	4	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	3	3	1.00	10	0.300
12	A	2	2	1.00	10	0.200
13	A	2	2	1.00	10	0.200
14	A	3	3	1.00	10	0.300
15	A	3	3	1.00	10	0.300
16	A	4	3	1.00	10	0.300
17	A	4	3	1.00	12	0.250
18	A	3	3	1.00	12	0.250
19	A	3	3	1.00	12	0.250
20	A	2	2	1.00	12	0.167
21	A	2	2	1.00	12	0.167
22	A	3	3	1.00	12	0.250
23	A	3	3	1.00	12	0.250
24	A	4	3	1.00	12	0.250
25	A	2	2	1.00	10	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	2	2	1.00	10	0.200
27	A	2	2	1.00	10	0.200
28	A	2	2	1.00	10	0.200
29	A	2	2	1.00	10	0.200
30	A	2	2	1.00	10	0.200
31	A	2	2	1.00	12	0.167
32	A	2	2	1.00	12	0.167
33	A	2	2	1.00	12	0.167
34	A	2	2	1.00	12	0.167
35	A	2	2	1.00	12	0.167
36	A	2	2	1.00	12	0.167
37	A	2	2	1.00	8	0.250
38	A	2	2	1.00	10	0.200
39	A	5	3	1.00	8	0.375
40	A	4	3	1.00	8	0.375
41	A	3	3	1.00	8	0.375
42	A	2	2	1.00	8	0.250
43	A	2	2	1.00	8	0.250
44	A	3	3	1.00	8	0.375
45	A	4	3	1.00	8	0.375
46	A	5	3	1.00	8	0.375
47	A	6	4	1.00	10	0.400
48	A	5	4	1.00	10	0.400
49	A	4	4	1.00	10	0.400
50	A	3	3	1.00	10	0.300
51	A	2	2	1.00	10	0.200
52	A	3	3	1.00	10	0.300
53	A	4	3	1.00	10	0.300
54	A	5	3	1.00	10	0.300
55	A	7	4	1.00	10	0.400
56	A	5	4	1.00	10	0.400
57	A	4	4	1.00	10	0.400
58	A	4	4	1.00	10	0.400
59	A	5	4	1.00	10	0.400
60	A	7	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	10	0.200
62	A	3	2	1.00	10	0.200
63	A	3	2	1.00	10	0.200
64	A	3	3	1.00	10	0.300
65	A	3	3	1.00	10	0.300
66	A	5	3	1.00	10	0.300
67	A	7	3	1.00	10	0.300
68	A	3	3	1.00	12	0.250
69	A	3	3	1.00	14	0.214
70	A	5	4	1.00	21	0.190
71	A	5	4	1.00	21	0.190
72	A	4	4	1.00	21	0.190
73	A	4	4	1.00	19	0.210
74	A	2	2	1.00	12	0.167
75	A	3	3	1.00	19	0.158
76	A	4	4	1.00	21	0.190
77	A	4	4	1.00	21	0.190
78	A	5	4	1.00	21	0.190
79	A	5	4	1.00	21	0.190
80	A	5	4	1.00	21	0.190
81	A	5	4	1.00	21	0.190
82	A	4	4	1.00	19	0.210
83	A	3	3	1.00	12	0.250
84	A	3	3	1.00	19	0.158
85	A	3	3	1.00	21	0.143
86	A	4	4	1.00	21	0.190
87	A	4	4	1.00	21	0.190
88	A	5	4	1.00	21	0.190
89	A	5	4	1.00	21	0.190
90	A	5	4	1.00	21	0.190
91	A	5	4	1.00	19	0.210
92	A	3	3	1.00	12	0.250
93	A	4	4	1.00	19	0.210
94	A	3	3	1.00	21	0.143
95	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	21	0.190
97	A	4	4	1.00	21	0.190
98	A	5	4	1.00	21	0.190
99	A	5	4	1.00	21	0.190
100	A	4	3	1.00	12	0.250
101	A	5	4	1.00	21	0.190
102	A	5	4	1.00	21	0.190
103	A	4	4	1.00	21	0.190
104	A	4	4	1.00	21	0.190
105	A	3	3	1.00	19	0.158
106	A	2	2	1.00	12	0.167
107	A	4	4	1.00	19	0.210
108	A	4	4	1.00	21	0.190
109	A	5	4	1.00	21	0.190
110	A	5	4	1.00	21	0.190
111	A	5	4	1.00	21	0.190
112	A	5	4	1.00	21	0.190
113	A	4	4	1.00	21	0.190
114	A	4	4	1.00	21	0.190
115	A	3	3	1.00	21	0.143
116	A	3	3	1.00	19	0.158
117	A	3	3	1.00	12	0.250
118	A	4	4	1.00	19	0.210
119	A	5	4	1.00	21	0.190
120	A	5	4	1.00	21	0.190
121	A	5	4	1.00	21	0.190
122	A	5	4	1.00	21	0.190
123	A	4	4	1.00	21	0.190
124	A	4	4	1.00	21	0.190
125	A	3	3	1.00	21	0.143
126	A	3	3	1.00	21	0.143
127	A	4	4	1.00	19	0.210
128	A	3	3	1.00	12	0.250
129	A	5	4	1.00	19	0.210
130	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	12	0.250
132	A	4	3	1.00	23	0.130
133	A	3	2	1.00	23	0.087
134	A	3	3	1.00	23	0.130
135	A	3	3	1.00	23	0.130
136	A	2	2	1.00	23	0.087
137	A	2	2	1.00	23	0.087
138	A	2	2	1.00	23	0.087
139	A	3	3	1.00	23	0.130
140	A	3	2	1.00	23	0.087
141	A	4	3	1.00	23	0.130
142	A	4	3	1.00	23	0.130
143	A	3	2	1.00	23	0.087
144	A	3	3	1.00	23	0.130
145	A	3	3	1.00	23	0.130
146	A	2	2	1.00	23	0.087
147	A	2	2	1.00	23	0.087
148	A	2	2	1.00	23	0.087
149	A	3	3	1.00	23	0.130
150	A	3	2	1.00	23	0.087
151	A	4	3	1.00	23	0.130
152	A	3	2	1.00	23	0.087
153	A	3	2	1.00	23	0.087
154	A	3	3	1.00	23	0.130
155	A	3	3	1.00	23	0.130
156	A	2	2	1.00	23	0.087
157	A	2	2	1.00	23	0.087
158	A	2	2	1.00	23	0.087
159	A	3	3	1.00	23	0.130
160	A	3	2	1.00	23	0.087
161	A	3	3	1.00	23	0.130
162	A	3	3	1.00	23	0.130
163	A	2	2	1.00	23	0.087
164	A	2	2	1.00	23	0.087
165	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	3	1.00	23	0.130
167	A	3	2	1.00	23	0.087
168	A	3	3	1.00	23	0.130
169	A	3	3	1.00	23	0.130
170	A	2	2	1.00	23	0.087
171	A	2	2	1.00	23	0.087
172	A	2	2	1.00	23	0.087
173	A	3	3	1.00	23	0.130
174	A	3	2	1.00	23	0.087
175	A	4	3	1.00	23	0.130
176	A	3	3	1.00	23	0.130
177	A	3	3	1.00	23	0.130
178	A	2	2	1.00	23	0.087
179	A	2	2	1.00	23	0.087
180	A	2	2	1.00	23	0.087
181	A	3	3	1.00	23	0.130
182	A	3	2	1.00	23	0.087
183	A	4	3	1.00	23	0.130
184	A	3	3	1.00	21	0.143
185	A	3	3	1.00	19	0.158
186	A	2	2	1.00	12	0.167
187	A	3	3	1.00	19	0.158
188	A	3	3	1.00	21	0.143
189	A	3	3	1.00	21	0.143
190	A	3	3	1.00	19	0.158
191	A	2	2	1.00	12	0.167
192	A	3	3	1.00	19	0.158
193	A	3	3	1.00	21	0.143
194	A	3	3	1.00	21	0.143
195	A	3	3	1.00	19	0.158
196	A	2	2	1.00	12	0.167
197	A	3	3	1.00	19	0.158
198	A	3	3	1.00	21	0.143
199	A	3	3	1.00	21	0.143
200	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	2	2	1.00	12	0.167
202	A	3	3	1.00	19	0.158
203	A	3	3	1.00	21	0.143
204	A	3	3	1.00	21	0.143
205	A	3	3	1.00	21	0.143
206	A	3	3	1.00	21	0.143
207	A	3	3	1.00	21	0.143
208	A	3	3	1.00	21	0.143
209	A	3	3	1.00	21	0.143
210	A	3	3	1.00	19	0.158
211	A	3	3	1.00	19	0.158
212	A	3	3	1.00	17	0.176
213	A	2	2	1.00	10	0.200
214	A	3	3	1.00	17	0.176
215	A	3	3	1.00	19	0.158
216	A	3	3	1.00	19	0.158
217	A	3	3	1.00	21	0.143
218	A	3	3	1.00	21	0.143
219	A	3	3	1.00	21	0.143
220	A	3	3	1.00	21	0.143
221	A	3	3	1.00	21	0.143
222	A	3	3	1.00	21	0.143
223	A	2	2	1.00	19	0.105
224	A	2	2	1.00	19	0.105
225	A	2	2	1.00	19	0.105
226	A	2	2	1.00	19	0.105
227	A	2	2	1.00	19	0.105
228	A	3	2	1.00	21	0.095
229	A	3	2	1.00	21	0.095
230	A	5	4	1.00	25	0.160
231	A	2	2	1.00	25	0.080
232	A	4	4	1.00	25	0.160
233	A	1	1	1.00	25	0.040
234	A	3	3	1.00	25	0.120
235	A	12	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	4	1.00	25	0.160
237	A	13	10	1.00	25	0.400
238	A	3	2	1.00	25	0.080
239	A	6	5	1.00	25	0.200
240	A	2	2	1.00	25	0.080
241	A	5	5	1.00	25	0.200
242	A	1	1	1.00	25	0.040
243	A	4	4	1.00	25	0.160
244	A	13	10	1.00	25	0.400
245	A	4	4	1.00	25	0.160
246	A	6	5	1.00	25	0.200
247	A	3	3	1.00	25	0.120
248	A	5	5	1.00	25	0.200
249	A	2	2	1.00	25	0.080
250	A	4	4	1.00	25	0.160
251	A	1	1	1.00	25	0.040
252	A	4	4	1.00	25	0.160
253	A	13	10	1.00	25	0.400
254	A	2	2	1.00	25	0.080
255	A	5	4	1.00	25	0.160
256	A	1	1	1.00	25	0.040
257	A	4	4	1.00	25	0.160
258	A	12	9	1.00	25	0.360
259	A	3	3	1.00	25	0.120
260	A	13	10	1.00	25	0.400
261	A	4	4	1.00	25	0.160
262	A	3	3	1.00	25	0.120
263	A	5	5	1.00	25	0.200
264	A	1	1	1.00	25	0.040
265	A	4	4	1.00	25	0.160
266	A	13	10	1.00	25	0.400
267	A	4	4	1.00	25	0.160
268	A	13	10	1.00	25	0.400
269	A	5	5	1.00	25	0.200
270	A	14	11	1.00	25	0.440

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	1	1	1.00	25	0.040
272	A	5	5	1.00	25	0.200
273	A	13	10	1.00	25	0.400
274	A	4	4	1.00	25	0.160
275	A	13	10	1.00	25	0.400
276	A	4	4	1.00	25	0.160
277	A	14	11	1.00	25	0.440
278	A	5	5	1.00	25	0.200
279	A	15	11	1.00	25	0.440
280	A	2	2	1.00	17	0.118
281	A	2	2	1.00	19	0.105
282	A	2	2	1.00	19	0.105
283	A	2	2	1.00	21	0.095
284	A	2	2	1.00	19	0.105
285	A	2	2	1.00	19	0.105
286	A	2	2	1.00	17	0.118
287	A	2	2	1.00	17	0.118
288	A	3	2	1.00	19	0.105
289	A	3	2	1.00	19	0.105
290	A	2	2	1.00	19	0.105
291	A	2	2	1.00	19	0.105
292	A	2	2	1.00	19	0.105
293	A	2	2	1.00	10	0.200
294	A	2	2	1.00	19	0.105
295	A	2	2	1.00	19	0.105
296	A	2	2	1.00	23	0.087
297	A	2	2	1.00	23	0.087
298	A	2	2	1.00	23	0.087
299	A	2	2	1.00	23	0.087

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] arctanh(sin(b*x+a))/b
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3855}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a+bx)\,dx = \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Maple [A]
time = 0.04, size = 19, normalized size = 1.73

method	result	size
derivativedivides	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
default	$\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$	19
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	35
risch	$-\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [A]
time = 0.30, size = 18, normalized size = 1.64

$$\frac{\log\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sec(b*x + a) + tan(b*x + a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.
time = 2.38, size = 28, normalized size = 2.55

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(8) = 16.
time = 0.99, size = 36, normalized size = 3.27

$$\begin{cases} \frac{\log\left(\tan\left(a+bx\right)+\sec\left(a+bx\right)\right)}{b} & \text{for}\: b \neq 0 \\ \frac{x\left(\tan\left(a\right)\sec\left(a\right)+\sec^{2}\left(a\right)\right)}{\tan\left(a\right)+\sec\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a + b*x) + sec(a + b*x))/b, Ne(b, 0)), (x*(tan(a)*sec(a)
 + sec(a)**2)/(tan(a) + sec(a)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(11) = 22.
time = 0.44, size = 44, normalized size = 4.00

$$\frac{\log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) - 2\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) - log(abs(1/sin(b*x + a) +
 sin(b*x + a) - 2)))/b
```

Mupad [B]
time = 0.40, size = 11, normalized size = 1.00

$$\frac{\mathrm{atanh}(\sin\left(a + b\,x\right))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x),x)
```

```
[Out] atanh(sin(a + b*x))/b
```

3.2 $\int \sec^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tan(a+bx)}{b}$$

[Out] tan(b*x+a)/b

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3852, 8}

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]^2,x]

[Out] Tan[a + b*x]/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 1.00

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Maple [A]
time = 0.19, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\tan(bx+a)}{b}$	11
default	$\frac{\tan(bx+a)}{b}$	11
risch	$\frac{2i}{b\left(e^{2i(bx+a)}+1\right)}$	20
norman	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] tan(b*x+a)/b
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] tan(b*x + a)/b
```

Fricas [A]
time = 2.03, size = 18, normalized size = 1.80

$$\frac{\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**2, x)
```

Giac [A]
time = 0.44, size = 10, normalized size = 1.00

$$\frac{\tan\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

Mupad [B]
time = 0.10, size = 10, normalized size = 1.00

$$\frac{\tan(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^2,x)
```

```
[Out] tan(a + b*x)/b
```

3.3 $\int \sec^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3853, 3855}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\,dx&=\frac{\sec(a+bx)\tan(a+bx)}{2b}+\frac{1}{2}\int \sec(a+bx)\,dx\\&=\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [A]
time = 0.20, size = 36, normalized size = 1.06

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)\tan(bx+a)}{2}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	36
default	$\frac{\frac{\sec(bx+a)\tan(bx+a)}{2}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	36
risch	$-\frac{i\left(e^{3i(bx+a)}-e^{i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^{2}}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{2b}$	78
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{2}}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2b}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*sec(b*x+a)*tan(b*x+a)+1/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [A]
time = 0.29, size = 46, normalized size = 1.35

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^{2}-1}-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(\sin\left(bx+a\right)-1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin
(b*x + a) - 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.
time = 2.06, size = 61, normalized size = 1.79

$$\frac{\cos\left(bx+a\right)^{2}\log\left(\sin\left(bx+a\right)+1\right)-\cos\left(bx+a\right)^{2}\log\left(-\sin\left(bx+a\right)+1\right)+2\,\sin\left(bx+a\right)}{4\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x + a
) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**3, x)
```

Giac [A]
time = 0.47, size = 48, normalized size = 1.41

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} - \log\left(|\sin(bx+a)+1|\right) + \log\left(|\sin(bx+a)-1|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(abs(sin(b*x + a) + 1)) + lo
g(abs(sin(b*x + a) - 1)))/b
```

Mupad [B]
time = 0.11, size = 36, normalized size = 1.06

$$\frac{\mathrm{atanh}(\sin(a+b\,x))}{2\,b} - \frac{\sin(a+b\,x)}{2\,b\,\left(\sin(a+b\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^3,x)
```

```
[Out] atanh(sin(a + b*x))/(2*b) - sin(a + b*x)/(2*b*(sin(a + b*x)^2 - 1))
```

3.4 $\int \sec^4(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {3852}

$$\frac{\tan^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 23, normalized size = 0.88

$$\frac{\tan(a+bx)+\frac{1}{3}\tan^3(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4,x]
```

```
[Out] (Tan[a + b*x] + Tan[a + b*x]^3/3)/b
```

Maple [A]
time = 0.15, size = 24, normalized size = 0.92

method	result	size
derivativedivides	$-\frac{\left(-\frac{2}{3}-\frac{\left(\sec^{2}(bx+a)\right)}{3}\right)\tan(bx+a)}{b}$	24
default	$-\frac{\left(-\frac{2}{3}-\frac{\left(\sec^{2}(bx+a)\right)}{3}\right)\tan(bx+a)}{b}$	24
risch	$\frac{4i\left(3\,\mathrm{e}^{2i(bx+a)}+1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{3}}$	33
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}-\frac{2\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}}{\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{3}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/b*(-2/3-1/3*sec(b*x+a)^2)*tan(b*x+a)
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.85

$$\frac{\tan\left(bx+a\right)^{3}+3\,\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4,x, algorithm="maxima")
[Out] 1/3*(tan(b*x + a)^3 + 3*tan(b*x + a))/b
```

Fricas [A]
time = 2.47, size = 31, normalized size = 1.19

$$\frac{\left(2\,\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4,x, algorithm="fricas")
[Out] 1/3*(2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sec^{4}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**4, x)
```

Giac [A]
time = 0.44, size = 22, normalized size = 0.85

$$\frac{\tan{(bx+a)}^3 + 3\,\tan{(bx+a)}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(tan(b*x + a)^3 + 3*tan(b*x + a))/b
```

Mupad [B]
time = 0.08, size = 21, normalized size = 0.81

$$\frac{\tan(a+b\,x)\,\left(\tan(a+b\,x)^2+3\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^4,x)
```

```
[Out] (tan(a + b*x)*(tan(a + b*x)^2 + 3))/(3*b)
```

3.5 $\int \sec^5(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

```
[Out] 3/8*arctanh(sin(b*x+a))/b+3/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)^3*tan(
b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3855}

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}+\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]^3*Tan[a + b*x])/(4*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\,dx &= \frac{\sec^3(a+bx)\tan(a+bx)}{4b}+\frac{3}{4}\int \sec^3(a+bx)\,dx\\ &= \frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}+\frac{3}{8}\int \sec(a+bx)\,dx\\ &= \frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.76

$$\frac{3\tanh^{-1}(\sin(a+bx))+\sec(a+bx)\left(3+2\sec^2(a+bx)\right)\tan(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]] + Sec[a + b*x]*(3 + 2*Sec[a + b*x]^2)*Tan[a + b*x]
)/(8*b)
```

Maple [A]
time = 0.25, size = 49, normalized size = 0.89

method	result
derivativedivides	$\frac{-\left(-\frac{\left(\sec^3(bx+a)\right)}{4}-\frac{3\sec(bx+a)}{8}\right)\tan(bx+a)+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{-\left(-\frac{\left(\sec^3(bx+a)\right)}{4}-\frac{3\sec(bx+a)}{8}\right)\tan(bx+a)+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$-\frac{i\left(3\,e^{7i(bx+a)}+11\,e^{5i(bx+a)}-11\,e^{3i(bx+a)}-3\,e^{i(bx+a)}\right)}{4b\left(e^{2i(bx+a)}+1\right)^4}-\frac{3\ln\left(e^{i(bx+a)}-i\right)}{8b}+\frac{3\ln\left(e^{i(bx+a)}+i\right)}{8b}$
norman	$\frac{\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{3\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{8b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-(-1/4*sec(b*x+a)^3-3/8*sec(b*x+a))*tan(b*x+a)+3/8*ln(sec(b*x+a)+tan(b
*x+a)))
```

Maxima [A]
time = 0.30, size = 71, normalized size = 1.29

$$-\frac{\frac{2\left(3\,\sin(bx+a)^3-5\,\sin(bx+a)\right)}{\sin(bx+a)^4-2\,\sin(bx+a)^2+1}-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(\sin\left(bx+a\right)-1\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x +
a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Fricas [A]
time = 2.60, size = 74, normalized size = 1.35

$$\frac{3\cos\left(bx+a\right)^{4}\log\left(\sin\left(bx+a\right)+1\right)-3\cos\left(bx+a\right)^{4}\log\left(-\sin\left(bx+a\right)+1\right)+2\left(3\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{16\,b\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b*
x + a) + 1) + 2*(3*cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{5}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**5, x)
```

Giac [A]
time = 0.41, size = 63, normalized size = 1.15

$$-\frac{\frac{2\left(3\sin\left(bx+a\right)^{3}-5\sin\left(bx+a\right)\right)}{\left(\sin\left(bx+a\right)^{2}-1\right)^{2}}-3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/16*(2*(3*sin(b*x + a)^3 - 5*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 - 3*log
(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B]
time = 0.12, size = 58, normalized size = 1.05

$$\frac{3\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{8\,b}+\frac{\frac{5\,\sin(a+b\,x)}{8}-\frac{3\,\sin(a+b\,x)^{3}}{8}}{b\,\left(\sin\left(a+b\,x\right)^{4}-2\,\sin\left(a+b\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^5,x)
```

```
[Out] (3*atanh(sin(a + b*x)))/(8*b) + ((5*sin(a + b*x))/8 - (3*sin(a + b*x)^3)/8)
/(b*(sin(a + b*x)^4 - 2*sin(a + b*x)^2 + 1))
```

3.6 $\int \sec^6(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\tan(a+bx)}{b}+\frac{2\tan^3(a+bx)}{3b}+\frac{\tan^5(a+bx)}{5b}$$

```
[Out] tan(b*x+a)/b+2/3*tan(b*x+a)^3/b+1/5*tan(b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3852}

$$\frac{\tan^5(a+bx)}{5b}+\frac{2\tan^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6,x]
```

```
[Out] Tan[a + b*x]/b + (2*Tan[a + b*x]^3)/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}+\frac{2\tan^3(a+bx)}{3b}+\frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 35, normalized size = 0.85

$$\frac{\tan(a+bx)+\frac{2}{3}\tan^3(a+bx)+\frac{1}{5}\tan^5(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6,x]
```

[Out] (Tan[a + b*x] + (2*Tan[a + b*x]^3)/3 + Tan[a + b*x]^5/5)/b

Maple [A]
time = 0.15, size = 34, normalized size = 0.83

method	result	size
derivativedivides	$-\frac{\left(-\frac{8}{15}-\frac{\left(\sec^{4}\left(b x+a\right)\right)}{5}-\frac{4\left(\sec^{2}\left(b x+a\right)\right)}{15}\right)\tan\left(b x+a\right)}{b}$	34
default	$-\frac{\left(-\frac{8}{15}-\frac{\left(\sec^{4}\left(b x+a\right)\right)}{5}-\frac{4\left(\sec^{2}\left(b x+a\right)\right)}{15}\right)\tan\left(b x+a\right)}{b}$	34
risch	$\frac{16 i \left(10\,{\mathrm e}^{4 i \left(b x+a\right)}+5\,{\mathrm e}^{2 i \left(b x+a\right)}+1\right)}{15 b \left({\mathrm e}^{2 i \left(b x+a\right)}+1\right)^{5}}$	44
norman	$\frac{-\frac{2\tan\left(\frac{b x}{2}+\frac{a}{2}\right)}{b}+\frac{8\left(\tan^{3}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{3 b}-\frac{116\left(\tan^{5}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{15 b}+\frac{8\left(\tan^{7}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{3 b}-\frac{2\left(\tan^{9}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{b}}{\left(\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)^{5}}$	96

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^6,x,method=_RETURNVERBOSE)

[Out] -1/b*(-8/15-1/5*sec(b*x+a)^4-4/15*sec(b*x+a)^2)*tan(b*x+a)

Maxima [A]
time = 0.29, size = 34, normalized size = 0.83

$$\frac{3\,\tan\left(bx+a\right)^{5}+10\,\tan\left(bx+a\right)^{3}+15\,\tan\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^6,x, algorithm="maxima")

[Out] 1/15*(3*tan(b*x + a)^5 + 10*tan(b*x + a)^3 + 15*tan(b*x + a))/b

Fricas [A]
time = 3.01, size = 41, normalized size = 1.00

$$\frac{\left(8\,\cos\left(bx+a\right)^{4}+4\,\cos\left(bx+a\right)^{2}+3\right)\sin\left(bx+a\right)}{15\,b\cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^6,x, algorithm="fricas")

[Out] 1/15*(8*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 3)*sin(b*x + a)/(b*cos(b*x + a)
^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^6(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6,x)
```

```
[Out] Integral(sec(a + b*x)**6, x)
```

Giac [A]
time = 0.43, size = 34, normalized size = 0.83

$$\frac{3\,\tan\left(bx+a\right)^{5}+10\,\tan\left(bx+a\right)^{3}+15\,\tan\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 10*tan(b*x + a)^3 + 15*tan(b*x + a))/b
```

Mupad [B]
time = 0.09, size = 31, normalized size = 0.76

$$\frac{\frac{\tan(a+b\,x)^5}{5}+\frac{2\tan(a+b\,x)^3}{3}+\tan(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^6,x)
```

```
[Out] (tan(a + b*x) + (2*tan(a + b*x)^3)/3 + tan(a + b*x)^5/5)/b
```

3.7 $\int \sec^7(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{5\sec(a+bx)\tan(a+bx)}{16b}+\frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

```
[Out] 5/16*arctanh(sin(b*x+a))/b+5/16*sec(b*x+a)*tan(b*x+a)/b+5/24*sec(b*x+a)^3*t
an(b*x+a)/b+1/6*sec(b*x+a)^5*tan(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3855}

$$\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}+\frac{5\tan(a+bx)\sec^3(a+bx)}{24b}+\frac{5\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^7,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) + (5*Sec[a + b*x]*Tan[a + b*x])/(16*b) + (
5*Sec[a + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^7(a+bx)\,dx &= \frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\frac{5}{6}\int \sec^5(a+bx)\,dx\\ &= \frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\frac{5}{8}\int \sec^3(a+bx)\,dx\\ &= \frac{5\sec(a+bx)\tan(a+bx)}{16b}+\frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\\ &= \frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{5\sec(a+bx)\tan(a+bx)}{16b}+\frac{5\sec^3(a+bx)\tan(a+bx)}{24b}+\end{aligned}$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 0.68

$$\frac{15\tanh^{-1}(\sin(a+bx))+\sec(a+bx)\left(15+10\sec^2(a+bx)+8\sec^4(a+bx)\right)\tan(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^7,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]] + Sec[a + b*x]*(15 + 10*Sec[a + b*x]^2 + 8*Sec[a
+ b*x]^4)*Tan[a + b*x])/(48*b)
```

Maple [A]
time = 0.24, size = 59, normalized size = 0.78

method	result
derivativedivides	$\frac{-\left(-\frac{\left(\sec^5(bx+a)\right)}{6}-\frac{5\left(\sec^3(bx+a)\right)}{24}-\frac{5\sec(bx+a)}{16}\right)\tan(bx+a)+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
default	$\frac{-\left(-\frac{\left(\sec^5(bx+a)\right)}{6}-\frac{5\left(\sec^3(bx+a)\right)}{24}-\frac{5\sec(bx+a)}{16}\right)\tan(bx+a)+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
risch	$-\frac{i\left(15\,e^{11i(bx+a)}+85\,e^{9i(bx+a)}+198\,e^{7i(bx+a)}-198\,e^{5i(bx+a)}-85\,e^{3i(bx+a)}-15\,e^{i(bx+a)}\right)}{24b\left(e^{2i(bx+a)}+1\right)^6}+\frac{5\ln\left(e^{i(bx+a)}+i\right)}{16b}-\frac{5\ln(e}{}$
norman	$\frac{\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{5\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{15\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{15\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{11\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}}{\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^6}-\frac{5\ln(\mathrm{ta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-(-1/6*sec(b*x+a)^5-5/24*sec(b*x+a)^3-5/16*sec(b*x+a))*tan(b*x+a)+5/16
*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.20

$$-\frac{\frac{2\left(15\sin(bx+a)^5-40\sin(bx+a)^3+33\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1}-15\log\left(\sin\left(bx+a\right)+1\right)+15\log\left(\sin\left(bx+a\right)-1\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*x
 + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 15*log(sin(b*x + a) +
1) + 15*log(sin(b*x + a) - 1))/b
```

Fricas [A]
time = 4.07, size = 84, normalized size = 1.11

$$\frac{15\cos\left(bx+a\right)^{6}\log\left(\sin\left(bx+a\right)+1\right)-15\cos\left(bx+a\right)^{6}\log\left(-\sin\left(bx+a\right)+1\right)+2\left(15\cos\left(bx+a\right)^{4}+10\cos\left(bx+a\right)^{2}+8\right)\sin\left(bx+a\right)}{96\,b\cos\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(15*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^6*log(-sin(
b*x + a) + 1) + 2*(15*cos(b*x + a)^4 + 10*cos(b*x + a)^2 + 8)*sin(b*x + a))
/(b*cos(b*x + a)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sec^{7}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7,x)
```

```
[Out] Integral(sec(a + b*x)**7, x)
```

Giac [A]
time = 0.43, size = 73, normalized size = 0.96

$$-\frac{\frac{2\left(15\sin(bx+a)^{5}-40\sin(bx+a)^{3}+33\sin(bx+a)\right)}{\left(\sin(bx+a)^{2}-1\right)^{3}}-15\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+15\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/96*(2*(15*sin(b*x + a)^5 - 40*sin(b*x + a)^3 + 33*sin(b*x + a))/(sin(b*x
 + a)^2 - 1)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) -
1)))/b
```

Mupad [B]
time = 0.16, size = 79, normalized size = 1.04

$$\frac{5\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b}-\frac{\frac{5\sin(a+b\,x)^{5}}{16}-\frac{5\sin(a+b\,x)^{3}}{6}+\frac{11\,\sin(a+b\,x)}{16}}{b\,\left(\sin\left(a+b\,x\right)^{6}-3\sin\left(a+b\,x\right)^{4}+3\sin\left(a+b\,x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^7,x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(16*b) - ((11*sin(a + b*x))/16 - (5*sin(a + b*x)^3)
/6 + (5*sin(a + b*x)^5)/16)/(b*(3*sin(a + b*x)^2 - 3*sin(a + b*x)^4 + sin(a
 + b*x)^6 - 1))
```

3.8 $\int \sec^8(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}$$

```
[Out] tan(b*x+a)/b+tan(b*x+a)^3/b+3/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3852}

$$\frac{\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{b}+\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^8,x]
```

```
[Out] Tan[a + b*x]/b + Tan[a + b*x]^3/b + (3*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]
^7/(7*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(a+bx)\,dx&=-\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx,x,-\tan(a+bx)\right)}{b}\\&=\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 43, normalized size = 0.81

$$\frac{\tan(a+bx)+\tan^3(a+bx)+\frac{3}{5}\tan^5(a+bx)+\frac{1}{7}\tan^7(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^8,x]
```

```
[Out] (Tan[a + b*x] + Tan[a + b*x]^3 + (3*Tan[a + b*x]^5)/5 + Tan[a + b*x]^7/7)/b
```

Maple [A]
time = 0.22, size = 44, normalized size = 0.83

method	result
derivativedivides	$-\frac{\left(-\frac{16}{35}-\frac{\left(\sec^{6}\left(bx+a\right)\right)}{7}-\frac{6\left(\sec^{4}\left(bx+a\right)\right)}{35}-\frac{8\left(\sec^{2}\left(bx+a\right)\right)}{35}\right)\tan\left(bx+a\right)}{b}$
default	$-\frac{\left(-\frac{16}{35}-\frac{\left(\sec^{6}\left(bx+a\right)\right)}{7}-\frac{6\left(\sec^{4}\left(bx+a\right)\right)}{35}-\frac{8\left(\sec^{2}\left(bx+a\right)\right)}{35}\right)\tan\left(bx+a\right)}{b}$
risch	$\frac{32i\left(35\,{\mathrm e}^{6i\left(bx+a\right)}+21\,{\mathrm e}^{4i\left(bx+a\right)}+7\,{\mathrm e}^{2i\left(bx+a\right)}+1\right)}{35b\left({\mathrm e}^{2i\left(bx+a\right)}+1\right)^{7}}$
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{86\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}+\frac{424\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{35b}-\frac{86\left(\tan^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}+\frac{4\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-2(\tan}{\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(-16/35-1/7*sec(b*x+a)^6-6/35*sec(b*x+a)^4-8/35*sec(b*x+a)^2)*tan(b*x+
a)
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.83

$$\frac{5\,\tan\left(bx+a\right)^{7}+21\,\tan\left(bx+a\right)^{5}+35\,\tan\left(bx+a\right)^{3}+35\,\tan\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*x
 + a))/b
```

Fricas [A]
time = 4.60, size = 51, normalized size = 0.96

$$\frac{\left(16\,\cos\left(bx+a\right)^{6}+8\,\cos\left(bx+a\right)^{4}+6\,\cos\left(bx+a\right)^{2}+5\right)\sin\left(bx+a\right)}{35\,b\cos\left(bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/35*(16*cos(b*x + a)^6 + 8*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 5)*sin(b*x
+ a)/(b*cos(b*x + a)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^8\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8,x)
```

```
[Out] Integral(sec(a + b*x)**8, x)
```

Giac [A]
time = 0.42, size = 44, normalized size = 0.83

$$\frac{5\,\tan\left(bx+a\right)^{7}+21\,\tan\left(bx+a\right)^{5}+35\,\tan\left(bx+a\right)^{3}+35\,\tan\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 21*tan(b*x + a)^5 + 35*tan(b*x + a)^3 + 35*tan(b*x
 + a))/b
```

Mupad [B]
time = 0.08, size = 39, normalized size = 0.74

$$\frac{\frac{\tan(a+b\,x)^7}{7}+\frac{3\tan(a+b\,x)^5}{5}+\tan(a+b\,x)^3+\tan(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^8,x)
```

```
[Out] (tan(a + b*x) + tan(a + b*x)^3 + (3*tan(a + b*x)^5)/5 + tan(a + b*x)^7/7)/b
```

3.9 $\int \sec^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{6\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{5b}+\frac{6\sqrt{\sec(a+bx)}\,\sin(a+bx)}{5b}+\frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b}$$

```
[Out] 2/5*sec(b*x+a)^(5/2)*sin(b*x+a)/b+6/5*sin(b*x+a)*sec(b*x+a)^(1/2)/b-6/5*(co
s(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2
^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2719}

$$\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}+\frac{6\sin(a+bx)\sqrt{\sec(a+bx)}}{5b}-\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(7/2),x]
```

```
[Out] (-6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b)
+ (6*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*Sec[a + b*x]^(5/2)*Sin[a +
 b*x])/(5*b)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{7}{2}}(a+bx)\,dx &= \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b} + \frac{3}{5}\int \sec^{\frac{3}{2}}(a+bx)\,dx \\ &= \frac{6\sqrt{\sec(a+bx)}\,\sin(a+bx)}{5b} + \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b} - \frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx \\ &= \frac{6\sqrt{\sec(a+bx)}\,\sin(a+bx)}{5b} + \frac{2\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{5b} - \frac{1}{5}\Big(3\sqrt{\cos(a+bx)}\,\sqrt{\sec(a \\ &= -\frac{6\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{5b} + \frac{6\sqrt{\sec(a+bx)}\,\sin(a+bx)}{5b} + \frac{2\,\mathrm{se}}{}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 59, normalized size = 0.69

$$\frac{\sec^{\frac{5}{2}}(a+bx)\left(-12\cos^{\frac{5}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|\,2\right)+7\sin(a+bx)+3\sin(3(a+bx))\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(7/2),x]
```

```
[Out] (Sec[a + b*x]^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2] + 7*S
in[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(97) = 194$.
time = 4.84, size = 358, normalized size = 4.21

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(24\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*b
*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1
/2*a)^3*(24*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*EllipticE(cos(1/2*b*
x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(
1/2)*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+12*Ell
ipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/
```

```
2*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+8*sin(1/2*b*x+1/2*a)^2*cos(1/2*b
*x+1/2*a)-3*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1
)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2))*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+
1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.89, size = 110, normalized size = 1.29

$$\frac{-3i\sqrt{2}\cos(bx+a)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+3i\sqrt{2}\cos(bx+a)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+\frac{2\left(3\cos(bx+a)^2+1\right)\sin(bx+a)}{\sqrt{\cos(bx+a)}}}{5\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*cos(b*x + a)^2*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*cos(b*x + a)^2*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))
 + 2*(3*cos(b*x + a)^2 + 1)*sin(b*x + a)/sqrt(cos(b*x + a)))/(b*cos(b*x + a
)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos\left(a+b\,x\right)}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(7/2),x)
[Out] int((1/cos(a + b*x))^(7/2), x)
```

3.10 $\int \sec^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b}+\frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b}$$

```
[Out] 2/3*sec(b*x+a)^(3/2)*sin(b*x+a)/b+2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*
a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a
)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2720}

$$\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) +
 (2*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(3*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(a+bx)\,dx = \frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b} + \frac{1}{3}\int \sqrt{\sec(a+bx)}\,dx$$
$$= \frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b} + \frac{1}{3}\left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$
$$= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b} + \frac{2\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{3b}$$

Mathematica [A]
time = 0.07, size = 46, normalized size = 0.74

$$\frac{2\sec^{\frac{3}{2}}(a+bx)\left(\cos^{\frac{3}{2}}(a+bx)F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(5/2),x]
```

```
[Out] (2*Sec[a + b*x]^(3/2)*(Cos[a + b*x]^(3/2)*EllipticF[(a + b*x)/2, 2] + Sin[a
 + b*x]))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(78) = 156.
time = 2.12, size = 213, normalized size = 3.44

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-2*sin(1/2*b*x+1/2*a)
^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-
1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))*((2*cos(1/2*b*x+1/2*a)^2-1)
*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(3/2)/sin(1/2*b*x+1/2*a)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.47, size = 88, normalized size = 1.42

$$\frac{-i\sqrt{2}\cos\left(bx+a\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right))+i\sqrt{2}\cos\left(bx+a\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right))+\frac{2\sin(bx+a)}{\sqrt{\cos\left(bx+a\right)}}}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*cos(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*si
n(b*x + a)) + I*sqrt(2)*cos(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a
) - I*sin(b*x + a)) + 2*sin(b*x + a)/sqrt(cos(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sec^{\frac{5}{2}}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(5/2),x)
```

```
[Out] Integral(sec(a + b*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(5/2),x)
```

```
[Out] int((1/cos(a + b*x))^(5/2), x)
```

3.11 $\int \sec^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{2\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}+\frac{2\sqrt{\sec(a+bx)}\,\sin(a+bx)}{b}$$

```
[Out] 2*sin(b*x+a)*sec(b*x+a)^(1/2)/b-2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/
2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1
/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3853, 3856, 2719}

$$\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (2
*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\sec(a+bx)}\,\sin(a+bx)}{b} - \int \frac{1}{\sqrt{\sec(a+bx)}}\,dx\\ &= \frac{2\sqrt{\sec(a+bx)}\,\sin(a+bx)}{b} - \left(\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx\\ &= -\frac{2\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b} + \frac{2\sqrt{\sec(a+bx)}\,\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.78

$$\frac{2\sqrt{\sec(a+bx)}\left(-\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + Si
n[a + b*x]))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(78) = 156$.
time = 2.85, size = 182, normalized size = 3.14

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}\right.\right.}\right.}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1
/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(
cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 73, normalized size = 1.26

$$\frac{-i\sqrt{2}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))+i\sqrt{2}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\,\sin(bx+a)))+\frac{2\sin(bx+a)}{\sqrt{\cos(bx+a)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
+ I*sin(b*x + a))) + I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sin(b*x + a)/sqrt(cos(b*x + a)))/
b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(3/2),x)
[Out] Integral(sec(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(3/2),x)
[Out] int((1/cos(a + b*x))^(3/2), x)
```

3.12 $\int \sqrt{\sec(a+bx)}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b
*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2720}

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(a+bx)}\,dx &= \left(\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 1.00

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(58) = 116$.
time = 1.58, size = 133, normalized size = 3.69

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\mathrm{EllipticF}\left(\mathrm{co}\right.}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin
(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))/sin(1/2*b*x+
1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 51, normalized size = 1.42

$$\frac{-i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*s
qrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(b*x + a)), x)
```

Mupad [B]
time = 0.13, size = 33, normalized size = 0.92

$$\frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\frac{1}{\cos\left(a+b\,x\right)}}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(1/2),x)
```

```
[Out] (2*cos(a + b*x)^(1/2)*(1/cos(a + b*x))^(1/2)*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.13 $\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2719}

$$\frac{2\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx &= \left(\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx\\ &= \frac{2\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sec[a + b*x]],x]
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[Sec[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(58) = 116$.
time = 1.62, size = 133, normalized size = 3.69

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\text{EllipticE}\left(\cos\cdots\right.}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,b}$
risch	$-\frac{i\sqrt{2}}{b\sqrt{\frac{e^{i(bx+a)}}{e^{2i(bx+a)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{e^{i(bx+a)}\left(e^{2i(bx+a)}+1\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(bx+a)}-i\right)}\sqrt{ie^{i(b\cdots}}}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1
/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sec(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.11, size = 57, normalized size = 1.58

$$\frac{i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) +
 I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(sec(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sec(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{1}{\cos(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(1/2),x)
[Out] int(1/(1/cos(a + b*x))^(1/2), x)
```

3.14 $\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{3b}+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}$$

```
[Out] 2/3*sin(b*x+a)/b/sec(b*x+a)^(1/2)+2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*
a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a
)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3854, 3856, 2720}

$$\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) +
 (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{1}{3}\int \sqrt{\sec(a+bx)}\,dx\\
&= \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{1}{3}\left(\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\
&= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b}+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)}\left(2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(2(a+bx))\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-3/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2
*(a + b*x)]))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(78) = 156$.
time = 2.01, size = 179, normalized size = 2.89

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\,(c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*x
+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(sin
(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2
*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/
sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 68, normalized size = 1.10

$$\frac{2\sqrt{\cos\left(bx+a\right)}\sin\left(bx+a\right)-i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right))+i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right))}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, co
s(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sec^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(3/2),x)
```

```
[Out] int(1/(1/cos(a + b*x))^(3/2), x)
```

3.15 $\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=62

$$\frac{6\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{5b}+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/5*sin(b*x+a)/b/sec(b*x+a)^(3/2)+6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*
a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a
)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3854, 3856, 2719}

$$\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{6\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-5/2),x]
```

```
[Out] (6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b) +
 (2*Sin[a + b*x])/(5*b*Sec[a + b*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx \\
&= \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)} + \frac{1}{5}\left(3\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx \\
&= \frac{6\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)\sqrt{\sec(a+bx)}}{5b} + \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 0.89

$$
\frac{\sqrt{\sec(a+bx)}\left(12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\sin(a+bx)+\sin(3(a+bx))\right)}{10b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-5/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Sin[
a + b*x] + Sin[3*(a + b*x)]))/(10*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(78) = 156.
time = 2.11, size = 202, normalized size = 3.26

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(-8\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+8\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2(\ldots}{5\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\ \mathrm{si}\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*sin(1/2*b*
x+1/2*a)^6*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*s
in(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*EllipticE(cos(1/2*b*x+1/2*a),2^(1/
2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2))/(-2*sin(
1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*
b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 74, normalized size = 1.19

$$\frac{2\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)+3i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right)))-3i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right)))}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*cos(b*x + a)^(3/2)*sin(b*x + a) + 3*I*sqrt(2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 3*I*sqrt(2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x +
 a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(5/2),x)
```

```
[Out] Integral(sec(a + b*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{1}{\cos(a+b\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(5/2),x)
```

```
[Out] int(1/(1/cos(a + b*x))^(5/2), x)
```

3.16 $\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)} dx$

Optimal. Leaf size=85

$$\frac{10\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{21b}+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}}$$

```
[Out] 2/7*sin(b*x+a)/b/sec(b*x+a)^(5/2)+10/21*sin(b*x+a)/b/sec(b*x+a)^(1/2)+10/21
*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*
x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3854, 3856, 2720}

$$\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}}+\frac{10\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-7/2),x]
```

```
[Out] (10*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(21*b)
 + (2*Sin[a + b*x])/(7*b*Sec[a + b*x]^(5/2)) + (10*Sin[a + b*x])/(21*b*Sqrt
[Sec[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}} + \frac{5}{21}\int \sqrt{\sec(a+bx)}\,dx$$
$$= \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx)}} + \frac{1}{21}\left(5\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{}{\sqrt{\cos(}}$$
$$= \frac{10\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{21b} + \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)} + \frac{10\sin(a+bx)}{21b\sqrt{\sec(a+bx}}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.72

$$\frac{\sqrt{\sec(a+bx)}\left(40\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+26\sin(2(a+bx))+3\sin(4(a+bx))\right)}{84b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-7/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 26*S
in[2*(a + b*x)] + 3*Sin[4*(a + b*x)]))/(84*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(97) = 194.
time = 2.58, size = 199, normalized size = 2.34

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(48\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-72\left(\mathrm{co}\right.\right.}{21\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(48*cos(1/2*b
*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(1/2*b*
x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)
*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b
*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1
/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 87, normalized size = 1.02

$$\frac{\frac{2\left(3\cos(bx+a)^3+5\cos(bx+a)\right)\sin(bx+a)}{\sqrt{\cos(bx+a)}} - 5i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + 5i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="fricas")
[Out] 1/21*(2*(3*cos(b*x + a)^3 + 5*cos(b*x + a))*sin(b*x + a)/sqrt(cos(b*x + a))
 - 5*I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) +
5*I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(7/2),x)
[Out] Integral(sec(a + b*x)**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{1}{\cos(a+b\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(7/2),x)
```

```
[Out] int(1/(1/cos(a + b*x))^(7/2), x)
```

3.17 $\int (c\sec(a+bx))^{7/2}\,dx$

Optimal. Leaf size=98

$$-\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{6c^3\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{5b}+\frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b}$$

```
[Out] 2/5*c*(c*sec(b*x+a))^(5/2)*sin(b*x+a)/b-6/5*c^4*(cos(1/2*a+1/2*b*x)^2)^(1/2
)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/
2)/(c*sec(b*x+a))^(1/2)+6/5*c^3*sin(b*x+a)*(c*sec(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$-\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{6c^3\sin(a+bx)\sqrt{c\sec(a+bx)}}{5b}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(7/2),x]
```

```
[Out] (-6*c^4*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b
*x]]) + (6*c^3*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/(5*b) + (2*c*(c*Sec[a + b
*x])^(5/2)*Sin[a + b*x])/(5*b)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (c\sec(a+bx))^{7/2}\,dx = \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b} + \frac{1}{5}\left(3c^2\right)\int (c\sec(a+bx))^{3/2}\,dx$$
$$= \frac{6c^3\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{5b} + \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b} - \frac{1}{5}\left(3c^4\right)\int \frac{}{\sqrt{c}}$$
$$= \frac{6c^3\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{5b} + \frac{2c(c\sec(a+bx))^{5/2}\sin(a+bx)}{5b} - \frac{\left(3c^4\right)\int\sqrt{c}}{5\sqrt{\cos(a+bx}}$$
$$= -\frac{6c^4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{6c^3\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{5b} + \frac{2c(c\sec(a}{}$$

Mathematica [A]
time = 0.18, size = 62, normalized size = 0.63

$$\frac{c(c\sec(a+bx))^{5/2}\left(-12\cos^{\frac{5}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+7\sin(a+bx)+3\sin(3(a+bx))\right)}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(7/2),x]
```

```
[Out] (c*(c*Sec[a + b*x])^(5/2)*(-12*Cos[a + b*x]^(5/2)*EllipticE[(a + b*x)/2, 2]
 + 7*Sin[a + b*x] + 3*Sin[3*(a + b*x)]))/(10*b)
```

Maple [C] Result contains complex when optimal does not.
time = 42.48, size = 354, normalized size = 3.61

method	result
default	$\frac{2(-1+\cos(bx+a))^2\left(3i\sin(bx+a)\left(\cos^3(bx+a)\right)\sqrt{\frac{1}{\cos(bx+a)+1}}\,\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)-3i\sin(bx+a)(\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/b*(-1+cos(b*x+a))^2*(3*I*sin(b*x+a)*cos(b*x+a)^3*(1/(cos(b*x+a)+1))^(1/
2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticE(I*(-1+cos(b*x+a))/sin(b*x+a)
,I)-3*I*sin(b*x+a)*cos(b*x+a)^3*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b
*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)+3*I*sin(b*x+a)*co
s(b*x+a)^2*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*Ellip
ticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)-3*I*sin(b*x+a)*cos(b*x+a)^2*(1/(cos(b*
x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a)
```

```
)/sin(b*x+a),I)-3*cos(b*x+a)^3+2*cos(b*x+a)^2+1)*cos(b*x+a)*(cos(b*x+a)+1)^
2*(c/cos(b*x+a))^(7/2)/sin(b*x+a)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 125, normalized size = 1.28

$$\frac{-3i\sqrt{2}\,c^{\frac{7}{2}}\cos\left(bx+a\right)^{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)))+3i\sqrt{2}\,c^{\frac{7}{2}}\cos\left(bx+a\right)^{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)))+2\left(3\,c^{3}\cos\left(bx+a\right)^{2}+c^{3}\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\,\sin\left(bx+a\right)}{5\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*c^(7/2)*cos(b*x + a)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*c^(7/2)*cos(b
*x + a)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) -
I*sin(b*x + a))) + 2*(3*c^3*cos(b*x + a)^2 + c^3)*sqrt(c/cos(b*x + a))*sin(
b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(7/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{c}{\cos\left(a+b\,x\right)}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(7/2),x)
[Out] int((c/cos(a + b*x))^(7/2), x)
```

3.18 $\int (c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{2c^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3b}+\frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b}$$

```
[Out] 2/3*c*(c*sec(b*x+a))^(3/2)*sin(b*x+a)/b+2/3*c^2*(cos(1/2*a+1/2*b*x)^2)^(1/2
)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)
*(c*sec(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2720}

$$\frac{2c^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3b}+\frac{2c\sin(a+bx)(c\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(
3*b) + (2*c*(c*Sec[a + b*x])^(3/2)*Sin[a + b*x])/(3*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (c\sec(a+bx))^{5/2}\,dx = \frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b} + \frac{1}{3}c^2\int \sqrt{c\sec(a+bx)}\,dx$$
$$= \frac{2c(c\sec(a+bx))^{3/2}\sin(a+bx)}{3b} + \frac{1}{3}\left(c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a}}$$
$$= \frac{2c^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3b} + \frac{2c(c\sec(a+bx))^{3/2}\sin(a+b}{3b}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 0.73

$$\frac{2c^2\sqrt{c\sec(a+bx)}\left(\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+\tan(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (2*c^2*Sqrt[c*Sec[a + b*x]]*(Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] +
 Tan[a + b*x]))/(3*b)
```

Maple [C] Result contains complex when optimal does not.
time = 41.75, size = 128, normalized size = 1.83

method	result
default	$-\frac{2(-1+\cos(bx+a))\left(i\sin(bx+a)\cos(bx+a)\sqrt{\frac{1}{\cos(bx+a)+1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)-\cos(bx+a)+1\right)\cos(}{3b\sin(bx+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b*(-1+cos(b*x+a))*(I*sin(b*x+a)*cos(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(c
os(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)-c
os(b*x+a)+1)*cos(b*x+a)*(cos(b*x+a)+1)^2*(c/cos(b*x+a))^(5/2)/sin(b*x+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 101, normalized size = 1.44

$$\frac{-i\sqrt{2}\,c^{\frac{5}{2}}\cos\left(bx+a\right)\text{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))+i\sqrt{2}\,c^{\frac{5}{2}}\cos\left(bx+a\right)\text{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right))+2\,c^{2}\sqrt{\frac{c}{\cos\left(bx+a\right)}}\,\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*c^(5/2)*cos(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a
) + I*sin(b*x + a)) + I*sqrt(2)*c^(5/2)*cos(b*x + a)*weierstrassPInverse(-4
, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*c^2*sqrt(c/cos(b*x + a))*sin(b*x +
a))/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sec\left(a+bx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2),x)
```

```
[Out] int((c/cos(a + b*x))^(5/2), x)
```

3.19 $\int (c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2c\sqrt{c\sec(a+bx)}\sin(a+bx)}{b}$$

```
[Out] -2*c^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+
1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)/(c*sec(b*x+a))^(1/2)+2*c*sin(b*x+a)*(c
*sec(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$\frac{2c\sin(a+bx)\sqrt{c\sec(a+bx)}}{b}-\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c^2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x
]]) + (2*c*Sqrt[c*Sec[a + b*x]]*Sin[a + b*x])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (c\sec(a+bx))^{3/2}\,dx = \frac{2c\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{b} - c^2\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$$
$$= \frac{2c\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{b} - \frac{c^2\int \sqrt{\cos(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\,\sqrt{c\sec(a+bx)}}$$
$$= -\frac{2c^2E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b\sqrt{\cos(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{2c\sqrt{c\sec(a+bx)}\,\sin(a+bx)}{b}$$

Mathematica [**A**]
time = 0.05, size = 48, normalized size = 0.73

$$\frac{2c\sqrt{c\sec(a+bx)}\left(-\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\sin(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]]*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])
+ Sin[a + b*x]))/b
```

Maple [**C**] Result contains complex when optimal does not.
time = 42.37, size = 320, normalized size = 4.85

method	result
default	$-\frac{2(\cos(bx+a)+1)^2(-1+\cos(bx+a))^2\left(i\sin(bx+a)\cos(bx+a)\sqrt{\frac{1}{\cos(bx+a)+1}}\,\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))^2*(I*sin(b*x+a)*cos(b*x+a)*(1/(cos(b*
x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a)
)/sin(b*x+a),I)-I*EllipticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*cos(
b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)+I*Ellipti
cF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos
(b*x+a)/(cos(b*x+a)+1))^(1/2)-I*EllipticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)*s
in(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)+cos(b*
x+a)-1)*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 84, normalized size = 1.27

$$\frac{-i\sqrt{2}\,c^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+i\sqrt{2}\,c^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+2c\sqrt{\frac{c}{\cos(bx+a)}}\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*c^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b
*x + a) + I*sin(b*x + a))) + I*sqrt(2)*c^(3/2)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*c*sqrt(c/cos(b*x
+ a))*sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos(a+b\,x)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2),x)
```

```
[Out] int((c/cos(a + b*x))^(3/2), x)
```

3.20 $\int \sqrt{c \sec(a+bx)}\, dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(a+bx)}\, F\left(\frac{1}{2}(a+bx)\middle|2\right) \sqrt{c\sec(a+bx)}}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b
*x),2^(1/2))*cos(b*x+a)^(1/2)*(c*sec(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2720}

$$\frac{2\sqrt{\cos(a+bx)}\, F\left(\frac{1}{2}(a+bx)\middle|2\right) \sqrt{c\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sec(a+bx)}\, dx &= \left(\sqrt{\cos(a+bx)}\, \sqrt{c\sec(a+bx)}\right) \int \frac{1}{\sqrt{\cos(a+bx)}}\, dx \\ &= \frac{2\sqrt{\cos(a+bx)}\, F\left(\frac{1}{2}(a+bx)\middle|2\right) \sqrt{c\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{2\sqrt{\cos(a+bx)}\, F\left(\frac{1}{2}(a+bx)\middle|2\right) \sqrt{c\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/b
```

Maple [C] Result contains complex when optimal does not.
time = 27.22, size = 98, normalized size = 2.58

method	result	size
default	$-\frac{2 i \sqrt{\frac{c}{\cos(bx+a)}}\,(-1+\cos(bx+a))\sqrt{\frac{1}{\cos(bx+a)+1}}\,\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)}, i\right)(\cos(bx+a)+1)^{2}}{b \sin(bx+a)^{2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/b*(c/cos(b*x+a))^(1/2)*(-1+cos(b*x+a))*(1/(cos(b*x+a)+1))^(1/2)*(cos(b
*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*(cos(
b*x+a)+1)^2/sin(b*x+a)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 57, normalized size = 1.50

$$\frac{-i\sqrt{2}\sqrt{c}\,\mathrm{weierstrassPInverse}(-4, 0, \cos(bx+a) + i\sin(bx+a)) + i\sqrt{2}\sqrt{c}\,\mathrm{weierstrassPInverse}(-4, 0, \cos(bx+a) - i\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a
)) + I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x
+ a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sec(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(b*x + a)), x)
```

Mupad [B]
time = 0.20, size = 35, normalized size = 0.92

$$\frac{2\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2),x)
```

```
[Out] (2*cos(a + b*x)^(1/2)*(c/cos(a + b*x))^(1/2)*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.21 $\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))/b/cos(b*x+a)^(1/2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2719}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx &= \frac{\int \sqrt{\cos(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\\ &= \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 29.64, size = 306, normalized size = 8.05

method	result
risch	$-\frac{i\sqrt{2}}{b\sqrt{\frac{c\,e^{i(bx+a)}}{e^{2i(bx+a)}+1}}}-\frac{i\left(-\frac{2\left(e^{2i(bx+a)}c+c\right)}{c\sqrt{e^{i(bx+a)}\left(e^{2i(bx+a)}c+c\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(bx+a)}-i\right)}\sqrt{ie^{}}}{}\right.}{}$
default	$\frac{2\left(i\sin(bx+a)\cos(bx+a)\sqrt{\frac{1}{\cos(bx+a)+1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\operatorname{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)-i\operatorname{EllipticE}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\sin(ba\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(I*sin(b*x+a)*cos(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+
a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)-I*EllipticE(I*(-1+co
s(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*cos(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos
(b*x+a)/(cos(b*x+a)+1))^(1/2)+I*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*s
in(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)-I*Elli
pticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(
cos(b*x+a)/(cos(b*x+a)+1))^(1/2)-cos(b*x+a)^2+cos(b*x+a))*(c/cos(b*x+a))^(1
/2)/sin(b*x+a)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*sec(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 66, normalized size = 1.74

$$\frac{i\sqrt{2}\,\sqrt{c}\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))-i\sqrt{2}\,\sqrt{c}\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\,\sin(bx+a)))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*
x + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sec(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sec(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{c}{\cos(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(1/2),x)
```

```
[Out] int(1/(c/cos(a + b*x))^(1/2), x)
```

3.22 $\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

```
[Out] 2/3*sin(b*x+a)/b/c/(c*sec(b*x+a))^(1/2)+2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/co
s(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*(c*
sec(b*x+a))^(1/2)/b/c^2
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3bc^2}+\frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(3*b*
c^2) + (2*Sin[a + b*x])/(3*b*c*Sqrt[c*Sec[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx &= \frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}} + \frac{\int \sqrt{c\sec(a+bx)}\,dx}{3c^2} \\
&= \frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3c^2} \\
&= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{3bc^2} + \frac{2\sin(a+bx)}{3bc\sqrt{c\sec(a+bx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.82

$$\frac{\sec^2(a+bx)\left(2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(2(a+bx))\right)}{3b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-3/2),x]
```

```
[Out] (Sec[a + b*x]^2*(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Sin[2*(a
+ b*x)]))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 27.16, size = 131, normalized size = 1.82

method	result
default	$-\frac{2(\cos(bx+a)+1)^2(-1+\cos(bx+a))\left(i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\sin(bx+a)\sqrt{\frac{1}{\cos(bx+a)+1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}-(\cos^2(bx+a))\right.}{3b\sin(bx+a)^3\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}\cos(bx+a)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))*(I*EllipticF(I*(-1+cos(b*x+a))/sin(
b*x+a),I)*sin(b*x+a)*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(
1/2)-cos(b*x+a)^2+cos(b*x+a))/sin(b*x+a)^3/(c/cos(b*x+a))^(3/2)/cos(b*x+a)^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 87, normalized size = 1.21

$$\frac{2\sqrt{\frac{c}{\cos(bx+a)}}\cos(bx+a)\sin(bx+a) - i\sqrt{2}\sqrt{c}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + i\sqrt{2}\sqrt{c}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(c/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - I*sqrt(2)*sqrt(c)*w
eierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*sqrt(c
)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/(b*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sec(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(3/2),x)
[Out] Integral((c*sec(a + b*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sec(b*x + a))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(3/2),x)
[Out] int(1/(c/cos(a + b*x))^(3/2), x)
```

3.23 $\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

```
[Out] 2/5*sin(b*x+a)/b/c/(c*sec(b*x+a))^(3/2)+6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/co
s(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/c^2/cos(b*x+a)^(1/
2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2719}

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(a + b*x)/2, 2])/(5*b*c^2*Sqrt[Cos[a + b*x]]*Sqrt[c*Sec[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Sec[a + b*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(c\sec(a+bx))^{5/2}}\,dx = \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{c\sec(a+bx)}}\,dx}{5c^2}$$
$$= \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}} + \frac{3\int \sqrt{\cos(a+bx)}\,dx}{5c^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}}$$
$$= \frac{6E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{5bc^2\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\sec(a+bx))^{3/2}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.83

$$\frac{\sqrt{c\sec(a+bx)}\left(12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\sin(a+bx)+\sin(3(a+bx))\right)}{10bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-5/2),x]
```

```
[Out] (Sqrt[c*Sec[a + b*x]]*(12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + Si
n[a + b*x] + Sin[3*(a + b*x)]))/(10*b*c^3)
```

Maple [C] Result contains complex when optimal does not.
time = 28.65, size = 323, normalized size = 4.49

method	result
default	$\frac{\frac{6i\sin(bx+a)\cos(bx+a)\sqrt{\frac{1}{\cos(bx+a)+1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)}{5}-\frac{6i\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\sin(bx+a)\cos}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/b*(3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b
*x+a)*cos(b*x+a)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)-3*I*(1/(cos(b*x+
a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a)*cos(b*x+a)*Ellipt
icE(I*(-1+cos(b*x+a))/sin(b*x+a),I)+3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a
)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I
)-3*I*(1/(cos(b*x+a)+1))^(1/2)*(cos(b*x+a)/(cos(b*x+a)+1))^(1/2)*sin(b*x+a)
*EllipticE(I*(-1+cos(b*x+a))/sin(b*x+a),I)-cos(b*x+a)^4-2*cos(b*x+a)^2+3*co
s(b*x+a))/sin(b*x+a)/(c/cos(b*x+a))^(5/2)/cos(b*x+a)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 95, normalized size = 1.32

$$\frac{2\sqrt{\frac{c}{\cos(bx+a)}}\cos(bx+a)^2\sin(bx+a) + 3i\sqrt{2}\sqrt{c}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))) - 3i\sqrt{2}\sqrt{c}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{5\,bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(c/cos(b*x + a))*cos(b*x + a)^2*sin(b*x + a) + 3*I*sqrt(2)*sqrt(
c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b
*x + a))) - 3*I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a))))/(b*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sec(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(5/2),x)
[Out] int(1/(c/cos(a + b*x))^(5/2), x)
```

3.24 $\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{21bc^4}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}+\frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}}$$

```
[Out] 2/7*sin(b*x+a)/b/c/(c*sec(b*x+a))^(5/2)+10/21*sin(b*x+a)/b/c^3/(c*sec(b*x+a
))^(1/2)+10/21*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(si
n(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*(c*sec(b*x+a))^(1/2)/b/c^4
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\frac{10\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{21bc^4}+\frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}}+\frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-7/2),x]
```

```
[Out] (10*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[c*Sec[a + b*x]])/(21*
b*c^4) + (2*Sin[a + b*x])/(7*b*c*(c*Sec[a + b*x])^(5/2)) + (10*Sin[a + b*x]
)/(21*b*c^3*Sqrt[c*Sec[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(c\sec(a+bx))^{7/2}}\,dx = \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{5\int \frac{1}{(c\sec(a+bx))^{3/2}}\,dx}{7c^2}$$
$$= \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}} + \frac{5\int \sqrt{c\sec(a+bx)}\,dx}{21c^4}$$
$$= \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{10\sin(a+bx)}{21bc^3\sqrt{c\sec(a+bx)}} + \frac{\left(5\sqrt{\cos(a+bx)}\sqrt{c\sec(a+bx)}\right.}{21c^4}$$
$$= \frac{10\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\sec(a+bx)}}{21bc^4} + \frac{2\sin(a+bx)}{7bc(c\sec(a+bx))^{5/2}} + \frac{}{21b}$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 0.66

$$\frac{\sqrt{c\sec(a+bx)}\left(40\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)+26\sin(2(a+bx))+3\sin(4(a+bx))\right)}{84bc^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-7/2),x]
```

```
[Out] (Sqrt[c*Sec[a + b*x]]*(40*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 26
*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)]))/(84*b*c^4)
```

Maple [C] Result contains complex when optimal does not.
time = 27.34, size = 153, normalized size = 1.53

method	result
default	$-\frac{2(\cos(bx+a)+1)^2(-1+\cos(bx+a))\left(5i\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(bx+a))}{\sin(bx+a)},i\right)\sin(bx+a)\sqrt{\frac{1}{\cos(bx+a)+1}}\sqrt{\frac{\cos(bx+a)}{\cos(bx+a)+1}}-3(\cos^4(bx+\right.}{21b\sin(bx+a)^3\left(\frac{c}{\cos(bx+a)}\right)^{\frac{7}{2}}\cos(bx+a)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))*(5*I*(1/(cos(b*x+a)+1))^(1/2)*(cos
(b*x+a)/(cos(b*x+a)+1))^(1/2)*EllipticF(I*(-1+cos(b*x+a))/sin(b*x+a),I)*sin
(b*x+a)-3*cos(b*x+a)^4+3*cos(b*x+a)^3-5*cos(b*x+a)^2+5*cos(b*x+a))/sin(b*x+
a)^3/(c/cos(b*x+a))^(7/2)/cos(b*x+a)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.03, size = 100, normalized size = 1.00

$$\frac{2\,\left(3\,\cos\left(bx+a\right)^{3}+5\,\cos\left(bx+a\right)\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\,\sin\left(bx+a\right)-5i\,\sqrt{2}\,\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+5i\,\sqrt{2}\,\sqrt{c}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)}{21\,bc^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="fricas")
[Out] 1/21*(2*(3*cos(b*x + a)^3 + 5*cos(b*x + a))*sqrt(c/cos(b*x + a))*sin(b*x +
a) - 5*I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*
x + a)) + 5*I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*s
in(b*x + a)))/(b*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sec\left(a+bx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(7/2),x)
[Out] Integral((c*sec(a + b*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate((c*sec(b*x + a))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(7/2),x)
[Out] int(1/(c/cos(a + b*x))^(7/2), x)
```

3.25 $\int \sec^{\frac{4}{3}}(a+bx)\,dx$

Optimal. Leaf size=51

$$\frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sqrt[3]{\sec(a+bx)}\,\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

```
[Out] 3*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*sec(b*x+a)^(1/3)*sin(b*x+a)/b/(
sin(b*x+a)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$\frac{3\sin(a+bx)\sqrt[3]{\sec(a+bx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sec[a + b*x]^(1/3)*Sin
[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{4}{3}}(a+bx)\,dx &= \sqrt[3]{\cos(a+bx)}\,\sqrt[3]{\sec(a+bx)}\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sqrt[3]{\sec(a+bx)}\,\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 1.08

$$\frac{3\csc(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(a+bx)\right)\sqrt[3]{\sec(a+bx)}\,\sqrt{-\tan^2(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(4/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b*x]^2]*Sec[a + b*
x]^(1/3)*Sqrt[-Tan[a + b*x]^2])/(4*b)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(4/3),x)
```

```
[Out] int(sec(b*x+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(4/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{\frac{4}{3}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(4/3),x)
```

```
[Out] Integral(sec(a + b*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(4/3),x)
```

```
[Out] int((1/cos(a + b*x))^(4/3), x)
```

3.26 $\int \sec^{\frac{2}{3}}(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{3\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\sec(a+bx)}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(1/3)/(
sin(b*x+a)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Sec[a
 + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{2}{3}}(a+bx)\,dx &= \cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\sec(a+bx)}\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 1.08

$$\frac{3\csc(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{2b\sqrt[3]{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(2/3),x]
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b*x]^2]*Sqrt[-Tan[
a + b*x]^2])/(2*b*Sec[a + b*x]^(1/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(2/3),x)
[Out] int(sec(b*x+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(2/3),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^{\frac{2}{3}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(2/3),x)
```

```
[Out] Integral(sec(a + b*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(2/3),x)
```

```
[Out] int((1/cos(a + b*x))^(2/3), x)
```

3.27 $\int \sqrt[3]{\sec(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2b\sec^{\frac{2}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/2*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(2/3)
/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3857, 2722}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}\,\sec^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*Sec
[a + b*x]^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\sec(a+bx)}\,dx &= \sqrt[3]{\cos(a+bx)}\,\sqrt[3]{\sec(a+bx)}\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2b\sec^{\frac{2}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.00

$$\frac{3\csc(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{b\sec^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(1/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b*x]^2]*Sqrt[-Tan[
a + b*x]^2])/(b*Sec[a + b*x]^(2/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \sec^{\frac{1}{3}}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^(1/3),x)
```

```
[Out] int(sec(b*x+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**(1/3),x)
```

```
[Out] Integral(sec(a + b*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^(1/3),x)
```

```
[Out] int((1/cos(a + b*x))^(1/3), x)
```

3.28 $\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4b\sec^{\frac{4}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/4*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(4/3)
/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}\sec^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*Sec
[a + b*x]^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{\sec(a+bx)}}\,dx &= \cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \sqrt[3]{\cos(a+bx)}\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4b\sec^{\frac{4}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 1.00

$$-\frac{3\csc(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{b\sec^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-1/3),x]
[Out] (-3*Csc[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(b*Sec[a + b*x]^(4/3))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(1/3),x)
[Out] int(1/sec(b*x+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^(-1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\sec\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(1/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(1/3),x)
```

```
[Out] int(1/(1/cos(a + b*x))^(1/3), x)
```

3.29 $\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5b\sec^{\frac{5}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/5*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(5/3
)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}\,\sec^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*Se
c[a + b*x]^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{2}{3}}(a+bx)}\,dx &= \sqrt[3]{\cos(a+bx)}\,\sqrt[3]{\sec(a+bx)}\int \cos^{\frac{2}{3}}(a+bx)\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5b\sec^{\frac{5}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 55, normalized size = 1.04

$$-\frac{3\csc(a+bx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{2b\sec^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-2/3),x]
[Out] (-3*Csc[a + b*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(2*b*Sec[a + b*x]^(5/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(2/3),x)
[Out] int(1/sec(b*x+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(2/3),x, algorithm="maxima")
[Out] integrate(sec(b*x + a)^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(2/3),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^(-2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{2}{3}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(2/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(2/3),x)
```

```
[Out] int(1/(1/cos(a + b*x))^(2/3), x)
```

3.30 $\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7b\sec^{\frac{7}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/7*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x+a)/b/sec(b*x+a)^(7/3
)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$-\frac{3\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}\,\sec^{\frac{7}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^(-4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*Se
c[a + b*x]^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx &= \cos^{\frac{2}{3}}(a+bx)\sec^{\frac{2}{3}}(a+bx)\int \cos^{\frac{4}{3}}(a+bx)\,dx\\ &= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7b\sec^{\frac{7}{3}}(a+bx)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 55, normalized size = 1.04

$$-\frac{3\csc(a+bx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{4b\sec^{\frac{7}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^(-4/3),x]
```

```
[Out] (-3*Csc[a + b*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(4*b*Sec[a + b*x]^(7/3))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(b*x+a)^(4/3),x)
```

```
[Out] int(1/sec(b*x+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^(-4/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sec^{\frac{4}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)**(4/3),x)
```

```
[Out] Integral(sec(a + b*x)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cos(a+b\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(a + b*x))^(4/3),x)
```

```
[Out] int(1/(1/cos(a + b*x))^(4/3), x)
```

3.31 $\int (c\sec(a+bx))^{4/3}\,dx$

Optimal. Leaf size=54

$$\frac{3c\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\,\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

[Out] 3*c*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*(c*sec(b*x+a))^(1/3)*sin(b*x+a)/b/(sin(b*x+a)^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$\frac{3c\sin(a+bx)\sqrt[3]{c\sec(a+bx)}\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(c*Sec[a + b*x])^(4/3),x]

[Out] (3*c*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*(c*Sec[a + b*x])^(1/3)*Sin[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^{4/3}\,dx &= \sqrt[3]{\frac{\cos(a+bx)}{c}}\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{4/3}}\,dx\\ &= \frac{3c\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\,\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 57, normalized size = 1.06

$$\frac{3\cot(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(a+bx)\right)(c\sec(a+bx))^{4/3}\sqrt{-\tan^2(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(4/3),x]
```

```
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[a + b*x]^2]*(c*Sec[a +
 b*x])^(4/3)*Sqrt[-Tan[a + b*x]^2])/(4*b)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(bx+a\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(4/3),x)
```

```
[Out] int((c*sec(b*x+a))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(1/3)*c*sec(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sec\left(a+bx\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(4/3),x)
```

```
[Out] int((c/cos(a + b*x))^(4/3), x)
```

3.32 $\int (c\sec(a+bx))^{2/3}\,dx$

Optimal. Leaf size=54

$$-\frac{3c\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{c\sec(a+bx)}\,\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3*c*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a))^(
1/3)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\,\sqrt[3]{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(2/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(c*
Sec[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^{2/3}\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int\frac{1}{\left(\frac{\cos(a+bx)}{c}\right)^{2/3}}\,dx\\ &= -\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 1.06

$$\frac{3\cot(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sqrt{-\tan^2(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(2/3),x]
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[a + b*x]^2]*(c*Sec[a +
 b*x])^(2/3)*Sqrt[-Tan[a + b*x]^2])/(2*b)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (c\sec(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(2/3),x)
[Out] int((c*sec(b*x+a))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(2/3),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(2/3),x, algorithm="fricas")
[Out] integral((c*sec(b*x + a))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec(a+bx))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(2/3),x)
```

```
[Out] int((c/cos(a + b*x))^(2/3), x)
```

3.33 $\int \sqrt[3]{c\sec(a+bx)}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2b(c\sec(a+bx))^{2/3}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/2*c*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a))
^(2/3)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}\,(c\sec(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(1/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin[a + b*x])/(2*b*(
c*Sec[a + b*x])^(2/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{c\sec(a+bx)}\,dx &= \sqrt[3]{\frac{\cos(a+bx)}{c}}\,\sqrt[3]{c\sec(a+bx)}\int \frac{1}{\sqrt[3]{\frac{\cos(a+bx)}{c}}}\,dx\\ &= -\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\,\sin(a+bx)}{2b\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.98

$$\frac{3\cot(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\,\sqrt{-\tan^2(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(1/3),x]
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[a + b*x]^2]*(c*Sec[a +
 b*x])^(1/3)*Sqrt[-Tan[a + b*x]^2])/b
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (c\sec(bx+a))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/3),x)
[Out] int((c*sec(b*x+a))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/3),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/3),x, algorithm="fricas")
[Out] integral((c*sec(b*x + a))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/3),x)
```

```
[Out] int((c/cos(a + b*x))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4b(c\sec(a+bx))^{4/3}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/4*c*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a))
^(4/3)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}\,(c\sec(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-1/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin[a + b*x])/(4*b*(
c*Sec[a + b*x])^(4/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{c\sec(a+bx)}}\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \sqrt[3]{\frac{\cos(a+bx)}{c}}\,dx\\
&= -\frac{3\cos^2(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sin(a+bx)}{4bc\sqrt{\sin^2(a+bx)}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.98

$$-\frac{3\cot(a+bx)\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{b\sqrt[3]{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-1/3),x]
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(b*(c*Sec[a + b*x])^(1/3))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sec\left(bx+a\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(1/3),x)
[Out] int(1/(c*sec(b*x+a))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="fricas")
[Out] integral((c*sec(b*x + a))^(2/3)/(c*sec(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c\sec\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(1/3),x)
```

```
[Out] int(1/(c/cos(a + b*x))^(1/3), x)
```

3.35 $\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5b(c\sec(a+bx))^{5/3}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/5*c*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a)
)^(5/3)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}\,(c\sec(a+bx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-2/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Sin[a + b*x])/(5*b*
(c*Sec[a + b*x])^(5/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sec(a+bx))^{2/3}}\,dx &= \sqrt[3]{\frac{\cos(a+bx)}{c}}\,\sqrt[3]{c\sec(a+bx)}\int\left(\frac{\cos(a+bx)}{c}\right)^{2/3}dx\\ &= -\frac{3\cos^2(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sqrt[3]{c\sec(a+bx)}\,\sin(a+bx)}{5bc\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 1.02

$$-\frac{3\cot(a+bx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{2b(c\sec(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-2/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(2*b*(c*Sec[a + b*x])^(2/3))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sec\left(bx+a\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(2/3),x)
```

```
[Out] int(1/(c*sec(b*x+a))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(1/3)/(c*sec(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sec\left(a+bx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(2/3),x)
```

```
[Out] int(1/(c/cos(a + b*x))^(2/3), x)
```

3.36 $\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3c\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7b(c\sec(a+bx))^{7/3}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/7*c*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x+a)/b/(c*sec(b*x+a)
)^(7/3)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3c\sin(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}\,(c\sec(a+bx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(-4/3),x]
```

```
[Out] (-3*c*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Sin[a + b*x])/(7*b*
(c*Sec[a + b*x])^(7/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sec(a+bx))^{4/3}}\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^{2/3}(c\sec(a+bx))^{2/3}\int \left(\frac{\cos(a+bx)}{c}\right)^{4/3}\,dx\\ &= -\frac{3\cos^3(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)(c\sec(a+bx))^{2/3}\sin(a+bx)}{7bc^2\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 1.02

$$-\frac{3\cot(a+bx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\sec^2(a+bx)\right)\sqrt{-\tan^2(a+bx)}}{4b(c\sec(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[a + b*x]^2]*Sqrt[-Ta
n[a + b*x]^2])/(4*b*(c*Sec[a + b*x])^(4/3))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sec\left(bx+a\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sec(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*sec(b*x+a))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^(2/3)/(c^2*sec(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sec\left(a+bx\right)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sec(a + b*x))**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sec(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c/cos(a + b*x))^(4/3),x)
```

```
[Out] int(1/(c/cos(a + b*x))^(4/3), x)
```

3.37 $\int \sec^n(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)\sec^{-1+n}(a+bx)\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*sec(b*x+a)^(-1+n)*sin
(b*x+a)/b/(1-n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3857, 2722}

$$-\frac{\sin(a+bx)\sec^{n-1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^n,x]
```

```
[Out] -((Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*Sec[a + b*x
]^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sec^n(a+bx)\,dx &= \cos^n(a+bx)\sec^n(a+bx)\int \cos^{-n}(a+bx)\,dx\\ &= -\frac{{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)\sec^{-1+n}(a+bx)\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 0.87

$$\frac{\csc(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(a+bx)\right)\sec^{-1+n}(a+bx)\sqrt{-\tan^2(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^n,x]
```

```
[Out] (Csc[a + b*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b*x]^2]*Sec[a
+ b*x]^(-1 + n)*Sqrt[-Tan[a + b*x]^2])/(b*n)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \sec^n(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^n,x)
```

```
[Out] int(sec(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sec^n(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**n,x)
```

```
[Out] Integral(sec(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{1}{\cos\left(a+b\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(a + b*x))^n,x)
```

```
[Out] int((1/cos(a + b*x))^n, x)
```

3.38 $\int (c\sec(a+bx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{c\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(c\sec(a+bx))^{-1+n}\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -c*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(c*sec(b*x+a))^(-1+
n)*sin(b*x+a)/b/(1-n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$-\frac{c\sin(a+bx)(c\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{b(1-n)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^n,x]
```

```
[Out] -((c*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(c*Sec[a
+ b*x])^(-1 + n)*Sin[a + b*x])/(b*(1 - n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (c\sec(a+bx))^n\,dx &= \left(\frac{\cos(a+bx)}{c}\right)^n(c\sec(a+bx))^n\int\left(\frac{\cos(a+bx)}{c}\right)^{-n}dx\\ &= -\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(c\sec(a+bx))^n\sin(a+bx)}{b(1-n)\sqrt{\sin^2(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.84

$$\frac{\cot(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(a+bx)\right)(c\sec(a+bx))^n\sqrt{-\tan^2(a+bx)}}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^n,x]
```

```
[Out] (Cot[a + b*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[a + b*x]^2]*(c*Sec
[a + b*x])^n*Sqrt[-Tan[a + b*x]^2])/(b*n)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (c\sec(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^n,x)
```

```
[Out] int((c*sec(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((c*sec(b*x + a))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**n,x)
```

```
[Out] Integral((c*sec(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{c}{\cos\left(a+b\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^n,x)
```

```
[Out] int((c/cos(a + b*x))^n, x)
```

3.39 $\int \sec^2(x)^{7/2} \, dx$

Optimal. Leaf size=50

$$\frac{5}{16}\sinh^{-1}(\tan(x))+\frac{5}{16}\sqrt{\sec^2(x)}\,\tan(x)+\frac{5}{24}\sec^2(x)^{3/2}\tan(x)+\frac{1}{6}\sec^2(x)^{5/2}\tan(x)$$

```
[Out] 5/16*arcsinh(tan(x))+5/24*(sec(x)^2)^(3/2)*tan(x)+1/6*(sec(x)^2)^(5/2)*tan(
x)+5/16*(sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4207, 201, 221}

$$\frac{1}{6}\tan(x)\sec^2(x)^{5/2}+\frac{5}{24}\tan(x)\sec^2(x)^{3/2}+\frac{5}{16}\tan(x)\sqrt{\sec^2(x)}+\frac{5}{16}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(7/2),x]
```

```
[Out] (5*ArcSinh[Tan[x]])/16 + (5*Sqrt[Sec[x]^2]*Tan[x])/16 + (5*(Sec[x]^2)^(3/2)
*Tan[x])/24 + ((Sec[x]^2)^(5/2)*Tan[x])/6
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)^{7/2}\,dx &= \text{Subst}\left(\int \left(1+x^2\right)^{5/2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{6}\sec^2(x)^{5/2}\tan(x) + \frac{5}{6}\text{Subst}\left(\int \left(1+x^2\right)^{3/2}\,dx, x, \tan(x)\right)\\ &= \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x) + \frac{5}{8}\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tan(x)\right)\\ &= \frac{5}{16}\sqrt{\sec^2(x)}\tan(x) + \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x) + \frac{5}{16}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\right.\\ &= \frac{5}{16}\sinh^{-1}(\tan(x)) + \frac{5}{16}\sqrt{\sec^2(x)}\tan(x) + \frac{5}{24}\sec^2(x)^{3/2}\tan(x) + \frac{1}{6}\sec^2(x)^{5/2}\tan(x)\end{aligned}$$

Mathematica [A]
time = 0.31, size = 74, normalized size = 1.48

$$\frac{1}{96}\cos(x)\sqrt{\sec^2(x)}\left(-30\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+30\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sec^6(x)(198\sin(x)+85\sin(3x)+15\sin(5x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(7/2),x]
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-30*Log[Cos[x/2] - Sin[x/2]] + 30*Log[Cos[x/2] + Si
n[x/2]] + (Sec[x]^6*(198*Sin[x] + 85*Sin[3*x] + 15*Sin[5*x]))/8))/96
```

Maple [A]
time = 0.40, size = 72, normalized size = 1.44

method	result
default	$\frac{\left(15\left(\cos^6(x)\right)\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)-15\left(\cos^6(x)\right)\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)+15\left(\cos^4(x)\right)\sin(x)+10\left(\cos^2(x)\right)\sin(x)+8\sin(x)\right)\cos(x)\sqrt{}}{6(\cos(2x)+1)^3}$
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\left(15e^{10ix}+85e^{8ix}+198e^{6ix}-198e^{4ix}-85e^{2ix}-15\right)}{24(e^{2ix}+1)^5}-\frac{5\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\ln(e^{ix}-i)\cos(x)}{8}+\frac{5\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\ln(}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/48*(15*cos(x)^6*ln(-(cos(x)-1-sin(x))/sin(x))-15*cos(x)^6*ln(-(cos(x)-1+s
in(x))/sin(x))+15*cos(x)^4*sin(x)+10*cos(x)^2*sin(x)+8*sin(x))*cos(x)*(1/co
s(x)^2)^(7/2)
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.84

$$\frac{1}{6}\left(\tan(x)^2+1\right)^{\frac{5}{2}}\tan(x)+\frac{5}{24}\left(\tan(x)^2+1\right)^{\frac{3}{2}}\tan(x)+\frac{5}{16}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{5}{16}\,\text{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/6*(tan(x)^2 + 1)^(5/2)*tan(x) + 5/24*(tan(x)^2 + 1)^(3/2)*tan(x) + 5/16*s
qrt(tan(x)^2 + 1)*tan(x) + 5/16*arcsinh(tan(x))
```

Fricas [A]
time = 3.29, size = 49, normalized size = 0.98

$$-\frac{15 \cos(x)^6 \log(\sin(x)+1) - 15 \cos(x)^6 \log(-\sin(x)+1) + 2\left(15 \cos(x)^4 + 10 \cos(x)^2 + 8\right)\sin(x)}{96 \cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/96*(15*cos(x)^6*log(sin(x) + 1) - 15*cos(x)^6*log(-sin(x) + 1) + 2*(15*c
os(x)^4 + 10*cos(x)^2 + 8)*sin(x))/cos(x)^6
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 0.44, size = 59, normalized size = 1.18

$$\frac{5 \log(\sin(x)+1)}{32 \operatorname{sgn}(\cos(x))} - \frac{5 \log(-\sin(x)+1)}{32 \operatorname{sgn}(\cos(x))} - \frac{15 \sin(x)^5 - 40 \sin(x)^3 + 33 \sin(x)}{48 \left(\sin(x)^2-1\right)^3 \operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 5/32*log(sin(x) + 1)/sgn(cos(x)) - 5/32*log(-sin(x) + 1)/sgn(cos(x)) - 1/48
*(15*sin(x)^5 - 40*sin(x)^3 + 33*sin(x))/((sin(x)^2 - 1)^3*sgn(cos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{1}{\cos(x)^2}\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2)^(7/2),x)
```

```
[Out] int((1/cos(x)^2)^(7/2), x)
```

3.40 $\int \sec^2(x)^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{3}{8}\sinh^{-1}(\tan(x))+\frac{3}{8}\sqrt{\sec^2(x)}\,\tan(x)+\frac{1}{4}\sec^2(x)^{3/2}\tan(x)$$

```
[Out] 3/8*arcsinh(tan(x))+1/4*(sec(x)^2)^(3/2)*tan(x)+3/8*(sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4207, 201, 221}

$$\frac{1}{4}\tan(x)\sec^2(x)^{3/2}+\frac{3}{8}\tan(x)\sqrt{\sec^2(x)}+\frac{3}{8}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(5/2),x]
```

```
[Out] (3*ArcSinh[Tan[x]])/8 + (3*Sqrt[Sec[x]^2]*Tan[x])/8 + ((Sec[x]^2)^(3/2)*Tan
[x])/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(x)^{5/2}\,dx = \text{Subst}\left(\int \left(1+x^2\right)^{3/2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{4}\sec^2(x)^{3/2}\tan(x) + \frac{3}{4}\text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{3}{8}\sqrt{\sec^2(x)}\tan(x) + \frac{1}{4}\sec^2(x)^{3/2}\tan(x) + \frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right)$$
$$= \frac{3}{8}\sinh^{-1}(\tan(x)) + \frac{3}{8}\sqrt{\sec^2(x)}\tan(x) + \frac{1}{4}\sec^2(x)^{3/2}\tan(x)$$

Mathematica [A]
time = 0.16, size = 68, normalized size = 1.89

$$\frac{1}{16}\cos(x)\sqrt{\sec^2(x)}\left(-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\sec^4(x)(11\sin(x)+3\sin(3x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(5/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[
x/2]] + (Sec[x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.
time = 0.29, size = 64, normalized size = 1.78

method	result
default	$\frac{\left(3\left(\cos^4(x)\right)\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)-3\left(\cos^4(x)\right)\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)+3\left(\cos^2(x)\right)\sin(x)+2\sin(x)\right)\cos(x)\sqrt{2}\,\sqrt{\frac{1}{\cos(2x)+1}}}{2(\cos(2x)+1)^2}$
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(3\,e^{6ix}+11\,e^{4ix}-11\,e^{2ix}-3)}{4(e^{2ix}+1)^3}+\frac{3\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)}{4}-\frac{3\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(3*cos(x)^4*ln(-(cos(x)-1-sin(x))/sin(x))-3*cos(x)^4*ln(-(cos(x)-1+sin(
x))/sin(x))+3*cos(x)^2*sin(x)+2*sin(x))*cos(x)*(1/cos(x)^2)^(5/2)
```

Maxima [A]
time = 0.52, size = 30, normalized size = 0.83

$$\frac{1}{4}\left(\tan(x)^2+1\right)^{\frac{3}{2}}\tan(x)+\frac{3}{8}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{3}{8}\,\text{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(tan(x)^2 + 1)^(3/2)*tan(x) + 3/8*sqrt(tan(x)^2 + 1)*tan(x) + 3/8*arcsi
nh(tan(x))
```

Fricas [A]
time = 2.70, size = 43, normalized size = 1.19

$$-\frac{3\cos(x)^4\log(\sin(x)+1) - 3\cos(x)^4\log(-\sin(x)+1) + 2\left(3\cos(x)^2+2\right)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(
x)^2 + 2)*sin(x))/cos(x)^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sec^2(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(5/2),x)
```

```
[Out] Integral((sec(x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.
time = 0.44, size = 53, normalized size = 1.47

$$\frac{3\log(\sin(x)+1)}{16\,\mathrm{sgn}(\cos(x))} - \frac{3\log(-\sin(x)+1)}{16\,\mathrm{sgn}(\cos(x))} - \frac{3\sin(x)^3 - 5\sin(x)}{8\left(\sin(x)^2-1\right)^2\mathrm{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/sgn(cos(x)) - 3/16*log(-sin(x) + 1)/sgn(cos(x)) - 1/8*
(3*sin(x)^3 - 5*sin(x))/((sin(x)^2 - 1)^2*sgn(cos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\frac{1}{\cos(x)^2}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2)^(5/2),x)
```

```
[Out] int((1/cos(x)^2)^(5/2), x)
```

3.41 $\int \sec^2(x)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\sinh^{-1}(\tan(x)) + \frac{1}{2}\sqrt{\sec^2(x)}\,\tan(x)$$

```
[Out] 1/2*arcsinh(tan(x))+1/2*(sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4207, 201, 221}

$$\frac{1}{2}\tan(x)\sqrt{\sec^2(x)} + \frac{1}{2}\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(3/2),x]
```

```
[Out] ArcSinh[Tan[x]]/2 + (Sqrt[Sec[x]^2]*Tan[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(x)^{3/2}\,dx = \text{Subst}\left(\int \sqrt{1+x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\sqrt{\sec^2(x)}\tan(x) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\sinh^{-1}(\tan(x)) + \frac{1}{2}\sqrt{\sec^2(x)}\tan(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(22) = 44.
time = 0.07, size = 52, normalized size = 2.36

$$\frac{1}{2}\cos(x)\sqrt{\sec^2(x)}\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sec(x)\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(3/2),x]
```

```
[Out] (Cos[x]*Sqrt[Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]
] + Sec[x]*Tan[x]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(16) = 32.
time = 0.20, size = 55, normalized size = 2.50

method	result	size
default	$-\frac{\left((\cos^2(x))\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)-(\cos^2(x))\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)-\sin(x)\right)\cos(x)\sqrt{2}\,\sqrt{\frac{1}{\cos(2x)+1}}}{\cos(2x)+1}$	55
risch	$-\frac{i\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}-1)}{e^{2ix}+1}-\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)+\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(cos(x)^2*ln(-(cos(x)-1+sin(x))/sin(x))-cos(x)^2*ln(-(cos(x)-1-sin(x))
/sin(x))-sin(x))*cos(x)*(1/cos(x)^2)^(3/2)
```

Maxima [A]
time = 0.51, size = 18, normalized size = 0.82

$$\frac{1}{2}\sqrt{\tan(x)^2+1}\,\tan(x)+\frac{1}{2}\,\text{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(tan(x)^2 + 1)*tan(x) + 1/2*arcsinh(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.
time = 2.74, size = 34, normalized size = 1.55

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)+2\sin(x)}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/cos(
x)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sec^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(3/2),x)
```

```
[Out] Integral((sec(x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.
time = 0.43, size = 44, normalized size = 2.00

$$\frac{\log(\sin(x)+1)}{4\,\mathrm{sgn}(\cos(x))}-\frac{\log(-\sin(x)+1)}{4\,\mathrm{sgn}(\cos(x))}-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)\mathrm{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/sgn(cos(x)) - 1/4*log(-sin(x) + 1)/sgn(cos(x)) - 1/2*si
n(x)/((sin(x)^2 - 1)*sgn(cos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int\left(\frac{1}{\cos(x)^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2)^(3/2),x)
```

```
[Out] int((1/cos(x)^2)^(3/2), x)
```

3.42 $\int \sqrt{\sec^2(x)}\, dx$

Optimal. Leaf size=3

$$\sinh^{-1}(\tan(x))$$

```
[Out] arcsinh(tan(x))
```

Rubi [A]
time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4207, 221}

$$\sinh^{-1}(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[x]^2],x]
```

```
[Out] ArcSinh[Tan[x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec^2(x)}\, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\, dx, x, \tan(x)\right)\\ &= \sinh^{-1}(\tan(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(3) = 6.
time = 0.01, size = 44, normalized size = 14.67

$$\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)\sqrt{\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[x]^2],x]
```

```
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[Sec[x]^2
]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(3) = 6$.
time = 0.19, size = 21, normalized size = 7.00

method	result	size
default	$-2\cos(x)\sqrt{2}\,\sqrt{\frac{1}{\cos(2x)+1}}\,\operatorname{arctanh}\left(\frac{-1+\cos(x)}{\sin(x)}\right)$	21
risch	$2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}+i)\cos(x)-2\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,\ln(e^{ix}-i)\cos(x)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*cos(x)*(1/cos(x)^2)^(1/2)*arctanh((-1+cos(x))/sin(x))
```

Maxima [A]
time = 0.54, size = 3, normalized size = 1.00

$$\operatorname{arsinh}(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(tan(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(3) = 6$.
time = 2.82, size = 17, normalized size = 5.67

$$-\frac{1}{2}\log(\sin(x)+1)+\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sec^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sec(x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(3) = 6$.
time = 0.45, size = 35, normalized size = 11.67

$$\frac{\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right)}{4\,\mathrm{sgn}(\cos(x))} - \frac{\log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right)}{4\,\mathrm{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(1/sin(x) + sin(x) + 2))/sgn(cos(x)) - 1/4*log(abs(1/sin(x) + si
n(x) - 2))/sgn(cos(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.33

$$\int \sqrt{\frac{1}{\cos(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)^2)^(1/2),x)
```

```
[Out] int((1/cos(x)^2)^(1/2), x)
```

3.43 $\int \frac{1}{\sqrt{\sec^2(x)}}\,dx$

Optimal. Leaf size=11

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

```
[Out] tan(x)/(sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 197}

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{\sqrt{\sec^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sec[x]^2],x]
[Out] Tan[x]/Sqrt[Sec[x]^2]
```

Maple [A]
time = 0.22, size = 14, normalized size = 1.27

method	result	size
default	$\frac{\sin(x)\sqrt{2}}{2\sqrt{\frac{1}{\cos(2x)+1}}\,\cos(x)}$	14
risch	$-\frac{i\mathrm{e}^{2ix}}{2\sqrt{\frac{\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)}+\frac{i}{2\sqrt{\frac{\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] sin(x)/(1/cos(x)^2)^(1/2)/cos(x)
```

Maxima [A]
time = 0.30, size = 11, normalized size = 1.00

$$\frac{\tan\left(x\right)}{\sqrt{\tan\left(x\right)^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="maxima")
[Out] tan(x)/sqrt(tan(x)^2 + 1)
```

Fricas [A]
time = 2.89, size = 4, normalized size = 0.36

$$-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="fricas")
[Out] -sin(x)
```

Sympy [A]
time = 0.19, size = 10, normalized size = 0.91

$$\frac{\tan\left(x\right)}{\sqrt{\sec^2\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(1/2),x)
```

```
[Out] tan(x)/sqrt(sec(x)**2)
```

Giac [A]
time = 0.43, size = 6, normalized size = 0.55

$$\mathrm{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sgn(cos(x))*sin(x)
```

Mupad [B]
time = 0.16, size = 12, normalized size = 1.09

$$\frac{\sqrt{2}\,\sin(2\,x)}{2\,\sqrt{2\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*sin(2*x))/(2*(2*cos(x)^2)^(1/2))
```

3.44 $\int \frac{1}{\sec^2(x)^{3/2}} \, dx$

Optimal. Leaf size=29

$$\frac{\tan(x)}{3\sec^2(x)^{3/2}} + \frac{2\tan(x)}{3\sqrt{\sec^2(x)}}$$

```
[Out] 1/3*tan(x)/(sec(x)^2)^(3/2)+2/3*tan(x)/(sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4207, 198, 197}

$$\frac{2\tan(x)}{3\sqrt{\sec^2(x)}} + \frac{\tan(x)}{3\sec^2(x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(-3/2),x]
```

```
[Out] Tan[x]/(3*(Sec[x]^2)^(3/2)) + (2*Tan[x])/(3*Sqrt[Sec[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^2(x)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{3\sec^2(x)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{3\sec^2(x)^{3/2}} + \frac{2\tan(x)}{3\sqrt{\sec^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.79

$$\frac{\sec(x)(9\sin(x)+\sin(3x))}{12\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(-3/2),x]
```

```
[Out] (Sec[x]*(9*Sin[x] + Sin[3*x]))/(12*Sqrt[Sec[x]^2])
```

Maple [A]
time = 0.20, size = 21, normalized size = 0.72

method	result	size
default	$\frac{\sin(x)\left(\cos^2(x)+2\right)\left(\cos(2x)+1\right)\sqrt{2}}{12\cos(x)^3\sqrt{\frac{1}{\cos(2x)+1}}}$	21
risch	$-\frac{i e^{4ix}}{24(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}}-\frac{3i e^{2ix}}{8\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{3i}{8\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)}+\frac{i e^{-2ix}}{24(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sin(x)*(cos(x)^2+2)/cos(x)^3/(1/cos(x)^2)^(3/2)
```

Maxima [A]
time = 0.31, size = 25, normalized size = 0.86

$$\frac{2\,\tan\left(x\right)}{3\,\sqrt{\tan\left(x\right)^2+1}}+\frac{\tan\left(x\right)}{3\,\left(\tan\left(x\right)^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*tan(x)/sqrt(tan(x)^2 + 1) + 1/3*tan(x)/(tan(x)^2 + 1)^(3/2)
```

Fricas [A]
time = 2.96, size = 10, normalized size = 0.34

$$-\frac{1}{3}\left(\cos\left(x\right)^{2}+2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A]
time = 0.32, size = 27, normalized size = 0.93

$$\frac{2\tan^{3}\left(x\right)}{3\left(\sec^{2}\left(x\right)\right)^{\frac{3}{2}}}+\frac{\tan\left(x\right)}{\left(\sec^{2}\left(x\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(3/2),x)
```

```
[Out] 2*tan(x)**3/(3*(sec(x)**2)**(3/2)) + tan(x)/(sec(x)**2)**(3/2)
```

Giac [A]
time = 0.44, size = 16, normalized size = 0.55

$$-\frac{1}{3}\operatorname{sgn}(\cos\left(x\right))\sin\left(x\right)^{3}+\operatorname{sgn}(\cos\left(x\right))\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*sgn(cos(x))*sin(x)^3 + sgn(cos(x))*sin(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{{\left(\frac{1}{{\cos\left(x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(1/cos(x)^2)^(3/2), x)
```

3.45 $\int \frac{1}{\sec^2(x)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{\tan(x)}{5\sec^2(x)^{5/2}}+\frac{4\tan(x)}{15\sec^2(x)^{3/2}}+\frac{8\tan(x)}{15\sqrt{\sec^2(x)}}$$

```
[Out] 1/5*tan(x)/(sec(x)^2)^(5/2)+4/15*tan(x)/(sec(x)^2)^(3/2)+8/15*tan(x)/(sec(x
)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4207, 198, 197}

$$\frac{8\tan(x)}{15\sqrt{\sec^2(x)}}+\frac{4\tan(x)}{15\sec^2(x)^{3/2}}+\frac{\tan(x)}{5\sec^2(x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(-5/2),x]
```

```
[Out] Tan[x]/(5*(Sec[x]^2)^(5/2)) + (4*Tan[x])/(15*(Sec[x]^2)^(3/2)) + (8*Tan[x])
/(15*Sqrt[Sec[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^2(x)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)^{7/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{5\sec^2(x)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{(1+x^2)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{5\sec^2(x)^{5/2}} + \frac{4\tan(x)}{15\sec^2(x)^{3/2}} + \frac{8}{15}\text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{5\sec^2(x)^{5/2}} + \frac{4\tan(x)}{15\sec^2(x)^{3/2}} + \frac{8\tan(x)}{15\sqrt{\sec^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.72

$$\frac{\sec(x)(150\sin(x)+25\sin(3x)+3\sin(5x))}{240\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(-5/2),x]
[Out] (Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/(240*Sqrt[Sec[x]^2])
```

Maple [A]
time = 0.22, size = 29, normalized size = 0.67

method	result
default	$\frac{\sin(x)(3(\cos^4(x))+4(\cos^2(x))+8)(\cos(2x)+1)^2\sqrt{2}}{120\cos(x)^5\sqrt{\frac{1}{\cos(2x)+1}}}$
risch	$-\frac{i e^{6ix}}{160(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}} - \frac{5i e^{2ix}}{16\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)} + \frac{5i}{16\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)} + \frac{5i e^{-2ix}}{96(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}} - \frac{}{240}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/15*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/cos(x)^5/(1/cos(x)^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.86

$$\frac{8\tan(x)}{15\sqrt{\tan(x)^2+1}} + \frac{4\tan(x)}{15(\tan(x)^2+1)^{\frac{3}{2}}} + \frac{\tan(x)}{5(\tan(x)^2+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*tan(x)/sqrt(tan(x)^2 + 1) + 4/15*tan(x)/(tan(x)^2 + 1)^(3/2) + 1/5*tan
(x)/(tan(x)^2 + 1)^(5/2)
```

Fricas [**A**]
time = 3.54, size = 18, normalized size = 0.42

$$-\frac{1}{15}\left(3\,\cos\left(x\right)^{4}+4\,\cos\left(x\right)^{2}+8\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [**A**]
time = 1.52, size = 44, normalized size = 1.02

$$\frac{8\tan^{5}\left(x\right)}{15\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{4\tan^{3}\left(x\right)}{3\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{\tan\left(x\right)}{\left(\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(5/2),x)
```

```
[Out] 8*tan(x)**5/(15*(sec(x)**2)**(5/2)) + 4*tan(x)**3/(3*(sec(x)**2)**(5/2)) +
tan(x)/(sec(x)**2)**(5/2)
```

Giac [**A**]
time = 0.42, size = 25, normalized size = 0.58

$$\frac{1}{5}\,\mathrm{sgn}(\cos\left(x\right))\sin\left(x\right)^{5}-\frac{2}{3}\,\mathrm{sgn}(\cos\left(x\right))\sin\left(x\right)^{3}+\mathrm{sgn}(\cos\left(x\right))\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*sgn(cos(x))*sin(x)^5 - 2/3*sgn(cos(x))*sin(x)^3 + sgn(cos(x))*sin(x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\left(\frac{1}{\cos(x)^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2)^(5/2),x)
```

```
[Out] int(1/(1/cos(x)^2)^(5/2), x)
```

3.46 $\int \frac{1}{\sec^2(x)^{7/2}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan(x)}{7\sec^2(x)^{7/2}}+\frac{6\tan(x)}{35\sec^2(x)^{5/2}}+\frac{8\tan(x)}{35\sec^2(x)^{3/2}}+\frac{16\tan(x)}{35\sqrt{\sec^2(x)}}$$

```
[Out] 1/7*tan(x)/(sec(x)^2)^(7/2)+6/35*tan(x)/(sec(x)^2)^(5/2)+8/35*tan(x)/(sec(x
)^2)^(3/2)+16/35*tan(x)/(sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4207, 198, 197}

$$\frac{16\tan(x)}{35\sqrt{\sec^2(x)}}+\frac{8\tan(x)}{35\sec^2(x)^{3/2}}+\frac{6\tan(x)}{35\sec^2(x)^{5/2}}+\frac{\tan(x)}{7\sec^2(x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2)^(-7/2),x]
```

```
[Out] Tan[x]/(7*(Sec[x]^2)^(7/2)) + (6*Tan[x])/(35*(Sec[x]^2)^(5/2)) + (8*Tan[x])
/(35*(Sec[x]^2)^(3/2)) + (16*Tan[x])/(35*Sqrt[Sec[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^2(x)^{7/2}}\,dx &= \text{Subst}\left(\int \frac{1}{(1+x^2)^{9/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(1+x^2)^{7/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{24}{35}\text{Subst}\left(\int \frac{1}{(1+x^2)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{8\tan(x)}{35\sec^2(x)^{3/2}} + \frac{16}{35}\text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{7\sec^2(x)^{7/2}} + \frac{6\tan(x)}{35\sec^2(x)^{5/2}} + \frac{8\tan(x)}{35\sec^2(x)^{3/2}} + \frac{16\tan(x)}{35\sqrt{\sec^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.65

$$\frac{\sec(x)(1225\sin(x) + 245\sin(3x) + 49\sin(5x) + 5\sin(7x))}{2240\sqrt{\sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2)^(-7/2),x]
[Out] (Sec[x]*(1225*Sin[x] + 245*Sin[3*x] + 49*Sin[5*x] + 5*Sin[7*x]))/(2240*Sqrt
[Sec[x]^2])
```

Maple [A]
time = 0.23, size = 35, normalized size = 0.61

method	result
default	$\frac{\sin(x)\left(5\left(\cos^6(x)\right)+6\left(\cos^4(x)\right)+8\left(\cos^2(x)\right)+16\right)\left(\cos(2x)+1\right)^3\sqrt{2}}{560\cos(x)^7\sqrt{\frac{1}{\cos(2x)+1}}}$
risch	$-\frac{i e^{8ix}}{896(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}} - \frac{35 i e^{2ix}}{128\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)} + \frac{35i}{128\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}\,(e^{2ix}+1)} + \frac{7 i e^{-2ix}}{128(e^{2ix}+1)\sqrt{\frac{e^{2ix}}{(e^{2ix}+1)^2}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/35*sin(x)*(5*cos(x)^6+6*cos(x)^4+8*cos(x)^2+16)/cos(x)^7/(1/cos(x)^2)^(7/
2)
```

Maxima [A]
time = 0.30, size = 49, normalized size = 0.86

$$\frac{16\tan(x)}{35\sqrt{\tan(x)^2+1}}+\frac{8\tan(x)}{35\left(\tan(x)^2+1\right)^{\frac{3}{2}}}+\frac{6\tan(x)}{35\left(\tan(x)^2+1\right)^{\frac{5}{2}}}+\frac{\tan(x)}{7\left(\tan(x)^2+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 16/35*tan(x)/sqrt(tan(x)^2 + 1) + 8/35*tan(x)/(tan(x)^2 + 1)^(3/2) + 6/35*t
an(x)/(tan(x)^2 + 1)^(5/2) + 1/7*tan(x)/(tan(x)^2 + 1)^(7/2)
```

Fricas [A]
time = 2.46, size = 24, normalized size = 0.42

$$-\frac{1}{35}\left(5\cos(x)^6+6\cos(x)^4+8\cos(x)^2+16\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)
```

Sympy [A]
time = 12.28, size = 60, normalized size = 1.05

$$\frac{16\tan^7(x)}{35\left(\sec^2(x)\right)^{\frac{7}{2}}}+\frac{8\tan^5(x)}{5\left(\sec^2(x)\right)^{\frac{7}{2}}}+\frac{2\tan^3(x)}{\left(\sec^2(x)\right)^{\frac{7}{2}}}+\frac{\tan(x)}{\left(\sec^2(x)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)**2)**(7/2),x)
```

```
[Out] 16*tan(x)**7/(35*(sec(x)**2)**(7/2)) + 8*tan(x)**5/(5*(sec(x)**2)**(7/2)) +
 2*tan(x)**3/(sec(x)**2)**(7/2) + tan(x)/(sec(x)**2)**(7/2)
```

Giac [A]
time = 0.44, size = 34, normalized size = 0.60

$$-\frac{1}{7}\operatorname{sgn}(\cos(x))\sin(x)^7+\frac{3}{5}\operatorname{sgn}(\cos(x))\sin(x)^5-\operatorname{sgn}(\cos(x))\sin(x)^3+\operatorname{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/7*sgn(cos(x))*sin(x)^7 + 3/5*sgn(cos(x))*sin(x)^5 - sgn(cos(x))*sin(x)^3
 + sgn(cos(x))*sin(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{1}{\cos(x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/cos(x)^2)^(7/2),x)
```

```
[Out] int(1/(1/cos(x)^2)^(7/2), x)
```

3.47 $\int \left(a\sec^2(x)\right)^{7/2}\,dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{5}{16}a^3\sqrt{a\sec^2(x)}\,\tan(x)+\frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x)+\frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x$$

```
[Out] 5/16*a^(7/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))+5/24*a^2*(a*sec(x)^
2)^(3/2)*tan(x)+1/6*a*(a*sec(x)^2)^(5/2)*tan(x)+5/16*a^3*(a*sec(x)^2)^(1/2)
*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 212}

$$\frac{5}{16}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{5}{16}a^3\tan(x)\sqrt{a\sec^2(x)}+\frac{5}{24}a^2\tan(x)\left(a\sec^2(x)\right)^{3/2}+\frac{1}{6}a\tan(x)\left(a\sec^2(x)\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(7/2),x]
```

```
[Out] (5*a^(7/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/16 + (5*a^3*Sqrt[a*S
ec[x]^2]*Tan[x])/16 + (5*a^2*(a*Sec[x]^2)^(3/2)*Tan[x])/24 + (a*(a*Sec[x]^2
)^(5/2)*Tan[x])/6
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\sec^2(x)\right)^{7/2}\,dx = a\text{Subst}\left(\int \left(a+ax^2\right)^{5/2}\,dx, x, \tan(x)\right)$$
$$= \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{6}\left(5a^2\right)\text{Subst}\left(\int \left(a+ax^2\right)^{3/2}\,dx, x, \tan(x)\right)$$
$$= \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \sqrt{a+ax^2}\,$$
$$= \frac{5}{16}a^3\sqrt{a\sec^2(x)}\,\tan(x) + \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{16}$$
$$= \frac{5}{16}a^3\sqrt{a\sec^2(x)}\,\tan(x) + \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan(x) + \frac{1}{6}a\left(a\sec^2(x)\right)^{5/2}\tan(x) + \frac{1}{16}$$
$$= \frac{5}{16}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right) + \frac{5}{16}a^3\sqrt{a\sec^2(x)}\,\tan(x) + \frac{5}{24}a^2\left(a\sec^2(x)\right)^{3/2}\tan($$

Mathematica [A]
time = 0.15, size = 78, normalized size = 0.93

$$\frac{1}{96}\cos^7(x)\left(a\sec^2(x)\right)^{7/2}\left(-30\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+30\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\sec^6(x)(198\sin(x)+85\sin(3x)+15\sin(5x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]^7*(a*Sec[x]^2)^(7/2)*(-30*Log[Cos[x/2] - Sin[x/2]] + 30*Log[Cos[x/2
] + Sin[x/2]] + (Sec[x]^6*(198*Sin[x] + 85*Sin[3*x] + 15*Sin[5*x]))/8))/96
```

Maple [A]
time = 0.36, size = 74, normalized size = 0.88

method	result
default	$\frac{\left(15\left(\cos^6(x)\right)\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)-15\left(\cos^6(x)\right)\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)+15\left(\cos^4(x)\right)\sin(x)+10\left(\cos^2(x)\right)\sin(x)+8\sin(x)\right)\cos(x)\left(\right.}{48}$
risch	$-\frac{ia^3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(15\,e^{10ix}+85\,e^{8ix}+198\,e^{6ix}-198\,e^{4ix}-85\,e^{2ix}-15\right)}{24\left(e^{2ix}+1\right)^5}+\frac{5a^3\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}+i\right)\cos(x)}{8}-\frac{5a^3\sqrt{\frac{a\,e^{2}}{\left(e^{2ix}+\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(15*cos(x)^6*ln(-(cos(x)-1-sin(x))/sin(x))-15*cos(x)^6*ln(-(cos(x)-1+s
in(x))/sin(x))+15*cos(x)^4*sin(x)+10*cos(x)^2*sin(x)+8*sin(x))*cos(x)*(a/co
s(x)^2)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2175 vs. $2(64) = 128$.
time = 4.49, size = 2175, normalized size = 25.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/96*(2040*a^3*cos(3*x)*sin(2*x) + 360*a^3*cos(x)*sin(2*x) - 360*a^3*cos(2*
x)*sin(x) - 60*a^3*sin(x) + 4*(15*a^3*sin(11*x) + 85*a^3*sin(9*x) + 198*a^3
*sin(7*x) - 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(12*x) -
 60*(6*a^3*sin(10*x) + 15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x)
+ 6*a^3*sin(2*x))*cos(11*x) + 24*(85*a^3*sin(9*x) + 198*a^3*sin(7*x) - 198*
a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(10*x) - 340*(15*a^3*sin
(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*cos(9*x) + 60*(
198*a^3*sin(7*x) - 198*a^3*sin(5*x) - 85*a^3*sin(3*x) - 15*a^3*sin(x))*cos(
8*x) - 792*(20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*cos(7*x) -
80*(198*a^3*sin(5*x) + 85*a^3*sin(3*x) + 15*a^3*sin(x))*cos(6*x) + 2376*(5*
a^3*sin(4*x) + 2*a^3*sin(2*x))*cos(5*x) - 300*(17*a^3*sin(3*x) + 3*a^3*sin(
x))*cos(4*x) + 15*(a^3*cos(12*x)^2 + 36*a^3*cos(10*x)^2 + 225*a^3*cos(8*x)^
2 + 400*a^3*cos(6*x)^2 + 225*a^3*cos(4*x)^2 + 36*a^3*cos(2*x)^2 + a^3*sin(1
2*x)^2 + 36*a^3*sin(10*x)^2 + 225*a^3*sin(8*x)^2 + 400*a^3*sin(6*x)^2 + 225
*a^3*sin(4*x)^2 + 180*a^3*sin(4*x)*sin(2*x) + 36*a^3*sin(2*x)^2 + 12*a^3*co
s(2*x) + a^3 + 2*(6*a^3*cos(10*x) + 15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*
a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(12*x) + 12*(15*a^3*cos(8*x) + 20*a
^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(10*x) + 30*(20*a^
3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(8*x) + 40*(15*a^3*
cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(6*x) + 30*(6*a^3*cos(2*x) + a^3)*cos(4
*x) + 2*(6*a^3*sin(10*x) + 15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4
*x) + 6*a^3*sin(2*x))*sin(12*x) + 12*(15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 1
5*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(10*x) + 30*(20*a^3*sin(6*x) + 15*a^3*s
in(4*x) + 6*a^3*sin(2*x))*sin(8*x) + 120*(5*a^3*sin(4*x) + 2*a^3*sin(2*x))*
sin(6*x))*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 15*(a^3*cos(12*x)^2 + 3
6*a^3*cos(10*x)^2 + 225*a^3*cos(8*x)^2 + 400*a^3*cos(6*x)^2 + 225*a^3*cos(4
*x)^2 + 36*a^3*cos(2*x)^2 + a^3*sin(12*x)^2 + 36*a^3*sin(10*x)^2 + 225*a^3*
```

```
sin(8*x)^2 + 400*a^3*sin(6*x)^2 + 225*a^3*sin(4*x)^2 + 180*a^3*sin(4*x)*sin
(2*x) + 36*a^3*sin(2*x)^2 + 12*a^3*cos(2*x) + a^3 + 2*(6*a^3*cos(10*x) + 15
*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*c
os(12*x) + 12*(15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*
cos(2*x) + a^3)*cos(10*x) + 30*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*c
os(2*x) + a^3)*cos(8*x) + 40*(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*cos(6
*x) + 30*(6*a^3*cos(2*x) + a^3)*cos(4*x) + 2*(6*a^3*sin(10*x) + 15*a^3*sin(
8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(12*x) + 12*(
15*a^3*sin(8*x) + 20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(1
0*x) + 30*(20*a^3*sin(6*x) + 15*a^3*sin(4*x) + 6*a^3*sin(2*x))*sin(8*x) + 1
20*(5*a^3*sin(4*x) + 2*a^3*sin(2*x))*sin(6*x))*log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1) - 4*(15*a^3*cos(11*x) + 85*a^3*cos(9*x) + 198*a^3*cos(7*x) - 19
8*a^3*cos(5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(12*x) + 60*(6*a^3*cos
(10*x) + 15*a^3*cos(8*x) + 20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*
x) + a^3)*sin(11*x) - 24*(85*a^3*cos(9*x) + 198*a^3*cos(7*x) - 198*a^3*cos(
5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(10*x) + 340*(15*a^3*cos(8*x) +
20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(9*x) - 60*(19
8*a^3*cos(7*x) - 198*a^3*cos(5*x) - 85*a^3*cos(3*x) - 15*a^3*cos(x))*sin(8*
x) + 792*(20*a^3*cos(6*x) + 15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(7*x
) + 80*(198*a^3*cos(5*x) + 85*a^3*cos(3*x) + 15*a^3*cos(x))*sin(6*x) - 792*
(15*a^3*cos(4*x) + 6*a^3*cos(2*x) + a^3)*sin(5*x) + 300*(17*a^3*cos(3*x) +
3*a^3*cos(x))*sin(4*x) - 340*(6*a^3*cos(2*x) + a^3)*sin(3*x))*sqrt(a)/(2*(6
*cos(10*x) + 15*cos(8*x) + 20*cos(6*x) + 15*cos(4*x) + 6*cos(2*x) + 1)*cos(
12*x) + cos(12*x)^2 + 12*(15*cos(8*x) + 20*cos(6*x) + 15*cos(4*x) + 6*cos(2
*x) + 1)*cos(10*x) + 36*cos(10*x)^2 + 30*(20*cos(6*x) + 15*cos(4*x) + 6*cos
(2*x) + 1)*cos(8*x) + 225*cos(8*x)^2 + 40*(15*cos(4*x) + 6*cos(2*x) + 1)*co
s(6*x) + 400*cos(6*x)^2 + 30*(6*cos(2*x) + 1)*cos(4*x) + 225*cos(4*x)^2 + 3
6*cos(2*x)^2 + 2*(6*sin(10*x) + 15*sin(8*x) + 20*sin(6*x) + 15*sin(4*x) + 6
*sin(2*x))*sin(12*x) + sin(12*x)^2 + 12*(15*sin(8*x) + 20*sin(6*x) + 15*sin
(4*x) + 6*sin(2*x))*sin(10*x) + 36*sin(10*x)^2 + 30*(20*sin(6*x) + 15*sin(4
*x) + 6*sin(2*x))*sin(8*x) + 225*sin(8*x)^2 + 120*(5*sin(4*x) + 2*sin(2*x))
*sin(6*x) + 400*sin(6*x)^2 + 225*sin(4*x)^2 + 180*sin(4*x)*sin(2*x) + 36*si
n(2*x)^2 + 12*cos(2*x) + 1)
```

Fricas [A]
time = 3.56, size = 65, normalized size = 0.77

$$-\frac{\left(15\,a^3\cos\left(x\right)^6\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(15\,a^3\cos\left(x\right)^4+10\,a^3\cos\left(x\right)^2+8\,a^3\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^2}}}{96\,\cos\left(x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(7/2),x, algorithm="fricas")
[Out] -1/96*(15*a^3*cos(x)^6*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(15*a^3*cos(x)^4
 + 10*a^3*cos(x)^2 + 8*a^3)*sin(x))*sqrt(a/cos(x)^2)/cos(x)^5
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 0.45, size = 79, normalized size = 0.94

$$\frac{1}{96}\left(15\,a^{3}\log\left(\sin\left(x\right)+1\right)\mathrm{sgn}(\cos\left(x\right))-15\,a^{3}\log\left(-\sin\left(x\right)+1\right)\mathrm{sgn}(\cos\left(x\right))-\frac{2\left(15\,a^{3}\mathrm{sgn}(\cos\left(x\right))\sin\left(x\right)^{5}-40\,a^{3}\mathrm{sgn}(\cos\left(x\right))\sin\left(x\right)^{3}+33\,a^{3}\mathrm{sgn}(\cos\left(x\right))\sin\left(x\right)\right)}{\left(\sin\left(x\right)^{2}-1\right)^{3}}\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/96*(15*a^3*log(sin(x) + 1)*sgn(cos(x)) - 15*a^3*log(-sin(x) + 1)*sgn(cos(
x)) - 2*(15*a^3*sgn(cos(x))*sin(x)^5 - 40*a^3*sgn(cos(x))*sin(x)^3 + 33*a^3
*sgn(cos(x))*sin(x))/(sin(x)^2 - 1)^3)*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{a}{{\cos\left(x\right)}^2}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^2)^(7/2),x)
```

```
[Out] int((a/cos(x)^2)^(7/2), x)
```

3.48 $\int \left(a\sec^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{3}{8}a^2\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)$$

```
[Out] 3/8*a^(5/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))+1/4*a*(a*sec(x)^2)^(
3/2)*tan(x)+3/8*a^2*(a*sec(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4207, 201, 223, 212}

$$\frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{3}{8}a^2\tan(x)\sqrt{a\sec^2(x)}+\frac{1}{4}a\tan(x)\left(a\sec^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/8 + (3*a^2*Sqrt[a*Se
c[x]^2]*Tan[x])/8 + (a*(a*Sec[x]^2)^(3/2)*Tan[x])/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a\sec^2(x)\right)^{5/2}\,dx &= a\text{Subst}\left(\int \left(a+ax^2\right)^{3/2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(x)\right)\\
&= \frac{3}{8}a^2\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x,\right.\\
&= \frac{3}{8}a^2\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x,\right.\\
&= \frac{3}{8}a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{3}{8}a^2\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{4}a\left(a\sec^2(x)\right)^{3/2}\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 1.11

$$\frac{1}{16}\cos^5(x)\left(a\sec^2(x)\right)^{5/2}\left(-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\sec^4(x)(11\sin(x)+3\sin(3x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(5/2),x]
```

```
[Out] (Cos[x]^5*(a*Sec[x]^2)^(5/2)*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2]
+ Sin[x/2]] + (Sec[x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/16
```

Maple [A]
time = 0.27, size = 66, normalized size = 1.02

method	result
default	$\frac{\left(3\left(\cos^4(x)\right)\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)-3\left(\cos^4(x)\right)\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)+3\left(\cos^2(x)\right)\sin(x)+2\sin(x)\right)\cos(x)\left(\frac{a}{\cos(x)^2}\right)^{\frac{5}{2}}}{8}$
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(3\,e^{6ix}+11\,e^{4ix}-11\,e^{2ix}-3\right)}{4\left(e^{2ix}+1\right)^3}+\frac{3a^2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}+i\right)\cos(x)}{4}-\frac{3a^2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}-i\right)\cos(x)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(3*cos(x)^4*ln(-(cos(x)-1-sin(x))/sin(x))-3*cos(x)^4*ln(-(cos(x)-1+sin(
x))/sin(x))+3*cos(x)^2*sin(x)+2*sin(x))*cos(x)*(a/cos(x)^2)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1111 vs. $2(49) = 98$.
time = 0.85, size = 1111, normalized size = 17.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(176*a^2*cos(3*x)*sin(2*x) + 48*a^2*cos(x)*sin(2*x) - 48*a^2*cos(2*x)*
sin(x) - 12*a^2*sin(x) + 4*(3*a^2*sin(7*x) + 11*a^2*sin(5*x) - 11*a^2*sin(3
*x) - 3*a^2*sin(x))*cos(8*x) - 24*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^2*
sin(2*x))*cos(7*x) + 16*(11*a^2*sin(5*x) - 11*a^2*sin(3*x) - 3*a^2*sin(x))*
cos(6*x) - 88*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*cos(5*x) - 24*(11*a^2*sin(3
*x) + 3*a^2*sin(x))*cos(4*x) + 3*(a^2*cos(8*x)^2 + 16*a^2*cos(6*x)^2 + 36*a
^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 16*a^2*sin(6*x)^2 + 36
*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*sin(2*x)^2 + 8*a^2*cos(
2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos
(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(6*x) + 12*(4*a^2*cos(
2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^2*sin(2*x))
*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6*x))*log(cos(x)^2 + s
in(x)^2 + 2*sin(x) + 1) - 3*(a^2*cos(8*x)^2 + 16*a^2*cos(6*x)^2 + 36*a^2*co
s(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 16*a^2*sin(6*x)^2 + 36*a^2*
sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*sin(2*x)^2 + 8*a^2*cos(2*x)
+ a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(8*x)
 + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(6*x) + 12*(4*a^2*cos(2*x)
+ a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(
8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6*x))*log(cos(x)^2 + sin(x)
^2 - 2*sin(x) + 1) - 4*(3*a^2*cos(7*x) + 11*a^2*cos(5*x) - 11*a^2*cos(3*x)
- 3*a^2*cos(x))*sin(8*x) + 12*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^2*cos(
2*x) + a^2)*sin(7*x) - 16*(11*a^2*cos(5*x) - 11*a^2*cos(3*x) - 3*a^2*cos(x)
)*sin(6*x) + 44*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*sin(5*x) + 24*(11*a
^2*cos(3*x) + 3*a^2*cos(x))*sin(4*x) - 44*(4*a^2*cos(2*x) + a^2)*sin(3*x))*
sqrt(a)/(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2
 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(2*x
) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(4*x
) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin(6*
x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^2 +
 8*cos(2*x) + 1)
```

Fricas [A]

time = 2.96, size = 56, normalized size = 0.86

$$-\frac{\left(3 a^{2} \cos \left(x\right)^{4} \log \left(-\frac{\sin\left(x\right)-1}{\sin\left(x\right)+1}\right)-2 \left(3 a^{2} \cos \left(x\right)^{2}+2 a^{2}\right) \sin \left(x\right)\right) \sqrt{\frac{a}{\cos \left(x\right)^{2}}}}{16 \cos \left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*a^2*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*a^2*cos(x)^2 +
 2*a^2)*sin(x))*sqrt(a/cos(x)^2)/cos(x)^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sec^{2}\left(x\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sec(x)**2)**(5/2), x)
```

Giac [A]
time = 0.42, size = 67, normalized size = 1.03

$$\frac{1}{16} \left(3 a^{2} \log \left(\sin \left(x\right)+1\right) \mathrm{sgn}(\cos \left(x\right))-3 a^{2} \log \left(-\sin \left(x\right)+1\right) \mathrm{sgn}(\cos \left(x\right))-\frac{2 \left(3 a^{2} \mathrm{sgn}(\cos \left(x\right)) \sin \left(x\right)^{3}-5 a^{2} \mathrm{sgn}(\cos \left(x\right)) \sin \left(x\right)\right)}{\left(\sin \left(x\right)^{2}-1\right)^{2}}\right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*a^2*log(sin(x) + 1)*sgn(cos(x)) - 3*a^2*log(-sin(x) + 1)*sgn(cos(x)
) - 2*(3*a^2*sgn(cos(x))*sin(x)^3 - 5*a^2*sgn(cos(x))*sin(x))/(sin(x)^2 - 1
)^2)*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{a}{{\cos \left(x\right)}^2}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^2)^(5/2),x)
```

```
[Out] int((a/cos(x)^2)^(5/2), x)
```

3.49 $\int (a \sec^2(x))^{3/2} \, dx$

Optimal. Leaf size=46

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{2}a\sqrt{a\sec^2(x)}\tan(x)$$

```
[Out] 1/2*a^(3/2)*arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))+1/2*a*(a*sec(x)^2)^(
1/2)*tan(x)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 212}

$$\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{2}a\tan(x)\sqrt{a\sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]])/2 + (a*Sqrt[a*Sec[x]^2
]*Tan[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a\sec^2(x)\right)^{3/2}\,dx &= a\text{Subst}\left(\int \sqrt{a+ax^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}a\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}a\sqrt{a\sec^2(x)}\,\tan(x)+\frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\tan(x)}{\sqrt{a\sec^2(x)}}\right)\\
&= \frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\tan(x)}{\sqrt{a\sec^2(x)}}\right)+\frac{1}{2}a\sqrt{a\sec^2(x)}\,\tan(x)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 1.20

$$\frac{1}{2}a\cos(x)\sqrt{a\sec^2(x)}\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sec(x)\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(3/2),x]
```

```
[Out] (a*Cos[x]*Sqrt[a*Sec[x]^2]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[
x/2]] + Sec[x]*Tan[x]))/2
```

Maple [A]
time = 0.22, size = 55, normalized size = 1.20

method	result	size
default	$\frac{\left(\left(\cos^2(x)\right)\ln\left(-\frac{\cos(x)-1-\sin(x)}{\sin(x)}\right)-\left(\cos^2(x)\right)\ln\left(-\frac{\cos(x)-1+\sin(x)}{\sin(x)}\right)+\sin(x)\right)\cos(x)\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}{2}$	55
risch	$-\frac{ia\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\left(e^{2ix}-1\right)}{e^{2ix}+1}+a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}+i\right)\cos\left(x\right)-a\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}\,\ln\left(e^{ix}-i\right)\cos\left(x\right)$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(cos(x)^2*ln(-(cos(x)-1-sin(x))/sin(x))-cos(x)^2*ln(-(cos(x)-1+sin(x))/
sin(x))+sin(x))*cos(x)*(a/cos(x)^2)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(34) = 68$.
time = 0.54, size = 324, normalized size = 7.04

$$-\frac{\left(8\,a\cos\left(3\,x\right)\sin\left(2\,x\right)-8\,a\cos\left(x\right)\sin\left(2\,x\right)+8\,a\cos\left(2\,x\right)\sin\left(x\right)-4\,\left(a\sin\left(3\,x\right)-a\sin\left(x\right)\right)\cos\left(4\,x\right)-\left(a\cos\left(4\,x\right)^{2}+4\,a\cos\left(2\,x\right)^{2}+a\sin\left(4\,x\right)^{2}+4\,a\sin\left(4\,x\right)\sin\left(2\,x\right)+4\,a\sin\left(2\,x\right)^{2}+2\,\left(2\,a\cos\left(2\,x\right)+a\right)\cos\left(4\,x\right)+4\,a\cos\left(2\,x\right)+a\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\sin\left(x\right)+1\right)+\left(a\cos\left(4\,x\right)^{2}+4\,a\cos\left(2\,x\right)^{2}+a\sin\left(4\,x\right)^{2}+4\,a\sin\left(4\,x\right)\sin\left(2\,x\right)+4\,a\sin\left(2\,x\right)^{2}+2\,\left(2\,a\cos\left(2\,x\right)+a\right)\cos\left(4\,x\right)+4\,a\cos\left(2\,x\right)+a\right)\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\sin\left(x\right)+1\right)+4\,\left(a\cos\left(3\,x\right)-a\cos\left(x\right)\right)\sin\left(4\,x\right)-4\,\left(2\,a\cos\left(2\,x\right)+a\right)\sin\left(3\,x\right)+4\,a\sin\left(x\right)\right)\sqrt{a}}{4\,\left(2\,\left(2\,\cos\left(2\,x\right)+1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^{2}+4\,\cos\left(2\,x\right)^{2}+\sin\left(4\,x\right)^{2}+4\,\sin\left(4\,x\right)\sin\left(2\,x\right)+4\,\sin\left(2\,x\right)^{2}+4\,\cos\left(2\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*cos(3*x)*sin(2*x) - 8*a*cos(x)*sin(2*x) + 8*a*cos(2*x)*sin(x) - 4
*(a*sin(3*x) - a*sin(x))*cos(4*x) - (a*cos(4*x)^2 + 4*a*cos(2*x)^2 + a*sin(
4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(2*a*cos(2*x) + a)*cos(
4*x) + 4*a*cos(2*x) + a)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (a*cos(4
*x)^2 + 4*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x
)^2 + 2*(2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*log(cos(x)^2 + sin(
x)^2 - 2*sin(x) + 1) + 4*(a*cos(3*x) - a*cos(x))*sin(4*x) - 4*(2*a*cos(2*x)
 + a)*sin(3*x) + 4*a*sin(x))*sqrt(a)/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x
)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*co
s(2*x) + 1)
```

Fricas [A]
time = 3.23, size = 39, normalized size = 0.85

$$-\frac{\left(a\cos\left(x\right)^{2}\log\left(-\frac{\sin\left(x\right)-1}{\sin\left(x\right)+1}\right)-2\,a\sin\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}}{4\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a*cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*a*sin(x))*sqrt(a/cos(x
)^2)/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**2)**(3/2), x)
```

Giac [A]
time = 0.43, size = 42, normalized size = 0.91

$$\frac{1}{4}\left(\log\left(\sin\left(x\right)+1\right)\operatorname{sgn}(\cos\left(x\right))-\log\left(-\sin\left(x\right)+1\right)\operatorname{sgn}(\cos\left(x\right))-\frac{2\operatorname{sgn}(\cos\left(x\right))\sin\left(x\right)}{\sin\left(x\right)^{2}-1}\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(sin(x) + 1)*sgn(cos(x)) - log(-sin(x) + 1)*sgn(cos(x)) - 2*sgn(cos
(x))*sin(x)/(sin(x)^2 - 1))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\left(\frac{a}{\cos\left(x\right)^{2}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^2)^(3/2),x)
```

```
[Out] int((a/cos(x)^2)^(3/2), x)
```

3.50 $\int \sqrt{a \sec^2(x)}\, dx$

Optimal. Leaf size=25

$$\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right)$$

```
[Out] arctanh(a^(1/2)*tan(x)/(a*sec(x)^2)^(1/2))*a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 223, 212}

$$\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \tan(x)}{\sqrt{a \sec^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sec[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTanh[(Sqrt[a]*Tan[x])/Sqrt[a*Sec[x]^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a \sec^2(x)}\, dx = a\text{Subst}\left(\int \frac{1}{\sqrt{a+ax^2}}\, dx, x, \tan(x)\right)$$
$$= a\text{Subst}\left(\int \frac{1}{1-ax^2}\, dx, x, \frac{\tan(x)}{\sqrt{a \sec^2(x)}}\right)$$
$$= \sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a}\, \tan(x)}{\sqrt{a \sec^2(x)}}\right)$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.84

$$\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)\sqrt{a \sec^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sec[x]^2],x]
[Out] Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])*Sqrt[a*Sec[x]
^2]
```

Maple [A]
time = 0.22, size = 23, normalized size = 0.92

method	result	size
default	$-2\cos\left(x\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\, \operatorname{arctanh}\left(\frac{-1+\cos\left(x\right)}{\sin\left(x\right)}\right)$	23
risch	$-2\sqrt{\frac{a\,\mathrm{e}^{2 i x}}{\left(\mathrm{e}^{2 i x}+1\right)^{2}}}\, \ln\left(\mathrm{e}^{i x}-i\right)\cos\left(x\right)+2\sqrt{\frac{a\,\mathrm{e}^{2 i x}}{\left(\mathrm{e}^{2 i x}+1\right)^{2}}}\, \ln\left(\mathrm{e}^{i x}+i\right)\cos\left(x\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*cos(x)*(a/cos(x)^2)^(1/2)*arctanh((-1+cos(x))/sin(x))
```

Maxima [A]
time = 0.54, size = 38, normalized size = 1.52

$$\frac{1}{2}\sqrt{a}\left(\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\sin\left(x\right)+1\right)-\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\sin\left(x\right)+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x
)^2 - 2*sin(x) + 1))
```

Fricas [A]
time = 3.31, size = 55, normalized size = 2.20

$$\left[-\frac{1}{2}\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\sin(x)}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(a/cos(x)^2)*cos(x)*log(-(sin(x) - 1)/(sin(x) + 1)), -sqrt(-a)*ar
ctan(sqrt(-a)*sqrt(a/cos(x)^2)*cos(x)*sin(x)/a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sec^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**2)**(1/2),x)
[Out] Integral(sqrt(a*sec(x)**2), x)
```

Giac [A]
time = 0.41, size = 31, normalized size = 1.24

$$\frac{1}{4}\sqrt{a}\left(\log\left(\left|\frac{1}{\sin(x)}+\sin(x)+2\right|\right)-\log\left(\left|\frac{1}{\sin(x)}+\sin(x)-2\right|\right)\right)\mathrm{sgn}(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(a)*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) -
2)))*sgn(cos(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{a}{\cos(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^2)^(1/2),x)
[Out] int((a/cos(x)^2)^(1/2), x)
```

3.51 $\int \frac{1}{\sqrt{a \sec^2(x)}} \, dx$

Optimal. Leaf size=13

$$\frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

```
[Out] tan(x)/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[a*Sec[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a \sec^2(x)}} \, dx &= a\text{Subst}\left(\int \frac{1}{\left(a + a x^2\right)^{3/2}} \, dx, x, \tan(x)\right) \\
&= \frac{\tan(x)}{\sqrt{a \sec^2(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(x)}{\sqrt{a \sec^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^2],x]
```

```
[Out] Tan[x]/Sqrt[a*Sec[x]^2]
```

Maple [A]
time = 0.25, size = 16, normalized size = 1.23

method	result	size
default	$\frac{\sin(x)}{\sqrt{\frac{a}{\cos(x)^2}}\,\cos(x)}$	16
risch	$-\frac{i\mathrm{e}^{2ix}}{2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)}+\frac{i}{2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,(\mathrm{e}^{2ix}+1)}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(x)/(a/cos(x)^2)^(1/2)/cos(x)
```

Maxima [A]
time = 0.56, size = 6, normalized size = 0.46

$$\frac{\sin(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sin(x)/sqrt(a)
```

Fricas [A]
time = 2.64, size = 16, normalized size = 1.23

$$\frac{\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^2)*cos(x)*sin(x)/a
```

Sympy [A]
time = 0.20, size = 12, normalized size = 0.92

$$\frac{\tan(x)}{\sqrt{a\sec^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(1/2),x)
```

```
[Out] tan(x)/sqrt(a*sec(x)**2)
```

Giac [A]
time = 0.44, size = 11, normalized size = 0.85

$$\frac{\sin(x)}{\sqrt{a}\,\operatorname{sgn}(\cos(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sin(x)/(sqrt(a)*sgn(cos(x)))
```

Mupad [B]
time = 0.21, size = 15, normalized size = 1.15

$$\frac{\sqrt{2}\,\sin(2\,x)}{2\,\sqrt{a}\,\sqrt{2\cos(x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*sin(2*x))/(2*a^(1/2)*(2*cos(x)^2)^(1/2))
```

3.52 $\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}$$

```
[Out] 1/3*tan(x)/(a*sec(x)^2)^(3/2)+2/3*tan(x)/a/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}+\frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(-3/2),x]
```

```
[Out] Tan[x]/(3*(a*Sec[x]^2)^(3/2)) + (2*Tan[x])/(3*a*Sqrt[a*Sec[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\sec^2(x)\right)^{3/2}}\,dx &= a\mathrm{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{2}{3}\mathrm{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{3\left(a\sec^2(x)\right)^{3/2}}+\frac{2\tan(x)}{3a\sqrt{a\sec^2(x)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.75

$$\frac{\sec^3(x)(9\sin(x)+\sin(3x))}{12\left(a\sec^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-3/2),x]
[Out] (Sec[x]^3*(9*Sin[x] + Sin[3*x]))/(12*(a*Sec[x]^2)^(3/2))
```

Maple [A]
time = 0.18, size = 23, normalized size = 0.64

method	result	size
default	$\frac{\sin(x)\left(\cos^2(x)+2\right)}{3\cos(x)^3\left(\frac{a}{\cos(x)^2}\right)^{\frac{3}{2}}}$	23
risch	$-\frac{i e^{4ix}}{24a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}-\frac{3i e^{2ix}}{8a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}+\frac{3i}{8a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}+\frac{i e^{-2ix}}{24a\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}$	149

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*sin(x)*(cos(x)^2+2)/cos(x)^3/(a/cos(x)^2)^(3/2)
```

Maxima [A]
time = 0.60, size = 14, normalized size = 0.39

$$\frac{\sin\left(3\,x\right)+9\,\sin\left(x\right)}{12\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*x) + 9*sin(x))/a^(3/2)
```

Fricas [A]
time = 3.06, size = 24, normalized size = 0.67

$$\frac{\left(\cos\left(x\right)^{3} + 2\,\cos\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\,\sin\left(x\right)}{3\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^3 + 2*cos(x))*sqrt(a/cos(x)^2)*sin(x)/a^2
```

Sympy [A]
time = 0.36, size = 31, normalized size = 0.86

$$\frac{2\tan^{3}\left(x\right)}{3\left(a\sec^{2}\left(x\right)\right)^{\frac{3}{2}}} + \frac{\tan\left(x\right)}{\left(a\sec^{2}\left(x\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(3/2),x)
```

```
[Out] 2*tan(x)**3/(3*(a*sec(x)**2)**(3/2)) + tan(x)/(a*sec(x)**2)**(3/2)
```

Giac [A]
time = 0.44, size = 26, normalized size = 0.72

$$-\frac{\sqrt{a}\,\sin\left(x\right)^{3} - 3\,\sqrt{a}\,\sin\left(x\right)}{3\,a^{2}\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(sqrt(a)*sin(x)^3 - 3*sqrt(a)*sin(x))/(a^2*sgn(cos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(\frac{a}{{\cos\left(x\right)}^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(a/cos(x)^2)^(3/2), x)
```

3.53 $\int \frac{1}{\left(a \sec^2(x)\right)^{5/2}} \, dx$

Optimal. Leaf size=55

$$\frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}}+\frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}}+\frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}$$

```
[Out] 1/5*tan(x)/(a*sec(x)^2)^(5/2)+4/15*tan(x)/a/(a*sec(x)^2)^(3/2)+8/15*tan(x)/
a^2/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}+\frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}}+\frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(-5/2),x]
```

```
[Out] Tan[x]/(5*(a*Sec[x]^2)^(5/2)) + (4*Tan[x])/(15*a*(a*Sec[x]^2)^(3/2)) + (8*T
an[x])/(15*a^2*Sqrt[a*Sec[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\sec^2(x)\right)^{5/2}}\,dx &= a\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{7/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}} + \frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}} + \frac{8\text{Subst}\left(\int \frac{1}{\left(a+ax^2\right)^{3/2}}\,dx, x, \tan(x)\right)}{15a}\\
&= \frac{\tan(x)}{5\left(a\sec^2(x)\right)^{5/2}} + \frac{4\tan(x)}{15a\left(a\sec^2(x)\right)^{3/2}} + \frac{8\tan(x)}{15a^2\sqrt{a\sec^2(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.65

$$
\frac{\cos(x)\sqrt{a\sec^2(x)}\,(150\sin(x)+25\sin(3x)+3\sin(5x))}{240a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-5/2),x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^2]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/(240*a^3)
```

Maple [A]
time = 0.25, size = 31, normalized size = 0.56

method	result
default	$\frac{\sin(x)\left(3\left(\cos^4(x)\right)+4\left(\cos^2(x)\right)+8\right)}{15\cos(x)^5\left(\frac{a}{\cos(x)^2}\right)^{\frac{5}{2}}}$
risch	$-\frac{i e^{6ix}}{160a^2\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}} - \frac{5i e^{2ix}}{16a^2\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}} + \frac{5i}{16a^2\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}} + \frac{5i e^{-2ix}}{96a^2\left(e^{2ix}+1\right)\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/cos(x)^5/(a/cos(x)^2)^(5/2)
```

Maxima [A]
time = 0.54, size = 22, normalized size = 0.40

$$
\frac{3\,\sin\left(5\,x\right)+25\,\sin\left(3\,x\right)+150\,\sin\left(x\right)}{240\,a^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="maxima")
[Out] 1/240*(3*sin(5*x) + 25*sin(3*x) + 150*sin(x))/a^(5/2)
```

Fricas [A]
time = 2.57, size = 32, normalized size = 0.58

$$\frac{\left(3\,\cos\left(x\right)^{5}+4\,\cos\left(x\right)^{3}+8\,\cos\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\,\sin\left(x\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/15*(3*cos(x)^5 + 4*cos(x)^3 + 8*cos(x))*sqrt(a/cos(x)^2)*sin(x)/a^3
```

Sympy [A]
time = 1.59, size = 49, normalized size = 0.89

$$\frac{8\tan^{5}\left(x\right)}{15\left(a\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{4\tan^{3}\left(x\right)}{3\left(a\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}+\frac{\tan\left(x\right)}{\left(a\sec^{2}\left(x\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(5/2),x)
[Out] 8*tan(x)**5/(15*(a*sec(x)**2)**(5/2)) + 4*tan(x)**3/(3*(a*sec(x)**2)**(5/2)
) + tan(x)/(a*sec(x)**2)**(5/2)
```

Giac [A]
time = 0.44, size = 36, normalized size = 0.65

$$\frac{3\,\sqrt{a}\,\sin\left(x\right)^{5}-10\,\sqrt{a}\,\sin\left(x\right)^{3}+15\,\sqrt{a}\,\sin\left(x\right)}{15\,a^{3}\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/15*(3*sqrt(a)*sin(x)^5 - 10*sqrt(a)*sin(x)^3 + 15*sqrt(a)*sin(x))/(a^3*sg
n(cos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{a}{{\cos\left(x\right)}^2}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^2)^(5/2),x)
[Out] int(1/(a/cos(x)^2)^(5/2), x)
```

3.54 $\int \frac{1}{\left(a\sec^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}}+\frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}}+\frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}}+\frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}$$

```
[Out] 1/7*tan(x)/(a*sec(x)^2)^(7/2)+6/35*tan(x)/a/(a*sec(x)^2)^(5/2)+8/35*tan(x)/
a^2/(a*sec(x)^2)^(3/2)+16/35*tan(x)/a^3/(a*sec(x)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4207, 198, 197}

$$\frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}+\frac{8\tan(x)}{35a^2\left(a\sec^2(x)\right)^{3/2}}+\frac{6\tan(x)}{35a\left(a\sec^2(x)\right)^{5/2}}+\frac{\tan(x)}{7\left(a\sec^2(x)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^2)^(-7/2),x]
```

```
[Out] Tan[x]/(7*(a*Sec[x]^2)^(7/2)) + (6*Tan[x])/(35*a*(a*Sec[x]^2)^(5/2)) + (8*T
an[x])/(35*a^2*(a*Sec[x]^2)^(3/2)) + (16*Tan[x])/(35*a^3*Sqrt[a*Sec[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a\sec^2(x))^{7/2}}\,dx = a\text{Subst}\left(\int \frac{1}{(a+ax^2)^{9/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{7\,(a\sec^2(x))^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(a+ax^2)^{7/2}}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan(x)}{7\,(a\sec^2(x))^{7/2}} + \frac{6\tan(x)}{35a\,(a\sec^2(x))^{5/2}} + \frac{24\text{Subst}\left(\int \frac{1}{(a+ax^2)^{5/2}}\,dx, x, \tan(x)\right)}{35a}$$
$$= \frac{\tan(x)}{7\,(a\sec^2(x))^{7/2}} + \frac{6\tan(x)}{35a\,(a\sec^2(x))^{5/2}} + \frac{8\tan(x)}{35a^2\,(a\sec^2(x))^{3/2}} + \frac{16\text{Subst}\left(\int \frac{1}{(a+ax^2)^{3/2}}\,dx, x\right.}{35a^2}$$
$$= \frac{\tan(x)}{7\,(a\sec^2(x))^{7/2}} + \frac{6\tan(x)}{35a\,(a\sec^2(x))^{5/2}} + \frac{8\tan(x)}{35a^2\,(a\sec^2(x))^{3/2}} + \frac{16\tan(x)}{35a^3\sqrt{a\sec^2(x)}}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 0.57

$$\frac{\cos(x)\sqrt{a\sec^2(x)}\,(1225\sin(x) + 245\sin(3x) + 49\sin(5x) + 5\sin(7x))}{2240a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^2)^(-7/2),x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^2]*(1225*Sin[x] + 245*Sin[3*x] + 49*Sin[5*x] + 5*Sin[
7*x]))/(2240*a^4)
```

Maple [A]
time = 0.26, size = 37, normalized size = 0.50

method	result
default	$\frac{\sin(x)\left(5\left(\cos^6(x)\right)+6\left(\cos^4(x)\right)+8\left(\cos^2(x)\right)+16\right)}{35\cos(x)^7\left(\frac{a}{\cos(x)^2}\right)^{\frac{7}{2}}}$
risch	$-\frac{i e^{8ix}}{896a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}} - \frac{35i e^{2ix}}{128a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}} + \frac{35i}{128a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}} + \frac{7i e^{-2ix}}{128a^3(e^{2ix}+1)\sqrt{\frac{a\,e^{2ix}}{(e^{2ix}+1)^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*sin(x)*(5*cos(x)^6+6*cos(x)^4+8*cos(x)^2+16)/cos(x)^7/(a/cos(x)^2)^(7/
2)
```

Maxima [A]
time = 0.54, size = 28, normalized size = 0.38

$$\frac{5\,\sin\left(7\,x\right)+49\,\sin\left(5\,x\right)+245\,\sin\left(3\,x\right)+1225\,\sin\left(x\right)}{2240\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2240*(5*sin(7*x) + 49*sin(5*x) + 245*sin(3*x) + 1225*sin(x))/a^(7/2)
```

Fricas [A]
time = 3.38, size = 38, normalized size = 0.51

$$\frac{\left(5\,\cos\left(x\right)^{7}+6\,\cos\left(x\right)^{5}+8\,\cos\left(x\right)^{3}+16\,\cos\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{2}}}\,\sin\left(x\right)}{35\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(x)^7 + 6*cos(x)^5 + 8*cos(x)^3 + 16*cos(x))*sqrt(a/cos(x)^2)*si
n(x)/a^4
```

Sympy [A]
time = 12.43, size = 66, normalized size = 0.89

$$\frac{16\tan^{7}\left(x\right)}{35\left(a\sec^{2}\left(x\right)\right)^{\frac{7}{2}}}+\frac{8\tan^{5}\left(x\right)}{5\left(a\sec^{2}\left(x\right)\right)^{\frac{7}{2}}}+\frac{2\tan^{3}\left(x\right)}{\left(a\sec^{2}\left(x\right)\right)^{\frac{7}{2}}}+\frac{\tan\left(x\right)}{\left(a\sec^{2}\left(x\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**2)**(7/2),x)
```

```
[Out] 16*tan(x)**7/(35*(a*sec(x)**2)**(7/2)) + 8*tan(x)**5/(5*(a*sec(x)**2)**(7/2
)) + 2*tan(x)**3/(a*sec(x)**2)**(7/2) + tan(x)/(a*sec(x)**2)**(7/2)
```

Giac [A]
time = 0.43, size = 45, normalized size = 0.61

$$-\frac{5\,\sqrt{a}\,\sin\left(x\right)^{7}-21\,\sqrt{a}\,\sin\left(x\right)^{5}+35\,\sqrt{a}\,\sin\left(x\right)^{3}-35\,\sqrt{a}\,\sin\left(x\right)}{35\,a^{4}\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/35*(5*sqrt(a)*sin(x)^7 - 21*sqrt(a)*sin(x)^5 + 35*sqrt(a)*sin(x)^3 - 35*
sqrt(a)*sin(x))/(a^4*sgn(cos(x)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cos(x)^2}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^2)^(7/2),x)
[Out] int(1/(a/cos(x)^2)^(7/2), x)
```

3.55 $\int \left(a \sec^3(x)\right)^{5/2} dx$

Optimal. Leaf size=117

$$-\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}+\frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\sin(x)+\frac{154}{585}a^2\sqrt{a\sec^3(x)}\tan(x)+\frac{22}{117}a^2$$

```
[Out] -154/195*a^2*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2
*x),2^(1/2))*(a*sec(x)^3)^(1/2)+154/195*a^2*cos(x)*sin(x)*(a*sec(x)^3)^(1/2
)+154/585*a^2*(a*sec(x)^3)^(1/2)*tan(x)+22/117*a^2*sec(x)^2*(a*sec(x)^3)^(1
/2)*tan(x)+2/13*a^2*sec(x)^4*(a*sec(x)^3)^(1/2)*tan(x)
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$\frac{154}{585}a^2\tan(x)\sqrt{a\sec^3(x)}+\frac{2}{13}a^2\tan(x)\sec^4(x)\sqrt{a\sec^3(x)}+\frac{22}{117}a^2\tan(x)\sec^2(x)\sqrt{a\sec^3(x)}-\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}+\frac{154}{195}a^2\sin(x)\cos(x)\sqrt{a\sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(5/2),x]
```

```
[Out] (-154*a^2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3])/195 + (154*a^2*C
os[x]*Sqrt[a*Sec[x]^3]*Sin[x])/195 + (154*a^2*Sqrt[a*Sec[x]^3]*Tan[x])/585
+ (22*a^2*Sec[x]^2*Sqrt[a*Sec[x]^3]*Tan[x])/117 + (2*a^2*Sec[x]^4*Sqrt[a*Se
c[x]^3]*Tan[x])/13
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int \left(a\sec^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{15}{2}}(x)\,dx}{\sec^{\frac{3}{2}}(x)}$$

$$= \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(x)}\,\tan(x) + \frac{\left(11a^2\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{11}{2}}(x)\,dx}{13\sec^{\frac{3}{2}}(x)}$$

$$= \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\,\tan(x) + \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(x)}\,\tan(x) + \frac{\left(77a^2\sqrt{a\sec^3(\right.}{117\,\mathrm{s}}$$

$$= \frac{154}{585}a^2\sqrt{a\sec^3(x)}\,\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(x)}\,\tan(x) + \frac{2}{13}a^2\sec^4(x)\sqrt{a\sec^3(}$$

$$= \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\,\sin(x) + \frac{154}{585}a^2\sqrt{a\sec^3(x)}\,\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(}$$

$$= \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\,\sin(x) + \frac{154}{585}a^2\sqrt{a\sec^3(x)}\,\tan(x) + \frac{22}{117}a^2\sec^2(x)\sqrt{a\sec^3(}$$

$$= -\frac{154}{195}a^2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)} + \frac{154}{195}a^2\cos(x)\sqrt{a\sec^3(x)}\,\sin(x) + \frac{154}{585}a^2\sqrt{a}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.50

$$-\frac{2}{585}a\sec(x)\left(a\sec^3(x)\right)^{3/2}\left(231\cos^{\frac{11}{2}}(x)E\left(\frac{x}{2}\middle|2\right) - 55\cos(x)\sin(x) - 77\cos^3(x)\sin(x) - 231\cos^5(x)\sin(x) - 45\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(5/2),x]
```

```
[Out] (-2*a*Sec[x]*(a*Sec[x]^3)^(3/2)*(231*Cos[x]^(11/2)*EllipticE[x/2, 2] - 55*C
os[x]*Sin[x] - 77*Cos[x]^3*Sin[x] - 231*Cos[x]^5*Sin[x] - 45*Tan[x]))/585
```

Maple [C] Result contains complex when optimal does not.
time = 0.90, size = 223, normalized size = 1.91

method	result

default	$-\frac{2(\cos(x)+1)^2(-1+\cos(x))^2\left(231i(\cos^7(x))\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)-231i(\cos^7(x))\sin(x)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585*(cos(x)+1)^2*(-1+cos(x))^2*(231*I*cos(x)^7*sin(x)*(1/(cos(x)+1))^(1/
2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)
^7*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+co
s(x))/sin(x),I)+231*I*cos(x)^6*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+
1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)^6*sin(x)*(1/(cos(x
)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)+231
*cos(x)^7-154*cos(x)^6-22*cos(x)^4-10*cos(x)^2-45)*cos(x)*(a/cos(x)^3)^(5/2
)/sin(x)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 102, normalized size = 0.87

$$\frac{231i\sqrt{2}\,a^{\frac{5}{2}}\cos(x)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)))-231i\sqrt{2}\,a^{\frac{5}{2}}\cos(x)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x)))+2\left(231a^2\cos(x)^6+77a^2\cos(x)^4+55a^2\cos(x)^2+45a^2\right)\sqrt{\frac{a}{\cos(x)^3}}\sin(x)}{585\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/585*(231*I*sqrt(2)*a^(5/2)*cos(x)^5*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(x) + I*sin(x))) - 231*I*sqrt(2)*a^(5/2)*cos(x)^5*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) - I*sin(x))) + 2*(231*a^2*c
os(x)^6 + 77*a^2*cos(x)^4 + 55*a^2*cos(x)^2 + 45*a^2)*sqrt(a/cos(x)^3)*sin(
x))/cos(x)^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec^3(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(5/2),x)
[Out] Integral((a*sec(x)**3)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(x)^3)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{a}{\cos\left(x\right)^3}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^3)^(5/2),x)
[Out] int((a/cos(x)^3)^(5/2), x)
```

3.56 $\int (a\sec^3(x))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{10}{21}a\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}+\frac{10}{21}a\sqrt{a\sec^3(x)}\,\sin(x)+\frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\,\tan(x)$$

```
[Out] 10/21*a*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2
^(1/2))*(a*sec(x)^3)^(1/2)+10/21*a*sin(x)*(a*sec(x)^3)^(1/2)+2/7*a*sec(x)*(
a*sec(x)^3)^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2720}

$$\frac{10}{21}a\sin(x)\sqrt{a\sec^3(x)}+\frac{2}{7}a\tan(x)\sec(x)\sqrt{a\sec^3(x)}+\frac{10}{21}a\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(3/2),x]
```

```
[Out] (10*a*Cos[x]^(3/2)*EllipticF[x/2, 2]*Sqrt[a*Sec[x]^3])/21 + (10*a*Sqrt[a*Se
c[x]^3]*Sin[x])/21 + (2*a*Sec[x]*Sqrt[a*Sec[x]^3]*Tan[x])/7
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
```

```
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\sec^3(x)\right)^{3/2}\,dx &= \frac{\left(a\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{9}{2}}(x)\,dx}{\sec^{\frac{3}{2}}(x)}\\ &= \frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\,\tan(x)+\frac{\left(5a\sqrt{a\sec^3(x)}\right)\int \sec^{\frac{5}{2}}(x)\,dx}{7\sec^{\frac{3}{2}}(x)}\\ &= \frac{10}{21}a\sqrt{a\sec^3(x)}\,\sin(x)+\frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\,\tan(x)+\frac{\left(5a\sqrt{a\sec^3(x)}\right)\int \sqrt{\sec(x}}{21\sec^{\frac{3}{2}}(x)}\\ &= \frac{10}{21}a\sqrt{a\sec^3(x)}\,\sin(x)+\frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\,\tan(x)+\frac{1}{21}\left(5a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}\right)\\ &= \frac{10}{21}a\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}+\frac{10}{21}a\sqrt{a\sec^3(x)}\,\sin(x)+\frac{2}{7}a\sec(x)\sqrt{a\sec^3(x)}\,\text{ta}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.66

$$\frac{2}{21}a\sec(x)\sqrt{a\sec^3(x)}\left(5\cos^{\frac{5}{2}}(x)F\left(\frac{x}{2}\middle|2\right)+5\cos(x)\sin(x)+3\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(3/2),x]
[Out] (2*a*Sec[x]*Sqrt[a*Sec[x]^3]*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*S
in[x] + 3*Tan[x]))/21
```

Maple [C] Result contains complex when optimal does not.
time = 0.57, size = 87, normalized size = 1.34

method	result
default	$-\frac{2(\cos(x)+1)^2(-1+\cos(x))\left(5i(\cos^3(x))\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\ \text{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)-5(\cos^3(x))+5(\cos^2(x))-3\right.}{21\sin(x)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(cos(x)+1)^2*(-1+cos(x))*(5*I*cos(x)^3*sin(x)*(1/(cos(x)+1))^(1/2)*(c
os(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-5*cos(x)^3+5*cos(
x)^2-3*cos(x)+3)*cos(x)*(a/cos(x)^3)^(3/2)/sin(x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.18, size = 74, normalized size = 1.14

$$\frac{5i\sqrt{2}\,a^{\frac{3}{2}}\cos\left(x\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(x\right)+i\,\sin\left(x\right))-5i\sqrt{2}\,a^{\frac{3}{2}}\cos\left(x\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(x\right)-i\,\sin\left(x\right))+2\left(5\,a\cos\left(x\right)^{2}+3\,a\right)\sqrt{\frac{a}{\cos\left(x\right)^{3}}}\,\sin\left(x\right)}{21\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="fricas")
[Out] 1/21*(5*I*sqrt(2)*a^(3/2)*cos(x)^2*weierstrassPInverse(-4, 0, cos(x) + I*si
n(x)) - 5*I*sqrt(2)*a^(3/2)*cos(x)^2*weierstrassPInverse(-4, 0, cos(x) - I*
sin(x)) + 2*(5*a*cos(x)^2 + 3*a)*sqrt(a/cos(x)^3)*sin(x))/cos(x)^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec^{3}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(3/2),x)
[Out] Integral((a*sec(x)**3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(x)^3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{a}{\cos\left(x\right)^3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^3)^(3/2),x)
```

```
[Out] int((a/cos(x)^3)^(3/2), x)
```

3.57 $\int \sqrt{a \sec^3(x)}\, dx$

Optimal. Leaf size=42

$$-2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}+2\cos(x)\sqrt{a\sec^3(x)}\sin(x)$$

```
[Out] -2*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2
))*(a*sec(x)^3)^(1/2)+2*cos(x)*sin(x)*(a*sec(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3853, 3856, 2719}

$$2\sin(x)\cos(x)\sqrt{a\sec^3(x)}-2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sec[x]^3],x]
```

```
[Out] -2*Cos[x]^(3/2)*EllipticE[x/2, 2]*Sqrt[a*Sec[x]^3] + 2*Cos[x]*Sqrt[a*Sec[x]
^3]*Sin[x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
```

```
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sec^3(x)}\,dx &= \frac{\sqrt{a\sec^3(x)}\int \sec^{\frac{3}{2}}(x)\,dx}{\sec^{\frac{3}{2}}(x)}\\ &= 2\cos(x)\sqrt{a\sec^3(x)}\,\sin(x)-\frac{\sqrt{a\sec^3(x)}\int \frac{1}{\sqrt{\sec(x)}}\,dx}{\sec^{\frac{3}{2}}(x)}\\ &= 2\cos(x)\sqrt{a\sec^3(x)}\,\sin(x)-\left(\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}\right)\int\sqrt{\cos(x)}\,dx\\ &= -2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)\sqrt{a\sec^3(x)}+2\cos(x)\sqrt{a\sec^3(x)}\,\sin(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.76

$$2\cos(x)\sqrt{a\sec^3(x)}\left(-\sqrt{\cos(x)}\,E\left(\frac{x}{2}\middle|2\right)+\sin(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sec[x]^3],x]
[Out] 2*Cos[x]*Sqrt[a*Sec[x]^3]*(-(Sqrt[Cos[x]]*EllipticE[x/2, 2]) + Sin[x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.50, size = 189, normalized size = 4.50

method	result
default	$-\frac{2(\cos(x)+1)^2(-1+\cos(x))^2\left(i\cos(x)\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)-i\cos(x)\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(cos(x)+1)^2*(-1+cos(x))^2*(I*cos(x)*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)
/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-I*cos(x)*sin(x)*(1/(co
s(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)+
I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*sin(x)*EllipticF(I*(-1+cos
(x))/sin(x),I)-I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*sin(x)*Elli
pticE(I*(-1+cos(x))/sin(x),I)+cos(x)-1)*cos(x)*(a/cos(x)^3)^(1/2)/sin(x)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 57, normalized size = 1.36

$$2\sqrt{\frac{a}{\cos(x)^3}}\cos(x)\sin(x)+i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)))-i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a/cos(x)^3)*cos(x)*sin(x) + I*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(x) + I*sin(x))) - I*sqrt(2)*sqrt(a)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) - I*sin(x)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**3)**(1/2),x)
[Out] Integral(sqrt(a*sec(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sec(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a}{\cos(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^3)^(1/2),x)
[Out] int((a/cos(x)^3)^(1/2), x)
```

3.58 $\int \frac{1}{\sqrt{a \sec^3(x)}} \, dx$

Optimal. Leaf size=44

$$\frac{2F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}$$

```
[Out] 2/3*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))/cos(x)^(3
/2)/(a*sec(x)^3)^(1/2)+2/3*tan(x)/(a*sec(x)^3)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2720}

$$\frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}+\frac{2F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sec[x]^3],x]
```

```
[Out] (2*EllipticF[x/2, 2])/(3*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (2*Tan[x])/(3*Sqr
t[a*Sec[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
```

```
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx &= \frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sec^3(x)}}\\ &= \frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}+\frac{\sec^{\frac{3}{2}}(x)\int \sqrt{\sec(x)}\,dx}{3\sqrt{a\sec^3(x)}}\\ &= \frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}+\frac{\int \frac{1}{\sqrt{\cos(x)}}\,dx}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}\\ &= \frac{2F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{2\tan(x)}{3\sqrt{a\sec^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{F\left(\frac{x}{2}\middle|2\right)}{\cos^{\frac{3}{2}}(x)}+\tan(x)\right)}{3\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^3],x]
[Out] (2*(EllipticF[x/2, 2]/Cos[x]^(3/2) + Tan[x]))/(3*Sqrt[a*Sec[x]^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.50, size = 76, normalized size = 1.73

method	result	size
default	$\frac{2(-1+\cos(x))\left(-i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sin(x)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)+\cos^2(x)-\cos(x)\right)(\cos(x)+1)^2}{3\cos(x)^2\sin(x)^3\sqrt{\frac{a}{\cos(x)^3}}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(-1+cos(x))*(-I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*sin(x)*E
llipticF(I*(-1+cos(x))/sin(x),I)+cos(x)^2-cos(x))*(cos(x)+1)^2/cos(x)^2/sin
(x)^3/(a/cos(x)^3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sec(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 58, normalized size = 1.32

$$\frac{2\sqrt{\frac{a}{\cos(x)^3}}\cos(x)^2\sin(x) + i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassPInverse}(-4, 0, \cos(x) + i\sin(x)) - i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassPInverse}(-4, 0, \cos(x) - i\sin(x))}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(a/cos(x)^3)*cos(x)^2*sin(x) + I*sqrt(2)*sqrt(a)*weierstrassPInv
erse(-4, 0, cos(x) + I*sin(x)) - I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4,
0, cos(x) - I*sin(x)))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sec^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*sec(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sec(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{a}{\cos(x)^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^3)^(1/2),x)
[Out] int(1/(a/cos(x)^3)^(1/2), x)
```

3.59 $\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}}+\frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}}$$

```
[Out] 14/15*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2))/a/cos(x
)^(3/2)/(a*sec(x)^3)^(1/2)+14/45*sin(x)/a/(a*sec(x)^3)^(1/2)+2/9*cos(x)^2*s
in(x)/a/(a*sec(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3854, 3856, 2719}

$$\frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}}+\frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{2\sin(x)\cos^2(x)}{9a\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(-3/2),x]
```

```
[Out] (14*EllipticE[x/2, 2])/(15*a*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (14*Sin[x])/(
45*a*Sqrt[a*Sec[x]^3]) + (2*Cos[x]^2*Sin[x])/(9*a*Sqrt[a*Sec[x]^3])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a\sec^3(x))^{3/2}}\,dx = \frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sec^3(x)}}$$

$$= \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}} + \frac{\left(7\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{5}{2}}(x)}\,dx}{9a\sqrt{a\sec^3(x)}}$$

$$= \frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}} + \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}} + \frac{\left(7\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sec(x)}}\,dx}{15a\sqrt{a\sec^3(x)}}$$

$$= \frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}} + \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}} + \frac{7\int \sqrt{\cos(x)}\,dx}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}$$

$$= \frac{14E\left(\frac{x}{2}\middle|2\right)}{15a\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}} + \frac{14\sin(x)}{45a\sqrt{a\sec^3(x)}} + \frac{2\cos^2(x)\sin(x)}{9a\sqrt{a\sec^3(x)}}$$

Mathematica [A]
time = 0.11, size = 43, normalized size = 0.59

$$\frac{\frac{84E\left(\frac{x}{2}|2\right)}{\cos^{\frac{3}{2}}(x)} + 33\sin(x) + 5\sin(3x)}{90a\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(-3/2),x]
```

```
[Out] ((84*EllipticE[x/2, 2])/Cos[x]^(3/2) + 33*Sin[x] + 5*Sin[3*x])/(90*a*Sqrt[a
*Sec[x]^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.55, size = 198, normalized size = 2.71

method	result

default	$-\frac{2\left(5\left(\cos^{6}(x)\right)-21i\cos(x)\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)+21i\cos(x)\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(5*cos(x)^6-21*I*cos(x)*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1
))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)+21*I*cos(x)*sin(x)*(1/(cos(x)+1)
)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)-21*I*(1
/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*sin(x)*EllipticF(I*(-1+cos(x))
/sin(x),I)+21*I*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*Ellip
ticE(I*(-1+cos(x))/sin(x),I)+2*cos(x)^4+14*cos(x)^2-21*cos(x))/cos(x)^5/sin
(x)/(a/cos(x)^3)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 73, normalized size = 1.00

$$\frac{2\left(5\cos(x)^{5}+7\cos(x)^{3}\right)\sqrt{\frac{a}{\cos(x)^{3}}}\sin(x)-21i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)))+21i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x)))}{45a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(x)^5 + 7*cos(x)^3)*sqrt(a/cos(x)^3)*sin(x) - 21*I*sqrt(2)*sq
rt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) + I*sin(x)))
 + 21*I*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(x) - I*sin(x))))/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sec^{3}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{a}{{\cos\left(x\right)}^3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^3)^(3/2),x)
```

```
[Out] int(1/(a/cos(x)^3)^(3/2), x)
```

3.60 $\int \frac{1}{\left(a\sec^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{26F\left(\frac{x}{2}\middle|2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}}+\frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}}+\frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}}+\frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}}$$

```
[Out] 26/77*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))/a^2/cos
(x)^(3/2)/(a*sec(x)^3)^(1/2)+78/385*cos(x)*sin(x)/a^2/(a*sec(x)^3)^(1/2)+26
/165*cos(x)^3*sin(x)/a^2/(a*sec(x)^3)^(1/2)+2/15*cos(x)^5*sin(x)/a^2/(a*sec
(x)^3)^(1/2)+26/77*tan(x)/a^2/(a*sec(x)^3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4208, 3854, 3856, 2720}

$$\frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}}+\frac{26F\left(\frac{x}{2}\middle|2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}}+\frac{2\sin(x)\cos^5(x)}{15a^2\sqrt{a\sec^3(x)}}+\frac{26\sin(x)\cos^3(x)}{165a^2\sqrt{a\sec^3(x)}}+\frac{78\sin(x)\cos(x)}{385a^2\sqrt{a\sec^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^3)^(-5/2),x]
```

```
[Out] (26*EllipticF[x/2, 2])/(77*a^2*Cos[x]^(3/2)*Sqrt[a*Sec[x]^3]) + (78*Cos[x]*
Sin[x])/(385*a^2*Sqrt[a*Sec[x]^3]) + (26*Cos[x]^3*Sin[x])/(165*a^2*Sqrt[a*S
ec[x]^3]) + (2*Cos[x]^5*Sin[x])/(15*a^2*Sqrt[a*Sec[x]^3]) + (26*Tan[x])/(77
*a^2*Sqrt[a*Sec[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a\sec^3(x))^{5/2}}\,dx = \frac{\sec^{\frac{3}{2}}(x)\int \frac{1}{\sec^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{\left(13\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{11}{2}}(x)}\,dx}{15a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{\left(39\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{7}{2}}(x)}\,dx}{55a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{\left(39\sec^{\frac{3}{2}}(x)\right)\int \frac{1}{\sec^{\frac{3}{2}}(x)}}{77a^2\sqrt{a\sec^3(x)}}$$

$$= \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}} + \frac{(}{}$$

$$= \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(x)}{15a^2\sqrt{a\sec^3(x)}} + \frac{26\tan(x)}{77a^2\sqrt{a\sec^3(x)}} + \frac{}{77}$$

$$= \frac{26F\left(\frac{x}{2}\middle|2\right)}{77a^2\cos^{\frac{3}{2}}(x)\sqrt{a\sec^3(x)}} + \frac{78\cos(x)\sin(x)}{385a^2\sqrt{a\sec^3(x)}} + \frac{26\cos^3(x)\sin(x)}{165a^2\sqrt{a\sec^3(x)}} + \frac{2\cos^5(x)\sin(}{15a^2\sqrt{a\sec^3(}}$$

Mathematica [A]
time = 0.11, size = 59, normalized size = 0.50

$$\frac{\cos(x)\sqrt{a\sec^3(x)}\left(24960\sqrt{\cos(x)}\,F\left(\frac{x}{2}\middle|2\right)+19122\sin(2x)+4406\sin(4x)+826\sin(6x)+77\sin(8x)\right)}{73920a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^3)^(-5/2),x]
```

```
[Out] (Cos[x]*Sqrt[a*Sec[x]^3]*(24960*Sqrt[Cos[x]]*EllipticF[x/2, 2] + 19122*Sin[
2*x] + 4406*Sin[4*x] + 826*Sin[6*x] + 77*Sin[8*x]))/(73920*a^3)
```

Maple [C] Result contains complex when optimal does not.
time = 0.49, size = 114, normalized size = 0.97

method	result
default	$\frac{2(-1+\cos(x))\left(77(\cos^8(x))-77(\cos^7(x))+91(\cos^6(x))-91(\cos^5(x))-195i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sin(x)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(x)}{\sin(x)}\right.\right.}{1155\cos(x)^8\sin(x)^3\left(\frac{a}{\cos(x)^3}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(-1+cos(x))*(77*cos(x)^8-77*cos(x)^7+91*cos(x)^6-91*cos(x)^5-195*I*(
1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*sin(x)*EllipticF(I*(-1+cos(x)
)/sin(x),I)+117*cos(x)^4-117*cos(x)^3+195*cos(x)^2-195*cos(x))*(cos(x)+1)^2
/cos(x)^8/sin(x)^3/(a/cos(x)^3)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.33, size = 79, normalized size = 0.68

$$\frac{2\left(77\cos(x)^8+91\cos(x)^6+117\cos(x)^4+195\cos(x)^2\right)\sqrt{\frac{a}{\cos(x)^3}}\sin(x)+195i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x))-195i\sqrt{2}\sqrt{a}\,\mathrm{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x))}{1155\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(2*(77*cos(x)^8 + 91*cos(x)^6 + 117*cos(x)^4 + 195*cos(x)^2)*sqrt(a/
cos(x)^3)*sin(x) + 195*I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4, 0, cos(x)
+ I*sin(x)) - 195*I*sqrt(2)*sqrt(a)*weierstrassPInverse(-4, 0, cos(x) - I*s
in(x)))/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sec^3(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sec(x)**3)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(x)^3)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cos(x)^3}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^3)^(5/2),x)
```

```
[Out] int(1/(a/cos(x)^3)^(5/2), x)
```

3.61 $\int \left(a \sec^4(x)\right)^{7/2} \, dx$

Optimal. Leaf size=163

$$a^3 \cos(x)\sqrt{a\sec^4(x)}\,\sin(x)+2a^3\sqrt{a\sec^4(x)}\,\sin^2(x)\tan(x)+3a^3\sqrt{a\sec^4(x)}\,\sin^2(x)\tan^3(x)+\frac{20}{7}a^3\sqrt{a\sec^4(x}$$

```
[Out] a^3*cos(x)*sin(x)*(a*sec(x)^4)^(1/2)+2*a^3*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(
x)+3*a^3*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x)^3+20/7*a^3*sin(x)^2*(a*sec(x)^4
)^(1/2)*tan(x)^5+5/3*a^3*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x)^7+6/11*a^3*sin(
x)^2*(a*sec(x)^4)^(1/2)*tan(x)^9+1/13*a^3*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x
)^11
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$a^3\sin(x)\cos(x)\sqrt{a\sec^4(x)}+\frac{1}{13}a^3\sin^2(x)\tan^{11}(x)\sqrt{a\sec^4(x)}+\frac{6}{11}a^3\sin^2(x)\tan^9(x)\sqrt{a\sec^4(x)}+\frac{5}{3}a^3\sin^2(x)\tan^7(x)\sqrt{a\sec^4(x)}+\frac{20}{7}a^3\sin^2(x)\tan^5(x)\sqrt{a\sec^4(x)}+3a^3\sin^2(x)\tan^3(x)\sqrt{a\sec^4(x)}+2a^3\sin^2(x)\tan(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(7/2),x]
```

```
[Out] a^3*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + 2*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]
 + 3*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3 + (20*a^3*Sqrt[a*Sec[x]^4]*Sin[
x]^2*Tan[x]^5)/7 + (5*a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/3 + (6*a^3*Sq
rt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^9)/11 + (a^3*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
]^11)/13
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a\sec^4(x)\right)^{7/2}\,dx = \left(a^3\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^{14}(x)\,dx$$
$$= -\left(\left(a^3\cos^2(x)\sqrt{a\sec^4(x)}\right)\text{Subst}\left(\int (1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right.\right.$$
$$= a^3\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)+2a^3\sqrt{a\sec^4(x)}\,\sin^2(x)\tan(x)+3a^3\sqrt{a\sec^4(x)}\,\sin^2($$

Mathematica [A]
time = 0.19, size = 54, normalized size = 0.33

$$\frac{\cos(x)(2048+2380\cos(2x)+1093\cos(4x)+378\cos(6x)+92\cos(8x)+14\cos(10x)+\cos(12x))\left(a\sec^4(x)\right)^{7/2}\sin(x)}{6006}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(7/2),x]
```

```
[Out] (Cos[x]*(2048 + 2380*Cos[2*x] + 1093*Cos[4*x] + 378*Cos[6*x] + 92*Cos[8*x]
+ 14*Cos[10*x] + Cos[12*x])*(a*Sec[x]^4)^(7/2)*Sin[x])/6006
```

Maple [A]
time = 0.45, size = 53, normalized size = 0.33

method	result	size
default	$\frac{(1024(\cos^{12}(x))+512(\cos^{10}(x))+384(\cos^8(x))+320(\cos^6(x))+280(\cos^4(x))+252(\cos^2(x))+231)\left(\frac{a}{\cos(x)^4}\right)^{\frac{7}{2}}\sin(x)\cos(x)}{3003}$	53
risch	$\frac{2048ia^3\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(1716\,e^{10ix}+1287\,e^{8ix}+715\,e^{6ix}+286\,e^{4ix}+13+79\cos(2x)+77i\sin(2x))}{3003(e^{2ix}+1)^{11}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3003*(1024*cos(x)^12+512*cos(x)^10+384*cos(x)^8+320*cos(x)^6+280*cos(x)^4
+252*cos(x)^2+231)*(a/cos(x)^4)^(7/2)*sin(x)*cos(x)
```

Maxima [A]
time = 0.50, size = 61, normalized size = 0.37

$$\frac{1}{13}\,a^{\frac{7}{2}}\tan(x)^{13}+\frac{6}{11}\,a^{\frac{7}{2}}\tan(x)^{11}+\frac{5}{3}\,a^{\frac{7}{2}}\tan(x)^9+\frac{20}{7}\,a^{\frac{7}{2}}\tan(x)^7+3\,a^{\frac{7}{2}}\tan(x)^5+2\,a^{\frac{7}{2}}\tan(x)^3+a^{\frac{7}{2}}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/13*a^(7/2)*tan(x)^13 + 6/11*a^(7/2)*tan(x)^11 + 5/3*a^(7/2)*tan(x)^9 + 20
/7*a^(7/2)*tan(x)^7 + 3*a^(7/2)*tan(x)^5 + 2*a^(7/2)*tan(x)^3 + a^(7/2)*tan
(x)
```

Fricas [A]
time = 3.25, size = 76, normalized size = 0.47

$$\frac{\left(1024\,a^{3}\cos\left(x\right)^{12}+512\,a^{3}\cos\left(x\right)^{10}+384\,a^{3}\cos\left(x\right)^{8}+320\,a^{3}\cos\left(x\right)^{6}+280\,a^{3}\cos\left(x\right)^{4}+252\,a^{3}\cos\left(x\right)^{2}+231\,a^{3}\right)\sqrt{\frac{a}{\cos\left(x\right)^{4}}}\,\sin\left(x\right)}{3003\,\cos\left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(1024*a^3*cos(x)^12 + 512*a^3*cos(x)^10 + 384*a^3*cos(x)^8 + 320*a^3
*cos(x)^6 + 280*a^3*cos(x)^4 + 252*a^3*cos(x)^2 + 231*a^3)*sqrt(a/cos(x)^4)
*sin(x)/cos(x)^11
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 0.45, size = 67, normalized size = 0.41

$$\frac{1}{3003}\left(231\,a^{3}\tan\left(x\right)^{13}+1638\,a^{3}\tan\left(x\right)^{11}+5005\,a^{3}\tan\left(x\right)^{9}+8580\,a^{3}\tan\left(x\right)^{7}+9009\,a^{3}\tan\left(x\right)^{5}+6006\,a^{3}\tan\left(x\right)^{3}+3003\,a^{3}\tan\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(7/2),x, algorithm="giac")
```

```
[Out] 1/3003*(231*a^3*tan(x)^13 + 1638*a^3*tan(x)^11 + 5005*a^3*tan(x)^9 + 8580*a
^3*tan(x)^7 + 9009*a^3*tan(x)^5 + 6006*a^3*tan(x)^3 + 3003*a^3*tan(x))*sqrt
(a)
```

Mupad [B]
time = 4.68, size = 589, normalized size = 3.61

$$\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,2048{}\mathrm{i}}{7\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^7\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}-\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,1536{}\mathrm{i}}{{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^8\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}+\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,10240{}\mathrm{i}}{3\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^9\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}-\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,4096{}\mathrm{i}}{{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^{10}\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}+\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,30720{}\mathrm{i}}{11\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^{11}\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}-\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,1024{}\mathrm{i}}{{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^{12}\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}+\frac{a^3\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{x\,2{}\mathrm{i}}+6\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+4\,{\mathrm{e}}^{x\,6{}\mathrm{i}}+{\mathrm{e}}^{x\,8{}\mathrm{i}}+1\right)\,2048{}\mathrm{i}}{13\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^{13}\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^4)^(7/2),x)
```

```
[Out] (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) +
4*exp(x*6i) + exp(x*8i) + 1)*2048i)/(7*(exp(x*2i) + 1)^7*(exp(x*2i) + 2*exp
(x*4i) + exp(x*6i))) - (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp
(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) + 1)*1536i)/((exp(x*2i) + 1)
^8*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) + (a^3*(a/(exp(-x*1i)/2 + exp(x*1
i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) + 1)*10
240i)/(3*(exp(x*2i) + 1)^9*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) - (a^3*(a
/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) + 4*exp(x
*6i) + exp(x*8i) + 1)*4096i)/((exp(x*2i) + 1)^10*(exp(x*2i) + 2*exp(x*4i) +
 exp(x*6i))) + (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) +
 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) + 1)*30720i)/(11*(exp(x*2i) + 1)^11*
(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) - (a^3*(a/(exp(-x*1i)/2 + exp(x*1i)/
2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i) + exp(x*8i) + 1)*1024i
)/((exp(x*2i) + 1)^12*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) + (a^3*(a/(exp
(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*(4*exp(x*2i) + 6*exp(x*4i) + 4*exp(x*6i)
+ exp(x*8i) + 1)*2048i)/(13*(exp(x*2i) + 1)^13*(exp(x*2i) + 2*exp(x*4i) + e
xp(x*6i)))
```

3.62 $\int (a\sec^4(x))^{5/2}\,dx$

Optimal. Leaf size=117

$$a^2\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)+\frac{4}{3}a^2\sqrt{a\sec^4(x)}\,\sin^2(x)\tan(x)+\frac{6}{5}a^2\sqrt{a\sec^4(x)}\,\sin^2(x)\tan^3(x)+\frac{4}{7}a^2\sqrt{a\sec^4(x)}$$

```
[Out] a^2*cos(x)*sin(x)*(a*sec(x)^4)^(1/2)+4/3*a^2*sin(x)^2*(a*sec(x)^4)^(1/2)*ta
n(x)+6/5*a^2*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x)^3+4/7*a^2*sin(x)^2*(a*sec(x
)^4)^(1/2)*tan(x)^5+1/9*a^2*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x)^7
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$a^2\sin(x)\cos(x)\sqrt{a\sec^4(x)}+\frac{1}{9}a^2\sin^2(x)\tan^7(x)\sqrt{a\sec^4(x)}+\frac{4}{7}a^2\sin^2(x)\tan^5(x)\sqrt{a\sec^4(x)}+\frac{6}{5}a^2\sin^2(x)\tan^3(x)\sqrt{a\sec^4(x)}+\frac{4}{3}a^2\sin^2(x)\tan(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(5/2),x]
```

```
[Out] a^2*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (4*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x
])/3 + (6*a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5 + (4*a^2*Sqrt[a*Sec[x]^
4]*Sin[x]^2*Tan[x]^5)/7 + (a^2*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^7)/9
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int (a\sec^4(x))^{5/2}\,dx &= \left(a^2\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^{10}(x)\,dx\\ &= -\left(\left(a^2\cos^2(x)\sqrt{a\sec^4(x)}\right)\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-\tan(x)\right)\right)\\ &= a^2\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)+\frac{4}{3}a^2\sqrt{a\sec^4(x)}\,\sin^2(x)\tan(x)+\frac{6}{5}a^2\sqrt{a\sec^4(x)}\,\sin^2(x\end{aligned}$$

Mathematica [A]
time = 0.11, size = 42, normalized size = 0.36

$$\frac{1}{315}\cos(x)(128+130\cos(2x)+46\cos(4x)+10\cos(6x)+\cos(8x))\left(a\sec^4(x)\right)^{5/2}\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(5/2),x]
```

```
[Out] (Cos[x]*(128 + 130*Cos[2*x] + 46*Cos[4*x] + 10*Cos[6*x] + Cos[8*x])*(a*Sec[
x]^4)^(5/2)*Sin[x])/315
```

Maple [A]
time = 0.29, size = 41, normalized size = 0.35

method	result	size
default	$\frac{\left(128\left(\cos^8(x)\right)+64\left(\cos^6(x)\right)+48\left(\cos^4(x)\right)+40\left(\cos^2(x)\right)+35\right)\cos(x)\sin(x)\left(\frac{a}{\cos(x)^4}\right)^{\frac{5}{2}}}{315}$	41
risch	$\frac{256ia^2\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\left(126\,e^{6ix}+84\,e^{4ix}+9+37\cos(2x)+35i\sin(2x)\right)}{315\left(e^{2ix}+1\right)^7}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*(128*cos(x)^8+64*cos(x)^6+48*cos(x)^4+40*cos(x)^2+35)*cos(x)*sin(x)*(
a/cos(x)^4)^(5/2)
```

Maxima [A]
time = 0.54, size = 43, normalized size = 0.37

$$\frac{1}{9}a^{\frac{5}{2}}\tan\left(x\right)^9+\frac{4}{7}a^{\frac{5}{2}}\tan\left(x\right)^7+\frac{6}{5}a^{\frac{5}{2}}\tan\left(x\right)^5+\frac{4}{3}a^{\frac{5}{2}}\tan\left(x\right)^3+a^{\frac{5}{2}}\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*a^(5/2)*tan(x)^9 + 4/7*a^(5/2)*tan(x)^7 + 6/5*a^(5/2)*tan(x)^5 + 4/3*a^
(5/2)*tan(x)^3 + a^(5/2)*tan(x)
```

Fricas [A]
time = 2.52, size = 58, normalized size = 0.50

$$\frac{\left(128\,a^2\cos\left(x\right)^8+64\,a^2\cos\left(x\right)^6+48\,a^2\cos\left(x\right)^4+40\,a^2\cos\left(x\right)^2+35\,a^2\right)\sqrt{\frac{a}{\cos\left(x\right)^4}}\sin\left(x\right)}{315\,\cos\left(x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*a^2*cos(x)^8 + 64*a^2*cos(x)^6 + 48*a^2*cos(x)^4 + 40*a^2*cos(x)
^2 + 35*a^2)*sqrt(a/cos(x)^4)*sin(x)/cos(x)^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec^4(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(5/2), x)
```

Giac [A]
time = 0.44, size = 49, normalized size = 0.42

$$\frac{1}{315}\left(35\,a^2\tan(x)^9+180\,a^2\tan(x)^7+378\,a^2\tan(x)^5+420\,a^2\tan(x)^3+315\,a^2\tan(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*a^2*tan(x)^9 + 180*a^2*tan(x)^7 + 378*a^2*tan(x)^5 + 420*a^2*tan(
x)^3 + 315*a^2*tan(x))*sqrt(a)
```

Mupad [B]
time = 2.35, size = 119, normalized size = 1.02

$$\frac{128\,a^{5/2}\,\left(\mathrm{e}^{x\,46\mathrm{i}}\,1\mathrm{i}+\mathrm{e}^{x\,48\mathrm{i}}\,9\mathrm{i}+\mathrm{e}^{x\,50\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{x\,52\mathrm{i}}\,84\mathrm{i}+\mathrm{e}^{x\,54\mathrm{i}}\,126\mathrm{i}\right)}{315\,\left(\frac{\mathrm{e}^{-x\,2\mathrm{i}}}{2}+\frac{\mathrm{e}^{x\,2\mathrm{i}}}{2}+1\right)\,\left(\mathrm{e}^{x\,48\mathrm{i}}+7\,\mathrm{e}^{x\,50\mathrm{i}}+21\,\mathrm{e}^{x\,52\mathrm{i}}+35\,\mathrm{e}^{x\,54\mathrm{i}}+35\,\mathrm{e}^{x\,56\mathrm{i}}+21\,\mathrm{e}^{x\,58\mathrm{i}}+7\,\mathrm{e}^{x\,60\mathrm{i}}+\mathrm{e}^{x\,62\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^4)^(5/2),x)
```

```
[Out] (128*a^(5/2)*(exp(x*46i)*1i + exp(x*48i)*9i + exp(x*50i)*36i + exp(x*52i)*8
4i + exp(x*54i)*126i))/(315*(exp(-x*2i)/2 + exp(x*2i)/2 + 1)*(exp(x*48i) +
7*exp(x*50i) + 21*exp(x*52i) + 35*exp(x*54i) + 35*exp(x*56i) + 21*exp(x*58i
) + 7*exp(x*60i) + exp(x*62i)))
```

3.63 $\int \left(a\sec^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$a\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)+\frac{2}{3}a\sqrt{a\sec^4(x)}\,\sin^2(x)\tan(x)+\frac{1}{5}a\sqrt{a\sec^4(x)}\,\sin^2(x)\tan^3(x)$$

```
[Out] a*cos(x)*sin(x)*(a*sec(x)^4)^(1/2)+2/3*a*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x)
+1/5*a*sin(x)^2*(a*sec(x)^4)^(1/2)*tan(x)^3
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4208, 3852}

$$a\sin(x)\cos(x)\sqrt{a\sec^4(x)}+\frac{1}{5}a\sin^2(x)\tan^3(x)\sqrt{a\sec^4(x)}+\frac{2}{3}a\sin^2(x)\tan(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(3/2),x]
```

```
[Out] a*Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x] + (2*a*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x])/3
 + (a*Sqrt[a*Sec[x]^4]*Sin[x]^2*Tan[x]^3)/5
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a\sec^4(x)\right)^{3/2}\,dx &= \left(a\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^6(x)\,dx\\ &= -\left(\left(a\cos^2(x)\sqrt{a\sec^4(x)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(x)\right)\right)\\ &= a\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)+\frac{2}{3}a\sqrt{a\sec^4(x)}\,\sin^2(x)\tan(x)+\frac{1}{5}a\sqrt{a\sec^4(x)}\,\sin^2(x)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 0.49

$$\frac{1}{15}\cos(x)(8+6\cos(2x)+\cos(4x))\left(a\sec^4(x)\right)^{3/2}\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(3/2),x]
[Out] (Cos[x]*(8 + 6*Cos[2*x] + Cos[4*x])*(a*Sec[x]^4)^(3/2)*Sin[x])/15
```

Maple [A]
time = 0.21, size = 29, normalized size = 0.48

method	result	size
default	$\frac{(8(\cos^4(x))+4(\cos^2(x))+3)\cos(x)\sin(x)\left(\frac{a}{\cos(x)^4}\right)^{\frac{3}{2}}}{15}$	29
risch	$\frac{16ia\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(5+11\cos(2x)+9i\sin(2x))}{15(e^{2ix}+1)^3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/15*(8*cos(x)^4+4*cos(x)^2+3)*cos(x)*sin(x)*(a/cos(x)^4)^(3/2)
```

Maxima [A]
time = 0.53, size = 25, normalized size = 0.41

$$\frac{1}{5}a^{\frac{3}{2}}\tan(x)^5+\frac{2}{3}a^{\frac{3}{2}}\tan(x)^3+a^{\frac{3}{2}}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(3/2),x, algorithm="maxima")
[Out] 1/5*a^(3/2)*tan(x)^5 + 2/3*a^(3/2)*tan(x)^3 + a^(3/2)*tan(x)
```

Fricas [A]
time = 2.54, size = 34, normalized size = 0.56

$$\frac{\left(8\,a\cos(x)^4+4\,a\cos(x)^2+3\,a\right)\sqrt{\frac{a}{\cos(x)^4}}\,\sin(x)}{15\,\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*a*cos(x)^4 + 4*a*cos(x)^2 + 3*a)*sqrt(a/cos(x)^4)*sin(x)/cos(x)^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec^4\left(x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(3/2), x)
```

Giac [**A**]
time = 0.45, size = 22, normalized size = 0.36

$$\frac{1}{15}\left(3\,\tan\left(x\right)^{5}+10\,\tan\left(x\right)^{3}+15\,\tan\left(x\right)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))*a^(3/2)
```

Mupad [**B**]
time = 0.56, size = 36, normalized size = 0.59

$$\frac{4\,a^{3/2}\,\sin\left(x\right)}{5\cos\left(x\right)^3}+\frac{a^{3/2}\,\sin\left(x\right)}{5\cos\left(x\right)^5}-\frac{8\,a^{3/2}\sin\left(x\right)^3}{15\cos\left(x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^4)^(3/2),x)
```

```
[Out] (4*a^(3/2)*sin(x))/(5*cos(x)^3) + (a^(3/2)*sin(x))/(5*cos(x)^5) - (8*a^(3/2
)*sin(x)^3)/(15*cos(x)^3)
```

3.64 $\int \sqrt{a \sec^4(x)}\, dx$

Optimal. Leaf size=15

$$\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)$$

```
[Out] cos(x)*sin(x)*(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 3852, 8}

$$\sin(x)\cos(x)\sqrt{a\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sec[x]^4],x]
```

```
[Out] Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\sec^4(x)}\,dx &= \left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\int \sec^2(x)\,dx\\
&= -\left(\left(\cos^2(x)\sqrt{a\sec^4(x)}\right)\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\right)\\
&= \cos(x)\sqrt{a\sec^4(x)}\,\sin(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\cos(x)\sqrt{a\sec^4(x)}\,\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sec[x]^4],x]
```

```
[Out] Cos[x]*Sqrt[a*Sec[x]^4]*Sin[x]
```

Maple [A]
time = 0.20, size = 14, normalized size = 0.93

method	result	size
default	$\cos\left(x\right)\sin\left(x\right)\sqrt{\frac{a}{\cos\left(x\right)^{4}}}$	14
risch	$2i\sqrt{\frac{a\,\mathrm{e}^{4ix}}{\left(\mathrm{e}^{2ix}+1\right)^{4}}}\,\left(1+\mathrm{e}^{-2ix}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)*sin(x)*(a/cos(x)^4)^(1/2)
```

Maxima [A]
time = 0.51, size = 6, normalized size = 0.40

$$\sqrt{a}\,\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*tan(x)
```

Fricas [A]
time = 2.90, size = 13, normalized size = 0.87

$$\sqrt{\frac{a}{\cos\left(x\right)^{4}}}\,\cos\left(x\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a/cos(x)^4)*cos(x)*sin(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec^4{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(x)**4), x)
```

Giac [**A**]
time = 0.42, size = 6, normalized size = 0.40

$$\sqrt{a}\, \tan{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*tan(x)
```

Mupad [**B**]
time = 0.11, size = 6, normalized size = 0.40

$$\sqrt{a}\, \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(x)^4)^(1/2),x)
```

```
[Out] a^(1/2)*tan(x)
```

3.65 $\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx$

Optimal. Leaf size=36

$$\frac{x\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{\tan(x)}{2\sqrt{a\sec^4(x)}}$$

```
[Out] 1/2*x*sec(x)^2/(a*sec(x)^4)^(1/2)+1/2*tan(x)/(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {4208, 2715, 8}

$$\frac{x\sec^2(x)}{2\sqrt{a\sec^4(x)}}+\frac{\tan(x)}{2\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sec[x]^4],x]
```

```
[Out] (x*Sec[x]^2)/(2*Sqrt[a*Sec[x]^4]) + Tan[x]/(2*Sqrt[a*Sec[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sec^4(x)}}\,dx = \frac{\sec^2(x)\int \cos^2(x)\,dx}{\sqrt{a\sec^4(x)}}$$
$$= \frac{\tan(x)}{2\sqrt{a\sec^4(x)}} + \frac{\sec^2(x)\int 1\,dx}{2\sqrt{a\sec^4(x)}}$$
$$= \frac{x\sec^2(x)}{2\sqrt{a\sec^4(x)}} + \frac{\tan(x)}{2\sqrt{a\sec^4(x)}}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 0.64

$$\frac{x\sec^2(x)+\tan(x)}{2\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sec[x]^4],x]
[Out] (x*Sec[x]^2 + Tan[x])/(2*Sqrt[a*Sec[x]^4])
```

Maple [A]
time = 0.28, size = 22, normalized size = 0.61

method	result	size
default	$\frac{\cos(x)\sin(x)+x}{2\cos(x)^2\sqrt{\frac{a}{\cos(x)^4}}}$	22
risch	$\frac{e^{2ix}x}{2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(e^{2ix}+1)^2} - \frac{ie^{4ix}}{8\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(e^{2ix}+1)^2} + \frac{i}{8\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(e^{2ix}+1)^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(cos(x)*sin(x)+x)/cos(x)^2/(a/cos(x)^4)^(1/2)
```

Maxima [A]
time = 0.52, size = 25, normalized size = 0.69

$$\frac{x}{2\sqrt{a}} + \frac{\tan(x)}{2\left(\sqrt{a}\tan(x)^2+\sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

[Out] 1/2*x/sqrt(a) + 1/2*tan(x)/(sqrt(a)*tan(x)^2 + sqrt(a))

Fricas [**A**]
time = 2.84, size = 27, normalized size = 0.75

$$\frac{\left(\cos\left(x\right)^{3}\sin\left(x\right)+x\cos\left(x\right)^{2}\right)\sqrt{\frac{a}{\cos\left(x\right)^{4}}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
[Out] 1/2*(cos(x)^3*sin(x) + x*cos(x)^2)*sqrt(a/cos(x)^4)/a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a\sec^{4}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(1/2),x)
[Out] Integral(1/sqrt(a*sec(x)**4), x)
```

Giac [**A**]
time = 0.42, size = 39, normalized size = 1.08

$$-\frac{1}{2}\,\sqrt{a}\left(\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-x}{a}-\frac{\tan\left(x\right)}{\left(\tan\left(x\right)^{2}+1\right)a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(a)*((pi*floor(x/pi + 1/2) - x)/a - tan(x)/((tan(x)^2 + 1)*a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{\frac{a}{\cos\left(x\right)^{4}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^4)^(1/2),x)
[Out] int(1/(a/cos(x)^4)^(1/2), x)
```

3.66 $\int \frac{1}{\left(a \sec^4(x)\right)^{3/2}} \, dx$

Optimal. Leaf size=86

$$\frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}}+\frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}}+\frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}}+\frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}$$

```
[Out] 5/16*x*sec(x)^2/a/(a*sec(x)^4)^(1/2)+5/24*cos(x)*sin(x)/a/(a*sec(x)^4)^(1/2
)+1/6*cos(x)^3*sin(x)/a/(a*sec(x)^4)^(1/2)+5/16*tan(x)/a/(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}}+\frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}+\frac{\sin(x)\cos^3(x)}{6a\sqrt{a\sec^4(x)}}+\frac{5\sin(x)\cos(x)}{24a\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(-3/2),x]
```

```
[Out] (5*x*Sec[x]^2)/(16*a*Sqrt[a*Sec[x]^4]) + (5*Cos[x]*Sin[x])/(24*a*Sqrt[a*Sec
[x]^4]) + (Cos[x]^3*Sin[x])/(6*a*Sqrt[a*Sec[x]^4]) + (5*Tan[x])/(16*a*Sqrt[
a*Sec[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a\sec^4(x))^{3/2}}\,dx = \frac{\sec^2(x)\int \cos^6(x)\,dx}{a\sqrt{a\sec^4(x)}}$$
$$= \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{(5\sec^2(x))\int \cos^4(x)\,dx}{6a\sqrt{a\sec^4(x)}}$$
$$= \frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}} + \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{(5\sec^2(x))\int \cos^2(x)\,dx}{8a\sqrt{a\sec^4(x)}}$$
$$= \frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}} + \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}} + \frac{(5\sec^2(x))\int 1\,dx}{16a\sqrt{a\sec^4(x)}}$$
$$= \frac{5x\sec^2(x)}{16a\sqrt{a\sec^4(x)}} + \frac{5\cos(x)\sin(x)}{24a\sqrt{a\sec^4(x)}} + \frac{\cos^3(x)\sin(x)}{6a\sqrt{a\sec^4(x)}} + \frac{5\tan(x)}{16a\sqrt{a\sec^4(x)}}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.44

$$\frac{\sec^6(x)(60x + 45\sin(2x) + 9\sin(4x) + \sin(6x))}{192\,(a\sec^4(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(-3/2),x]
```

```
[Out] (Sec[x]^6*(60*x + 45*Sin[2*x] + 9*Sin[4*x] + Sin[6*x]))/(192*(a*Sec[x]^4)^(
3/2))
```

Maple [A]
time = 0.24, size = 41, normalized size = 0.48

method	result
default	$\frac{8(\cos^5(x))\sin(x)+10(\cos^3(x))\sin(x)+15\cos(x)\sin(x)+15x}{48\cos(x)^6\left(\frac{a}{\cos(x)^4}\right)^{\frac{3}{2}}}$
risch	$\frac{5\,e^{2ix}x}{16a(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}} - \frac{ie^{8ix}}{384a(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}} - \frac{3ie^{6ix}}{128a(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}} - \frac{15ie^{4ix}}{128a(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(8*cos(x)^5*sin(x)+10*cos(x)^3*sin(x)+15*cos(x)*sin(x)+15*x)/cos(x)^6/
(a/cos(x)^4)^(3/2)
```

Maxima [A]
time = 0.51, size = 58, normalized size = 0.67

$$\frac{15\,\tan\left(x\right)^{5}+40\,\tan\left(x\right)^{3}+33\,\tan\left(x\right)}{48\,\left(a^{\frac{3}{2}}\tan\left(x\right)^{6}+3\,a^{\frac{3}{2}}\tan\left(x\right)^{4}+3\,a^{\frac{3}{2}}\tan\left(x\right)^{2}+a^{\frac{3}{2}}\right)}+\frac{5\,x}{16\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*tan(x)^5 + 40*tan(x)^3 + 33*tan(x))/(a^(3/2)*tan(x)^6 + 3*a^(3/2)*
tan(x)^4 + 3*a^(3/2)*tan(x)^2 + a^(3/2)) + 5/16*x/a^(3/2)
```

Fricas [A]
time = 2.80, size = 43, normalized size = 0.50

$$\frac{\left(15\,x\cos\left(x\right)^{2}+\left(8\,\cos\left(x\right)^{7}+10\,\cos\left(x\right)^{5}+15\,\cos\left(x\right)^{3}\right)\sin\left(x\right)\right)\sqrt{\frac{a}{\cos\left(x\right)^{4}}}}{48\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*x*cos(x)^2 + (8*cos(x)^7 + 10*cos(x)^5 + 15*cos(x)^3)*sin(x))*sqrt
(a/cos(x)^4)/a^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sec^{4}\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(-3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cos(x)^4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^4)^(3/2),x)
```

```
[Out] int(1/(a/cos(x)^4)^(3/2), x)
```

3.67 $\int \frac{1}{\left(a\sec^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}}+\frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}}+\frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}}+\frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}}+\frac{63\,\mathrm{tan}}{256a^2\sqrt{a}}$$

```
[Out] 63/256*x*sec(x)^2/a^2/(a*sec(x)^4)^(1/2)+21/128*cos(x)*sin(x)/a^2/(a*sec(x)
^4)^(1/2)+21/160*cos(x)^3*sin(x)/a^2/(a*sec(x)^4)^(1/2)+9/80*cos(x)^5*sin(x
)/a^2/(a*sec(x)^4)^(1/2)+1/10*cos(x)^7*sin(x)/a^2/(a*sec(x)^4)^(1/2)+63/256
*tan(x)/a^2/(a*sec(x)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 2715, 8}

$$\frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{63\tan(x)}{256a^2\sqrt{a\sec^4(x)}}+\frac{\sin(x)\cos^7(x)}{10a^2\sqrt{a\sec^4(x)}}+\frac{9\sin(x)\cos^5(x)}{80a^2\sqrt{a\sec^4(x)}}+\frac{21\sin(x)\cos^3(x)}{160a^2\sqrt{a\sec^4(x)}}+\frac{21\sin(x)\cos(x)}{128a^2\sqrt{a\sec^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[x]^4)^(-5/2),x]
```

```
[Out] (63*x*Sec[x]^2)/(256*a^2*Sqrt[a*Sec[x]^4]) + (21*Cos[x]*Sin[x])/(128*a^2*Sq
rt[a*Sec[x]^4]) + (21*Cos[x]^3*Sin[x])/(160*a^2*Sqrt[a*Sec[x]^4]) + (9*Cos[
x]^5*Sin[x])/(80*a^2*Sqrt[a*Sec[x]^4]) + (Cos[x]^7*Sin[x])/(10*a^2*Sqrt[a*S
ec[x]^4]) + (63*Tan[x])/(256*a^2*Sqrt[a*Sec[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a\sec^4(x))^{5/2}}\,dx = \frac{\sec^2(x)\int \cos^{10}(x)\,dx}{a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{(9\sec^2(x))\int \cos^8(x)\,dx}{10a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{(63\sec^2(x))\int \cos^6(x)\,dx}{80a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{(21\sec^2(x))\int \cos^4(x)\,d}{32a^2\sqrt{a\sec^4(x)}}$$
$$= \frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}} + \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{(6}{}$$
$$= \frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}} + \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + \frac{\cos^7(x)\sin(x)}{10a^2\sqrt{a\sec^4(x)}} + \frac{}{25}$$
$$= \frac{63x\sec^2(x)}{256a^2\sqrt{a\sec^4(x)}} + \frac{21\cos(x)\sin(x)}{128a^2\sqrt{a\sec^4(x)}} + \frac{21\cos^3(x)\sin(x)}{160a^2\sqrt{a\sec^4(x)}} + \frac{9\cos^5(x)\sin(x)}{80a^2\sqrt{a\sec^4(x)}} + $$

Mathematica [A]
time = 0.10, size = 55, normalized size = 0.42

$$\frac{\cos^2(x)\sqrt{a\sec^4(x)}\,(2520x + 2100\sin(2x) + 600\sin(4x) + 150\sin(6x) + 25\sin(8x) + 2\sin(10x))}{10240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[x]^4)^(-5/2),x]
```

```
[Out] (Cos[x]^2*Sqrt[a*Sec[x]^4]*(2520*x + 2100*Sin[2*x] + 600*Sin[4*x] + 150*Sin
[6*x] + 25*Sin[8*x] + 2*Sin[10*x]))/(10240*a^3)
```

Maple [A]
time = 0.45, size = 57, normalized size = 0.43

method	result
default	$\frac{128(\cos^9(x))\sin(x)+144(\cos^7(x))\sin(x)+168(\cos^5(x))\sin(x)+210(\cos^3(x))\sin(x)+315\cos(x)\sin(x)+315x}{1280\cos(x)^{10}\left(\frac{a}{\cos(x)^4}\right)^{\frac{5}{2}}}$
risch	$\frac{63\,e^{2ix}x}{256a^2(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}} - \frac{ie^{12ix}}{10240a^2(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}} - \frac{5ie^{10ix}}{4096a^2(e^{2ix}+1)^2\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}} - \frac{105ie^{4ix}}{1024a^2(e^{2ix}+1)^2\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sec(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(128*cos(x)^9*sin(x)+144*cos(x)^7*sin(x)+168*cos(x)^5*sin(x)+210*cos
(x)^3*sin(x)+315*cos(x)*sin(x)+315*x)/cos(x)^10/(a/cos(x)^4)^(5/2)
```

Maxima [A]
time = 0.51, size = 88, normalized size = 0.67

$$\frac{315 \tan(x)^9 + 1470 \tan(x)^7 + 2688 \tan(x)^5 + 2370 \tan(x)^3 + 965 \tan(x)}{1280 \left(a^{\frac{5}{2}} \tan(x)^{10} + 5 a^{\frac{5}{2}} \tan(x)^8 + 10 a^{\frac{5}{2}} \tan(x)^6 + 10 a^{\frac{5}{2}} \tan(x)^4 + 5 a^{\frac{5}{2}} \tan(x)^2 + a^{\frac{5}{2}}\right)} + \frac{63 x}{256 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/1280*(315*tan(x)^9 + 1470*tan(x)^7 + 2688*tan(x)^5 + 2370*tan(x)^3 + 965*
tan(x))/(a^(5/2)*tan(x)^10 + 5*a^(5/2)*tan(x)^8 + 10*a^(5/2)*tan(x)^6 + 10*
a^(5/2)*tan(x)^4 + 5*a^(5/2)*tan(x)^2 + a^(5/2)) + 63/256*x/a^(5/2)
```

Fricas [A]
time = 3.23, size = 55, normalized size = 0.42

$$\frac{\left(315 x \cos(x)^2 + \left(128 \cos(x)^{11} + 144 \cos(x)^9 + 168 \cos(x)^7 + 210 \cos(x)^5 + 315 \cos(x)^3\right) \sin(x)\right) \sqrt{\frac{a}{\cos(x)^4}}}{1280 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(315*x*cos(x)^2 + (128*cos(x)^11 + 144*cos(x)^9 + 168*cos(x)^7 + 210
*cos(x)^5 + 315*cos(x)^3)*sin(x))*sqrt(a/cos(x)^4)/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \sec^4(x)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sec(x)**4)**(-5/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sec(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{a}{\cos(x)^4}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/cos(x)^4)^(5/2),x)
```

```
[Out] int(1/(a/cos(x)^4)^(5/2), x)
```

3.68 $\int ((b\sec(c+dx))^p)^n \, dx$

Optimal. Leaf size=81

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)((b\sec(c+dx))^p)^n\sin(c+dx)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -cos(d*x+c)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(d*x+c)^2)*((b*
sec(d*x+c))^p)^n*sin(d*x+c)/d/(-n*p+1)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4208, 3857, 2722}

$$-\frac{\sin(c+dx)\cos(c+dx)((b\sec(c+dx))^p)^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^p)^n,x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d*
x]^2]*((b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]^2
]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int ((b\sec(c+dx))^p)^n\,dx = \left((b\sec(c+dx))^{-np}((b\sec(c+dx))^p)^n\right)\int (b\sec(c+dx))^{np}\,dx$$
$$= \left(\left(\frac{\cos(c+dx)}{b}\right)^{np}((b\sec(c+dx))^p)^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-np}dx$$
$$= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)((b\sec(c+dx))^p)^n\sin(}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 0.85

$$\frac{\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{np}{2};\frac{1}{2}(2+np);\sec^2(c+dx)\right)((b\sec(c+dx))^p)^n\sqrt{-\tan^2(c+dx)}}{dnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^p)^n,x]
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]*
((b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int ((b\sec(dx+c))^p)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*sec(d*x+c))^p)^n,x)
[Out] int(((b*sec(d*x+c))^p)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="maxima")
[Out] integrate(((b*sec(d*x + c))^p)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="fricas")
[Out] integral(((b*sec(d*x + c))^p)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(b \sec \left(c+dx\right)\right)^{p}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))**p)**n,x)
[Out] Integral(((b*sec(c + d*x))**p)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*sec(d*x+c))^p)^n,x, algorithm="giac")
[Out] integrate(((b*sec(d*x + c))^p)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left({\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^p\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b/cos(c + d*x))^p)^n,x)
[Out] int(((b/cos(c + d*x))^p)^n, x)
```

3.69 $\int (a(b\sec(c+dx))^p)^n \, dx$

Optimal. Leaf size=83

$$-\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n\sin(c+dx)}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -cos(d*x+c)*hypergeom([1/2, -1/2*n*p+1/2],[-1/2*n*p+3/2],cos(d*x+c)^2)*(a*(
b*sec(d*x+c))^p)^n*sin(d*x+c)/d/(-n*p+1)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4208, 3857, 2722}

$$-\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-np);\frac{1}{2}(3-np);\cos^2(c+dx)\right)(a(b\sec(c+dx))^p)^n}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Sec[c + d*x])^p)^n,x]
```

```
[Out] -((Cos[c + d*x]*Hypergeometric2F1[1/2, (1 - n*p)/2, (3 - n*p)/2, Cos[c + d*
x]^2]*(a*(b*Sec[c + d*x])^p)^n*Sin[c + d*x])/(d*(1 - n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
& !IntegerQ[p]
```

Rubi steps

$$\int (a(b\sec(c+dx))^p)^n \, dx = \left((b\sec(c+dx))^{-np} (a(b\sec(c+dx))^p)^n\right) \int (b\sec(c+dx))^{np} \, dx$$
$$= \left(\left(\frac{\cos(c+dx)}{b}\right)^{np} (a(b\sec(c+dx))^p)^n\right) \int \left(\frac{\cos(c+dx)}{b}\right)^{-np} dx$$
$$= -\frac{\cos(c+dx)\, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1-np); \frac{1}{2}(3-np); \cos^2(c+dx)\right) (a(b\sec(c+dx))^p)^n \sin}{d(1-np)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 0.86

$$\frac{\cot(c+dx)\, {}_2F_1\left(\frac{1}{2}, \frac{np}{2}; \frac{1}{2}(2+np); \sec^2(c+dx)\right) (a(b\sec(c+dx))^p)^n \sqrt{-\tan^2(c+dx)}}{dnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Sec[c + d*x])^p)^n,x]
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, (n*p)/2, (2 + n*p)/2, Sec[c + d*x]^2]*
(a*(b*Sec[c + d*x])^p)^n*Sqrt[-Tan[c + d*x]^2])/(d*n*p)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a(b\sec(dx+c))^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sec(d*x+c))^p)^n,x)
[Out] int((a*(b*sec(d*x+c))^p)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="maxima")
[Out] integrate(((b*sec(d*x + c))^p*a)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="fricas")
[Out] integral(((b*sec(d*x + c))^p*a)^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(b\sec(c+dx))^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))**p)**n,x)
[Out] Integral((a*(b*sec(c + d*x))**p)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sec(d*x+c))^p)^n,x, algorithm="giac")
[Out] integrate(((b*sec(d*x + c))^p*a)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\left(\frac{b}{\cos(c+d\,x)}\right)^p\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b/cos(c + d*x))^p)^n,x)
[Out] int((a*(b/cos(c + d*x))^p)^n, x)
```

3.70 $\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd}+\frac{2(b\sec(c+dx))^{7/2}\sin(}{7b^3d}$$

```
[Out] 10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*(b*sec(d*x+c))^(7/2)*sin(d*x+
c)/b^3/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^3d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21bd}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b*d) + (2*(b*Sec[c + d*x]
)^(7/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^4}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^2}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d} + \frac{5}{21}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21bd} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^3d} + \frac{1}{21}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10(b\sec(c+dx)}{21b}$$

Mathematica [A]
time = 0.23, size = 69, normalized size = 0.71

$$\frac{\sec^2(c+dx)\sqrt{b\sec(c+dx)}\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [C] Result contains complex when optimal does not.
time = 42.84, size = 152, normalized size = 1.57

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-5\right.}{21d\cos(dx+c)^3\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c
```

```
)-1)/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(b/cos(d*x
+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 114, normalized size = 1.18

$$\frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(5\cos(dx+c)^2+3\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{21\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*sqr
t(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec(c+dx)}\,\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
[Out] int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

3.71 $\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d}$$

```
[Out] 2/5*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/b^2/d-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+6/5*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^2d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])^(5/
2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

`EqQ[n^2, 1/4]`

Rubi steps

$$\int \sec^3(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^3}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d} - \frac{1}{5}(3b$$
$$= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^2d} - \frac{}{5\sqrt{}}$$
$$= -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} +$$

Mathematica [A]
time = 0.19, size = 69, normalized size = 0.73

$$\frac{\sec^2(c+dx)\sqrt{b\sec(c+dx)}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]]*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c +
 d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] Result contains complex when optimal does not.
time = 39.44, size = 356, normalized size = 3.75

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(3i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*cos(d*x+c)^3*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d
```

```
*x+c),I)*sin(d*x+c)-3*I*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+3*
I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-3*I*cos(d*x+c)^2*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)
-1)/sin(d*x+c),I)*sin(d*x+c)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*(b/cos(d*x+c)
)^(1/2)/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 120, normalized size = 1.26

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2 \operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 3i\sqrt{2}\sqrt{b}\cos(dx+c)^2 \operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2\left(3\cos(dx+c)^2+1\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{5d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(c+dx)}\,\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.72 $\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*
sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3bd}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d)
 + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^2}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{1}{3}\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3bd} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2(b\sec(c+dx))^{3/}}{3bd}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.74

$$\frac{2(b\sec(c+dx))^{3/2}\left(\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
[Out] (2*(b*Sec[c + d*x])^(3/2)*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + S
in[c + d*x]))/(3*b*d)
```

Maple [C] Result contains complex when optimal does not.
time = 41.90, size = 130, normalized size = 1.88

method	result
default	$-\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\,(\cos(dx+c)-1)\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\cos(}{3d\sin(dx+c)^3\cos(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(b/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)
-1)/sin(d*x+c),I)-cos(d*x+c)+1)*(cos(d*x+c)+1)^2/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 98, normalized size = 1.42

$$\frac{-i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(c+dx\right)}\,\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos \left(c+d\,x\right)}}}{{\cos \left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((b/cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.73 $\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=63

$$-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] -2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*sin(d*x+c)*(b*sec
(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)\sqrt{b\sec(c+dx)}\,dx &= \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b}\\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} - b\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{b\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}\\
&= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 47, normalized size = 0.75

$$\frac{2\sqrt{b\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 46.71, size = 314, normalized size = 4.98

method	result
default	$-\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\,(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(d\ldots}{\sin(\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(b/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(I*sin(d*x+c)*c
os(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*cos(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/
sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
```

```
n(d*x+c)+I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*EllipticE(I*(cos(d*x+c)-1)
/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)+cos(d*x+c)-1)/sin(d*x+c)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 83, normalized size = 1.32

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x +
c))*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec(c+dx)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((b/cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.74 $\int \sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Maple [C] Result contains complex when optimal does not.
time = 31.42, size = 98, normalized size = 2.58

method	result	size
default	$-\frac{2 i \sqrt{\frac{b}{\cos(dx+c)}}\,(\cos(dx+c)-1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)}, i\right)(\cos(dx+c)+1)^2}{d\sin(dx+c)^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*(b/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(cos(d*
x+c)+1)^2/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.64, size = 57, normalized size = 1.50

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c)), x)
```

Mupad [B]
time = 0.23, size = 35, normalized size = 0.92

$$\frac{2\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*(b/cos(c + d*x))^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/d
```

3.75 $\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=39

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3856, 2719}

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{b\sec(c+dx)}\,dx = b\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{b\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 1.00

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 27.28, size = 303, normalized size = 7.77

method	result
default	$\frac{2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\cdots\right.}{\cdots}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*cos(d*x+c)*Elliptic
E(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Ellipti
cE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*(b/cos(d*x+c))^(1/2)/
sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 63, normalized size = 1.62

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec(c+dx)}\,\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cos(c+d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(b/cos(c + d*x))^(1/2), x)
```

3.76 $\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*b*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*
sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d)
 + (2*b*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)\sqrt{b\sec(c+dx)}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{1}{3}\int \sqrt{b\sec(c+dx)}\,dx \\
&= \frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}} \\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.76

$$
\frac{\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin
[2*(c + d*x)]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 31.47, size = 123, normalized size = 1.84

method	result
default	$\frac{2(\cos(dx+c)-1)\left(-i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)+\cos^2(dx+c)-\cos(dx+c)\right)(\cos(dx+}{3d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(cos(d*x+c)-1)*(-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+cos(d*x+c)^2-c
os(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 84, normalized size = 1.25

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec(c+dx)}\,\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\cos(c+d\,x)^2\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2), x)
```

3.77 $\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)+6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*b^2*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)\sqrt{b\sec(c+dx)}\,dx &= b^3\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{1}{5}(3b)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{(3b)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.81

$$
\frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] Result contains complex when optimal does not.
time = 29.36, size = 315, normalized size = 4.50

method	result
default	$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\left(3i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-3i\cos(dx+c)\operatorname{EllipticE}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(b/cos(d*x+c))^(1/2)*(3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*si
n(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*EllipticF(I*(cos
```

```
(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^4-2*
cos(d*x+c)^2+3*cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 92, normalized size = 1.31

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2), x)
```

3.78 $\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*b*sin(d*x+c)/d/(b*sec(d*x+c
))^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
d) + (2*b^3*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b*Sin[c + d*x]
)/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sqrt{b\sec(c+dx)}\,dx &= b^4\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{1}{7}\left(5b^2\right)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{5}{21}\int \sqrt{b\sec(c+dx)}\,dx\\ &= \frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{1}{21}\left(5\sqrt{\cos(c+dx)}\sqrt{b\,\mathrm{s}\right.\\ &= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^3\sin(c+dx)}{7d(b\sec(c+dx))^{5}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.66

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[b*Sec[c + d*x]],x]
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Maple [C] Result contains complex when optimal does not.
time = 28.64, size = 145, normalized size = 1.53

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-3\left(\cos^4(dx+c)\right)+3\left(\cos^3(dx+c)\right)-\right.}{21d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/21/d*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-3*cos(d*x+c
```

```
)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d*x
+c))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 97, normalized size = 1.02

$$\frac{2\left(3\cos\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(c+dx\right)}\,\cos^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*cos(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2), x)
```

3.79 $\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/9*b^4*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b^2*sin(d*x+c)/d/(b*sec(d*x
+c))^(3/2)+14/15*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (14*b*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^2*Sin[c +
d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\sqrt{b\sec(c+dx)}\,dx &= b^5\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{1}{9}\left(7b^3\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{1}{15}(7b)\int \frac{1}{\sqrt{b\sec(c+dx}}\\
&= \frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{(7b)\int\sqrt{\cos(c+dx}}{15\sqrt{\cos(c+dx)}\sqrt{b\sec}}\\
&= \frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^4\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^2\sin}{45d(b\sec(}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 71, normalized size = 0.72

$$\frac{\sqrt{b\sec(c+dx)}\left(84\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))\right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Co
s[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Maple [C] Result contains complex when optimal does not.
time = 28.07, size = 325, normalized size = 3.32

method	result
default	$-\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\left(21i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-21i\sin(dx+c)\cos(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(b/cos(d*x+c))^(1/2)*(21*I*sin(d*x+c)*cos(d*x+c)*EllipticE(I*(cos(d
*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
```

```
^(1/2)-21*I*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*cos(d*x+c)^6+2
1*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*I*sin(d*x+c)*EllipticF(I*(cos(d*
x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 105, normalized size = 1.07

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+7\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)+21i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-21i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
+ c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2), x)
```

3.80 $\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2(b\sec(c+dx))^{7/2}\,\text{si}}{7b^2d}$$

```
[Out] 10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(b*sec(d*x+c))^(7/2)*sin(d*x+c)
/b^2/d+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^2d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}+\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(2
1*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*Sec[c + d*x]
)^(7/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(b\sec(c+dx))^{3/2}\,dx &= \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^3}\\ &= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d}+\frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b}\\ &= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d}+\frac{1}{21}\\ &= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^2d}+\frac{1}{21}\\ &= \frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{10(b\sec(c+dx)}{21}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 64, normalized size = 0.67

$$\frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b*d)
```

Maple [C] Result contains complex when optimal does not.
time = 45.44, size = 152, normalized size = 1.60

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-5(\right.}{21d\sin(dx+c)^3\cos(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c
```

```
)-1)/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(b/cos(d*x
+c))^(3/2)/sin(d*x+c)^3/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 117, normalized size = 1.23

$$\frac{-5i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(5\,b\cos\left(dx+c\right)^{2}+3\,b\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{21\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b*cos(d*x + c)^2 + 3*b)
*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.81 $\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=98

$$-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd}$$

```
[Out] 2/5*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/b/d-6/5*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+6/5*b*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5bd}+\frac{6b\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^2}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} + \frac{3}{5}\int (b\sec(c+dx))^{3/2}\,dx$$
$$= \frac{6b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} - \frac{1}{5}(3$$
$$= \frac{6b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5bd} - \frac{}{5\sqrt{}}$$
$$= -\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{6b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.19, size = 64, normalized size = 0.65

$$\frac{(b\sec(c+dx))^{5/2}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
[Out] ((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*b*d)
```

Maple [C] Result contains complex when optimal does not.
time = 42.89, size = 356, normalized size = 3.63

method	result
default	$\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(3i\left(\cos^3(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sin(dx+c)-3i(c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*sin(d*x+c)*cos(d*x+c)^3*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c
```

```
)-1)/sin(d*x+c),I)-3*I*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*I
*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-3*I*sin(d*x+c)*cos(d*x+c)
^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cos(d*x+c)-1)/sin(d*x+c),I)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)*(b/cos(d*x+c))
^(3/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 121, normalized size = 1.23

$$\frac{-3i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{5\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*b^(3/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(3/2)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*b*cos(d*x + c)^2 + b)*sqrt(b/cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] int((b/cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.82 $\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}$$

```
[Out] 2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*
sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(b\sec(c+dx))^{3/2}\,dx &= \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b}\\ &= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}b\int \sqrt{b\sec(c+dx)}\,dx\\ &= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\left(b\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int\\ &= \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2(b\sec(c+dx))^{3/2}}{3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + T
an[c + d*x]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 42.55, size = 122, normalized size = 1.82

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\cos(dx\right.}{3d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/
sin(d*x+c),I)-cos(d*x+c)+1)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 99, normalized size = 1.48

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\,b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*b*sqrt(b/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{3}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/cos(c + d*x),x)
[Out] int((b/cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.83 $\int (b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

```
[Out] -2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*b*sin(d*x+c)*(b
*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$\frac{2b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} - b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{b^2\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

$$= -\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
+ Sin[c + d*x]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 43.45, size = 322, normalized size = 4.88

method	result
default	$\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-i\sin(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x
+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)*
EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-cos(d*x+c)+1
)*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 84, normalized size = 1.27

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\,b\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b*sqrt(b/cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2),x)
[Out] Integral((b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2),x)
[Out] int((b/cos(c + d*x))^(3/2), x)
```

3.84 $\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=39

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3856, 2720}

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(b\sec(c+dx))^{3/2}\,dx &= b\int \sqrt{b\sec(c+dx)}\,dx\\
&= \left(b\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 1.00

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Maple [C] Result contains complex when optimal does not.
time = 31.97, size = 98, normalized size = 2.51

method	result	size
default	$-\frac{2i\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}(\cos(dx+c)-1)\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)(\cos(dx+c)+1)^{3}\sqrt{\frac{1}{\cos(dx+c)+1}}}{d\sin(dx+c)^{2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*(b/cos(d*x+c))^(3/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*(cos(d*x+c)-1
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(cos(d*x+c)+1)^3*(1/(cos(d*x+c)+
1))^(1/2)/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 57, normalized size = 1.46

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b^{\frac{3}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^{\frac{3}{2}}\cos (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cos (c+d\,x)\,{\left(\frac{b}{\cos (c+d\,x)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)*(b/cos(c + d*x))^(3/2), x)
```

3.85 $\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=41

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3856, 2719}

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{3/2}\,dx = b^2\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$
$$= \frac{b^2\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 1.00

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Maple [C] Result contains complex when optimal does not.
time = 28.62, size = 309, normalized size = 7.54

method	result
default	$\frac{2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-i\cos(dx+c)\,\text{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\right.}{}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*cos(d*x+c)*Elliptic
E(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*Ellipti
cE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
```

```
d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*cos(d*x+c)*(b/cos(d*x
+c))^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.28, size = 63, normalized size = 1.54

$$\frac{i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2), x)
```

3.86 $\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*
d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos^3(c+dx)(b\sec(c+dx))^{3/2}\,dx = b^3\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{1}{3}b\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}} + \frac{1}{3}\left(b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}$$
$$= \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b^2\sin(c+dx)}{3d\sqrt{b\sec(c+dx}}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.74

$$\frac{b\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + S
in[2*(c + d*x)]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 29.22, size = 129, normalized size = 1.84

method	result
default	$-\frac{2(\cos(dx+c)+1)^2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}(\cos(dx+c)-1)\cos(dx+c)\left(i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \sin(}{3d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)*(cos(d*x+c)-1)*cos(d*x+c)*(I*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2+cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 85, normalized size = 1.21

$$\frac{2 b \sqrt{\frac{b}{\cos(dx+c)}} \cos(dx+c)\sin(dx+c) - i\sqrt{2}\, b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + i\sqrt{2}\, b^{\frac{3}{2}} \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*b^(3/2)
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*b^(3
/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^3 \left(\frac{b}{\cos(c+d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2), x)
```

3.87 $\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)+6/5*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(
1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (6*b^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^3*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(b\sec(c+dx))^{3/2}\,dx &= b^4\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{1}{5}\left(3b^2\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{\left(3b^2\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^3\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.81

$$\frac{b\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] Result contains complex when optimal does not.
time = 30.39, size = 319, normalized size = 4.43

method	result
default	$-\frac{2\left(-3i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+3i\cos(dx+c)\,\text{EllipticE}\left(\frac{i(\cos(dx+\ldots}{\sin(dx+\ldots}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(-3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*EllipticE(I
*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+
```

```
c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(
d*x+c))*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.98, size = 93, normalized size = 1.29

$$\frac{2\,b\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+3i\,\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\,\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*b^(
3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2), x)
```

3.88 $\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*b^4*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*b^2*sin(d*x+c)/d/(b*sec(d*x
+c))^(1/2)+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(2
1*d) + (2*b^4*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b^2*Sin[c +
d*x])/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos^5(c+dx)(b\sec(c+dx))^{3/2}\,dx = b^5\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}\left(5b^3\right)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}(5b)\int \sqrt{b\sec(c+$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^2\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\Big(5b\sqrt{\cos(c+dx)}$$
$$= \frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^4\sin(c+}{7d(b\sec(c+d}$$

Mathematica [A]
time = 0.09, size = 64, normalized size = 0.65

$$\frac{b\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Maple [C] Result contains complex when optimal does not.
time = 30.08, size = 151, normalized size = 1.54

method	result
default	$\frac{2(\cos(dx+c)+1)^2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}(\cos(dx+c)-1)\cos(dx+c)\left(-5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{si}\right.}{21d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)*(cos(d*x+c)-1)*cos(d*x+c)*(-5*
I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(c
```

```
os(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+3*cos(d*x+c)^4-3*cos(d*x+c)^3+5*cos(d
*x+c)^2-5*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 99, normalized size = 1.01

$$\frac{-5i\,\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5i\,\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+2\,\left(3\,b\cos\left(dx+c\right)^{3}+5\,b\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(3*b*cos(d*x + c)^3 + 5*b*cos(d*x + c))*sqrt(b/cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^5\,\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2), x)
```

3.89 $\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=100

$$\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/9*b^5*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b^3*sin(d*x+c)/d/(b*sec(d*x
+c))^(3/2)+14/15*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (14*b^2*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b^5*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^3*Sin[c
+ d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(b\sec(c+dx))^{3/2}\,dx &= b^6\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx\\ &= \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{1}{9}\left(7b^4\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{1}{15}\left(7b^2\right)\int \frac{1}{\sqrt{b\sec(c\cdots}}\\ &= \frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{\left(7b^2\right)\int\sqrt{\cos(c+\cdots}}{15\sqrt{\cos(c+dx)}\sqrt{b\cdots}}\\ &= \frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^5\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^3\cdots}{45d(b\,\mathrm{se}\cdots}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 72, normalized size = 0.72

$$\frac{b\sqrt{b\sec(c+dx)}\left(84\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))\right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Maple [C] Result contains complex when optimal does not.
time = 32.34, size = 331, normalized size = 3.31

method	result
default	$-\frac{2\left(5\left(\cos^6(dx+c)\right)+21i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-21i\sin(dx+c)\cos\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(5*cos(d*x+c)^6+21*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d
*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

```
-21*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21*I*sin(d*x+c)*Ellipt
icE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-21*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+2*cos(d*x+c)^4+
14*cos(d*x+c)^2-21*cos(d*x+c))*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 107, normalized size = 1.07

$$\frac{21i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(5b\cos(dx+c)^4+7b\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{45d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(21*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b*co
s(d*x + c)^4 + 7*b*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*cos(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^6\,\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2), x)
```

3.90 $\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{10b(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d}+\frac{2(b\sec(c+dx))^{7/2}\text{s}}{7bd}$$

```
[Out] 10/21*b*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(b*sec(d*x+c))^(7/2)*sin(d*x+
c)/b/d+10/21*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7bd}+\frac{10b\sin(c+dx)(b\sec(c+dx))^{3/2}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(21*d) + (10*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*(b*Sec[c +
d*x])^(7/2)*Sin[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(c+dx)(b\sec(c+dx))^{5/2}\,dx &= \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^2}\\
&= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd} + \frac{5}{7}\int (b\sec(c+dx))^{5/2}\,dx\\
&= \frac{10b(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd} +\\
&= \frac{10b(b\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7bd} +\\
&= \frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{10b(b\sec(c+
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 61, normalized size = 0.62

$$\frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [C] Result contains complex when optimal does not.
time = 41.40, size = 152, normalized size = 1.55

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-}{21d\sin(dx+c)^3\cos(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*cos(d*x+c)^3*sin(d*x+c)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c
```

```
)-1)/sin(d*x+c),I)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(b/cos(d*x
+c))^(5/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 121, normalized size = 1.23

$$\frac{-5i\,\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\,\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(5\,b^{2}\cos\left(dx+c\right)^{2}+3\,b^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{21\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b^2*cos(d*x + c)^2 + 3*
b^2)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(5/2)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
[Out] int((b/cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.91 $\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=97

$$-\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/d-6/5*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)
/(b*sec(d*x+c))^(1/2)+6/5*b^2*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3853, 3856, 2719}

$$-\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(b*Sec[c + d*x
])^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{5/2}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}(3b)\int (b\sec(c+dx))^{3/2}\,dx$$
$$= \frac{6b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{1}{5}$$
$$= \frac{6b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{}{5}$$
$$= -\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{6b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.18, size = 61, normalized size = 0.63

$$\frac{(b\sec(c+dx))^{5/2}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
[Out] ((b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] +
 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] Result contains complex when optimal does not.
time = 44.94, size = 348, normalized size = 3.59

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(3i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/5/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^3*(1/(cos(d*x+c)+1
```

```
))^(1/2)*sin(d*x+c)-3*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*Elli
pticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*
I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*I*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)^2*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*(b/cos(d*x+c)
)^(5/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 125, normalized size = 1.29

$$\frac{-3i\sqrt{2}\,b^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,b^{\frac{5}{2}}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3b^2\cos(dx+c)^2+b^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{5d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(5/2)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)^2 + b^2)*sqrt(b/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{5}{2}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

```
[Out] int((b/cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.92 $\int (b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/3*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2720}

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*d) + (2*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(c+dx))^{5/2}\,dx = \frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}\,dx$$
$$= \frac{2b(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\left(b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(}}$$
$$= \frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{2b(b\sec(c+dx))^{3/2}\sin(c+}{3d}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.73

$$\frac{2b^2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Tan[c + d*x]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 43.88, size = 128, normalized size = 1.83

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\cos(dx+c)+1\right)\cos(d}{3d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)-1)*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-cos
(d*x+c)+1)*cos(d*x+c)*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 101, normalized size = 1.44

$$\frac{-i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,b^{2}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*b^2*sqrt(b/cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((b/cos(c + d*x))^(5/2), x)
```

3.93 $\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=68

$$-\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

```
[Out] -2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*b^2*sin(d*x+c)*
(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b\int (b\sec(c+dx))^{3/2}\,dx \\
&= \frac{2b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} - b^3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{b^3\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} \\
&= -\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.74

$$
\frac{2b^2\sqrt{b\sec(c+dx)}\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
) + Sin[c + d*x]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 49.26, size = 322, normalized size = 4.74

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-i\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/
sin(d*x+c),I)-I*cos(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+I*EllipticF(
```

```
I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*
x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)-
1)*cos(d*x+c)^2*(b/cos(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.64, size = 86, normalized size = 1.26

$$\frac{-i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,b^{2}\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*b^2*sqrt(b/cos(d*
x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(b/cos(c + d*x))^(5/2), x)
```

3.94 $\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=41

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] 2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3856, 2720}

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^2\int \sqrt{b\sec(c+dx)}\,dx\\ &= \left(b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.00

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/d
```

Maple [C] Result contains complex when optimal does not.
time = 29.45, size = 98, normalized size = 2.39

method	result	size
default	$-\frac{2i\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{5}{2}}(\cos(dx+c)-1)\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)(\cos(dx+c)+1)^{5}\left(\frac{1}{\cos(dx+c)+1}\right)^{\frac{3}{2}}}{d\sin(dx+c)^{2}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*(b/cos(d*x+c))^(5/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*(cos(d*x+c)-1
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(cos(d*x+c)+1)^5*(1/(cos(d*x+c)+
1))^(3/2)/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.38, size = 57, normalized size = 1.39

$$\frac{-i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2), x)
```

3.95 $\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=41

$$\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3856, 2719}

$$\frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{b^3\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{2b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.93

$$
\frac{2\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)(b\sec(c+dx))^{5/2}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]*(b*Sec[c + d*x])^(5/2))/d
```

Maple [C] Result contains complex when optimal does not.
time = 30.36, size = 311, normalized size = 7.59

method	result
default	$-\frac{2\left(i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\right.}{}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,b^2e^{-i(dx+c)}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{d}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*EllipticF(I*(cos(d
*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c
)-1)/sin(d*x+c),I)*sin(d*x+c)+cos(d*x+c)^2-cos(d*x+c))*cos(d*x+c)^2*(b/cos(
d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 63, normalized size = 1.54

$$\frac{i\sqrt{2}\,b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^3\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2), x)
```

3.96 $\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=72

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)+2/3*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(
3*d) + (2*b^3*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^4\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{1}{3}b^2\int \sqrt{b\sec(c+dx)}\,dx\\ &= \frac{2b^3\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{1}{3}\left(b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}\\ &= \frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2b^3\sin(c+dx}{3d\sqrt{b\sec(c+d}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.75

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[2*(c + d*x)]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 29.70, size = 131, normalized size = 1.82

method	result
default	$-\frac{2(\cos(dx+c)+1)^2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}(\cos(dx+c)-1)(\cos^2(dx+c))\left(i\,\text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{si}\right.}{3d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(5/2)*(cos(d*x+c)-1)*cos(d*x+c)^2*(I
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)^2+cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.10, size = 87, normalized size = 1.21

$$\frac{2\,b^2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-i\,\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+i\,\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*b^(5/
2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*b^
(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2), x)
```

3.97 $\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=72

$$\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*b^4*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)+6/5*b^3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(
1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*b^3*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^5\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{1}{5}\left(3b^3\right)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{\left(3b^3\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{6b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^4\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 0.83

$$
\frac{b^2\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(b*Sec[c + d*x])^(5/2),x]
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ Sin[c + d*x] + Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] Result contains complex when optimal does not.
time = 30.70, size = 321, normalized size = 4.46

method	result
default	$-\frac{2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^2(dx+c)\right)\left(-3i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+3i\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/5/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(-3*I*cos(d*x+c)*sin(d*x+c)*Ellipt
icF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+3*I*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin
(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*s
```

```
in(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1
)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 95, normalized size = 1.32

$$\frac{2\,b^{2}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+3i\,\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3i\,\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*b
^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) - 3*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2), x)
```

3.98 $\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}+\frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*b^5*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*b^3*sin(d*x+c)/d/(b*sec(d*x
+c))^(1/2)+10/21*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(21*d) + (2*b^5*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*b^3*Sin[c
+ d*x])/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos^6(c+dx)(b\sec(c+dx))^{5/2}\,dx = b^6\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}\left(5b^4\right)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\left(5b^2\right)\int \sqrt{b\sec(c+}$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10b^3\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{1}{21}\left(5b^2\sqrt{\cos(c+dx)}\right.$$
$$= \frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21d} + \frac{2b^5\sin(c+}{7d(b\sec(c+}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.66

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ 26*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*d)
```

Maple [C] Result contains complex when optimal does not.
time = 28.70, size = 153, normalized size = 1.53

method	result
default	$\frac{2(\cos(dx+c)+1)^2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}(\cos(dx+c)-1)\left(\cos^2(dx+c)\right)\left(-5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{21d\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(5/2)*(cos(d*x+c)-1)*cos(d*x+c)^2*(-
5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*
```

```
(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+3*cos(d*x+c)^4-3*cos(d*x+c)^3+5*cos
(d*x+c)^2-5*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 103, normalized size = 1.03

$$\frac{-5i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(3\,b^{2}\cos(dx+c)^{3}+5\,b^{2}\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(3*b^2*cos(d*x + c)^3 + 5*b^2*cos(d*x + c))*sqrt(b/cos(d
*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^6\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2), x)
```

3.99 $\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=100

$$\frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/9*b^6*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b^4*sin(d*x+c)/d/(b*sec(d*x
+c))^(3/2)+14/15*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (14*b^3*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b^6*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b^4*Sin[c
+ d*x])/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(b\sec(c+dx))^{5/2}\,dx &= b^7\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx\\ &= \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{1}{9}\left(7b^5\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\ &= \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{1}{15}\left(7b^3\right)\int \frac{1}{\sqrt{b\sec(c+\ldots}}\\ &= \frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{\left(7b^3\right)\int\sqrt{\cos(c+\ldots}}{15\sqrt{\cos(c+dx)}\sqrt{b\ldots}}\\ &= \frac{14b^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^6\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b^4\ldots}{45d(b\,\mathrm{se}\ldots}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 74, normalized size = 0.74

$$\frac{b^2\sqrt{b\sec(c+dx)}\left(84\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^2(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))\right)}{90d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*(84*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
+ Cos[c + d*x]^2*(33*Sin[c + d*x] + 5*Sin[3*(c + d*x)])))/(90*d)
```

Maple [C] Result contains complex when optimal does not.
time = 30.57, size = 333, normalized size = 3.33

method	result
default	$-\frac{2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\cos^2(dx+c)\right)\left(5\left(\cos^6(dx+c)\right)+21i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{s}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(5*cos(d*x+c)^6+21*I*cos(d*x+c)*E
llipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)
```

```
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*I*cos(d*x+c)*EllipticF(I*(cos(d*x+c)-
1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+21*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*I*EllipticF(I*(cos(
d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 111, normalized size = 1.11

$$\frac{21i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(5\,b^2\cos(dx+c)^4+7\,b^2\cos(dx+c)^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(21*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b^2*
cos(d*x + c)^4 + 7*b^2*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*cos(d*x + c)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^7\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2), x)
```

3.100 $\int (b\sec(c+dx))^{7/2}\,dx$

Optimal. Leaf size=98

$$-\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6b^3\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*b*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/d-6/5*b^4*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+6/5*b^3*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3853, 3856, 2719}

$$-\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6b^3\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2b\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-6*b^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*b^3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(b*Sec[c + d
*x])^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(c+dx))^{7/2}\,dx = \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int (b\sec(c+dx))^{3/2}\,dx$$
$$= \frac{6b^3\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{1}{5}\left(3b^4\right)\int \frac{}{\sqrt{}}$$
$$= \frac{6b^3\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b(b\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{(3b^4)\int \sqrt{}}{5\sqrt{\cos(c+d}}$$
$$= -\frac{6b^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{6b^3\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2b(b\sec(}{}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.63

$$\frac{b(b\sec(c+dx))^{5/2}\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (b*(b*Sec[c + d*x])^(5/2)*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
 + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [C] Result contains complex when optimal does not.
time = 49.86, size = 354, normalized size = 3.61

method	result
default	$\frac{2(\cos(dx+c)-1)^2\left(3i\left(\cos^3(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sin(dx+c)-3i\left(\cos^3(dx+c)\right)\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(cos(d*x+c)-1)^2*(3*I*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I
)-3*I*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*I*sin(d*x+c)*cos(d
*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
E(I*(cos(d*x+c)-1)/sin(d*x+c),I)-3*I*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin
```

```
(d*x+c),I)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)*cos(d*x+c)*(cos(d*x+c)+1)^2*(b/
cos(d*x+c))^(7/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 125, normalized size = 1.28

$$\frac{-3i\sqrt{2}\,b^{\frac{7}{2}}\cos\left(dx+c\right)^{2}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,b^{\frac{7}{2}}\cos\left(dx+c\right)^{2}\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,b^{3}\cos\left(dx+c\right)^{2}+b^{3}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{5\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*b^(7/2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(7/2)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*b^3*cos(d*x + c)^2 + b^3)*sqrt(b/cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(7/2),x)
[Out] int((b/cos(c + d*x))^(7/2), x)
```

3.101 $\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd}+\frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d}+\frac{2(b\sec(c+dx))^{7/2}\sin($$

```
[Out] 10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*(b*sec(d*x+c))^(7/2)*sin(d*
x+c)/b^4/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^4d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^2d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^2*d) + (2*(b*Sec[c +
d*x])^(7/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^5}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^3}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d} + \frac{5\int \sqrt{b\sec(c-}{21b}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^2d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^4d} + \frac{\left(5\sqrt{\cos(c+}\right.}{}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+}{21b^2d}$$

Mathematica [**A**]
time = 0.10, size = 69, normalized size = 0.69

$$\frac{\sec^3(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*
(c + d*x)] + 6*Tan[c + d*x]))/(21*d*Sqrt[b*Sec[c + d*x]])
```

Maple [**C**] Result contains complex when optimal does not.
time = 47.64, size = 152, normalized size = 1.52

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(5i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-5(\cos^3(dx+c))+\right.}{21d\sin(dx+c)^3\cos(dx+c)^4\sqrt{\frac{b}{\cos(dx+c)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^
```

```
3-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)*(cos(d*x+c)+1)^2/sin(d*x+c)
^3/cos(d*x+c)^4/(b/cos(d*x+c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.54, size = 117, normalized size = 1.17

$$\frac{-5i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(5\cos\left(dx+c\right)^{2}+3\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{21\,bd\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*sqr
t(b/cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{5}\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(1/2)), x)
```

3.102 $\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5bd}+\frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d}$$

```
[Out] 2/5*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/b^3/d-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)
/(b*sec(d*x+c))^(1/2)+6/5*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^3d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5bd}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b*d) + (2*(b*Sec[c + d*x])^(5/
2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^4}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b^2}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5bd} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d} - \frac{3}{5}\int \frac{1}{\sqrt{b\sec(c+}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5bd} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^3d} - \frac{3\int \sqrt{\cos(}}{5\sqrt{\cos(c+dx)}}$$
$$= -\frac{6E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5bd} + \frac{2(b\sec(c+}{}$$

Mathematica [A]
time = 0.25, size = 61, normalized size = 0.63

$$\frac{-\frac{6E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}} + 2(3+\sec^2(c+dx))\tan(c+dx)}{5d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 47.76, size = 356, normalized size = 3.67

method	result
default	$-\frac{2(\cos(dx+c)-1)^2\left(3i\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-3i\left(\cos^3(dx+c)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(cos(d*x+c)-1)^2*(3*I*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),
I)-3*I*cos(d*x+c)^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*I*(1/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*Ellipti
cF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-3*I*cos(d*x+c)^2*sin(d*x+c)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/si
n(d*x+c),I)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*(cos(d*x+c)+1)^2/cos(d*x+c)^3/
(b/cos(d*x+c))^(1/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 123, normalized size = 1.27

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2 \,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 3i\sqrt{2}\sqrt{b}\cos(dx+c)^2 \,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2\left(3\cos(dx+c)^2+1\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{5\,bd\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x +
c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(1/2)), x)
```

3.103 $\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{3bd}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d}$$

```
[Out] 2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(
b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^2d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b*
d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^3}\\
&= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d}+\frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b}\\
&= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^2d}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+d}}}{3b}\\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+dx}{3b^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 51, normalized size = 0.71

$$\frac{2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\tan(c+dx)\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan
[c + d*x]))/(3*b*d)
```

Maple [C] Result contains complex when optimal does not.
time = 47.07, size = 130, normalized size = 1.81

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\cos(dx+c)+1\right)(\cos(}{3d\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)-1)*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-cos
(d*x+c)+1)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 101, normalized size = 1.40

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3\,bd\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c))/
(b*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(1/2)), x)
```

3.104 $\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\sin(c+dx)}{bd}$$

```
[Out] -2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*sin(d*x+c)*(b*sec(d
*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{bd}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
+ (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b^2}\\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{bd} - \int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}\\
&= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{bd}
\end{aligned}
$$

Mathematica [**A**]
time = 0.09, size = 48, normalized size = 0.74

$$
\frac{-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2\tan(c+dx)}{d\sqrt{b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(d*Sqr
t[b*Sec[c + d*x]])
```

Maple [**C**] Result contains complex when optimal does not.
time = 47.33, size = 319, normalized size = 4.91

method	result
default	$\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-i\sin(c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x
+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+
```

```
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos
(d*x+c)-1)/sin(d*x+c),I)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-cos(d*x+c)+1
)*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 86, normalized size = 1.32

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x +
c))*sin(d*x + c))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/2)), x)
```

3.105 $\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{bd}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\int \sqrt{b\sec(c+dx)}\,dx}{b}$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Mathematica [**A**]
time = 0.02, size = 41, normalized size = 1.00

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b*d)
```

Maple [**C**] Result contains complex when optimal does not.
time = 31.97, size = 98, normalized size = 2.39

method	result	size
default	$-\frac{2i\left(\frac{1}{\cos(dx+c)+1}\right)^{\frac{3}{2}}(\cos(dx+c)-1)\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)(\cos(dx+c)+1)^2}{d\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*(1/(cos(d*x+c)+1))^(3/2)*(cos(d*x+c)-1)*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(1/2)/(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)/sin(d*x+c)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 60, normalized size = 1.46

$$\frac{-i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/2)), x)
```

3.106 $\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3856, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\ &= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sec[c + d*x]],x]
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 29.86, size = 306, normalized size = 8.05

method	result
risch	$-\frac{i\sqrt{2}}{d\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(\cdots}}}{\cdots}\right.}{\cdots}$
default	$\frac{2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*sin(d*x+c)*cos(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
cos(d*x+c)-1)/sin(d*x+c),I)+I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x
+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*sin(d*x
+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))*(b/cos(d*x+c))^(1/2)/
sin(d*x+c)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 66, normalized size = 1.74

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(1/(b/cos(c + d*x))^(1/2), x)
```

3.107 $\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd}+\frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*se
c(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b*
d) + (2*Sin[c + d*x])/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx\\
&= \frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b}\\
&= \frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b}\\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3bd}+\frac{2\sin(c+dx)}{3d\sqrt{b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.87

$$\frac{b\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (b*Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(
c + d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 32.52, size = 126, normalized size = 1.83

method	result
default	$\frac{2(\cos(dx+c)-1)\left(-i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)+\cos^2(dx+c)-\cos(dx+c)\right)(\cos(d}{3db\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(cos(d*x+c)-1)*(-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+cos(d*x+c)^2-c
os(d*x+c))*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 87, normalized size = 1.26

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)/(b/cos(c + d*x))^(1/2), x)
```

3.108 $\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*b*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(
b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
 + (2*b*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx\\
&= \frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{3}{5}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\
&= \frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{3\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}\\
&= \frac{6E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.90

$$
\frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b*d)
```

Maple [C] Result contains complex when optimal does not.
time = 32.36, size = 316, normalized size = 4.72

method	result
default	$-\frac{2\left(-3i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+3i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+}{\sin(dx+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(-3*I*cos(d*x+c)*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*cos(d*x+c)*
sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-
```

```
1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(
d*x+c))*(b/cos(d*x+c))^(1/2)/b/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 95, normalized size = 1.42

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/2), x)
```

3.109 $\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd}+\frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/d/(b*sec(d*x+c))
^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2720}

$$\frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b*d) + (2*b^2*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^3\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{1}{7}(5b)\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx \\
&= \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b} \\
&= \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21d\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)}{21b} \\
&= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21bd} + \frac{2b^2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10}{21d\sqrt{}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.68

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[b*Sec[c + d*x]],x]
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b*d)
```

Maple [C] Result contains complex when optimal does not.
time = 28.48, size = 148, normalized size = 1.53

method	result
default	$\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(-5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)+3(\cos^4(dx+c))\right.}{21db\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(-5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d
*x+c)+3*cos(d*x+c)^4-3*cos(d*x+c)^3+5*cos(d*x+c)^2-5*cos(d*x+c))*(b/cos(d*x
+c))^(1/2)/b/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.12, size = 100, normalized size = 1.03

$$\frac{2\left(3\cos\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))}{21\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)))/(b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^3/(b/cos(c + d*x))^(1/2), x)
```

3.110 $\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/9*b^3*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*b*sin(d*x+c)/d/(b*sec(d*x+c
))^(3/2)+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*b^3*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*b*Sin[c + d*x]
)/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= b^4\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{1}{9}\left(7b^2\right)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7}{15}\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7\int \sqrt{\cos(c+dx)}\,dx}{15\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^3\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14b\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 70, normalized size = 0.74

$$\frac{\frac{336E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 4\cos(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))}{360d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] ((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*Si
n[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 30.84, size = 328, normalized size = 3.45

method	result
default	$2\left(21i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-21i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-}{\sin(dx+c)}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/d*(21*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*I*EllipticE(
I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)^6+21*I*EllipticF(I*(cos(d
*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)-21*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cos(d*x+c)^4-1
4*cos(d*x+c)^2+21*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 108, normalized size = 1.14

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+7\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{45\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
+ c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(c+dx\right)}{\sqrt{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4/(b/cos(c + d*x))^(1/2), x)
```

3.111 $\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d}+\frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d}$$

```
[Out] 10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/7*(b*sec(d*x+c))^(7/2)*sin(d*
x+c)/b^5/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^5d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^3d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^2*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^3*d) + (2*(b*Sec[c
+ d*x])^(7/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^6}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^4}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d} + \frac{5\int \sqrt{b\sec(c}{21b}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^3d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^5d} + \frac{\left(5\sqrt{\cos(c+\right.}{}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{21b^2d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+}{21b^3d}$$

Mathematica [A]
time = 0.18, size = 69, normalized size = 0.69

$$\frac{\sec^4(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*
(c + d*x)] + 6*Tan[c + d*x]))/(21*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 48.15, size = 152, normalized size = 1.52

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i(\cos^3(dx+c))\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\right.}{21d\cos(dx+c)^5\sin(dx+c)^3\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d
```

```
*x+c)^3*sin(d*x+c)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)/cos(d*x+c)
^5/sin(d*x+c)^3/(b/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 117, normalized size = 1.17

$$\frac{-5i\,\sqrt{2}\,\sqrt{b}\,\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\,\sqrt{2}\,\sqrt{b}\,\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(5\,\cos\left(dx+c\right)^{2}+3\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{21\,b^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*sqr
t(b/cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{6}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^6\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(3/2)), x)
```

3.112 $\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6\sqrt{b\sec(c+dx)}\sin(c+dx)}{5b^2d}+\frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d}$$

```
[Out] 2/5*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/b^4/d-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+6/5*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^4d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5b^2d}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b^2*d) + (2*(b*Sec[c + d*x])
^(5/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^5}$$
$$= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b^3}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d} - \frac{3\int \frac{1}{\sqrt{b\sec(c+}}}{5b}$$
$$= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^4d} - \frac{3\int \sqrt{\cos}}{5b\sqrt{\cos(c+dx}}$$
$$= -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^2d} + \frac{2(b\sec(c}{}$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.64

$$\frac{-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2(3+\sec^2(c+dx))\tan(c+dx)}{5bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*b*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 48.59, size = 356, normalized size = 3.56

method	result
default	$\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(3i(\cos^3(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sin(dx+c)-3i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*sin(d*x+c)*cos(d*x+c)^3*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c
)-1)/sin(d*x+c),I)-3*I*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*I
*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-3*I*sin(d*x+c)*cos(d*x+c)
^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cos(d*x+c)-1)/sin(d*x+c),I)-3*cos(d*x+c)^3+2*cos(d*x+c)^2+1)/sin(d*x+c)^5/(
b/cos(d*x+c))^(3/2)/cos(d*x+c)^4
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 123, normalized size = 1.23

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3\cos(dx+c)^2+1\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{5\,b^2d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x +
c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(3/2)), x)
```

3.113 $\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d}$$

```
[Out] 2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(
b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^3d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
2*d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^4}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^2}$$
$$= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^3d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2(b\sec(c+dx))^{3/2}\sin(c+d}{3b^3d}$$

Mathematica [A]
time = 0.12, size = 56, normalized size = 0.78

$$\frac{2\sec^3(c+dx)\left(\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sin(c+dx)\right)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(3/2),x]
[Out] (2*Sec[c + d*x]^3*(Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + Sin[c + d
*x]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 48.37, size = 125, normalized size = 1.74

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\cos(dx+c)+1\right)(\cos(}{3d\,b^3\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(cos(d*x+c)-1)*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-cos
(d*x+c)+1)*(cos(d*x+c)+1)^2*(b/cos(d*x+c))^(3/2)/b^3/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 101, normalized size = 1.40

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3\,b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c))/
(b^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(3/2)), x)
```

3.114 $\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^2d}$$

```
[Out] -2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*sin(d*x+c)*(b*sec
(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{b^2d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]
) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b^3} \\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^2d} - \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b} \\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^2d} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} \\
&= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.75

$$
\frac{-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2\tan(c+dx)}{bd\sqrt{b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b*d*S
qrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 51.00, size = 322, normalized size = 4.74

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-i\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/
sin(d*x+c),I)-I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
```

```
(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+I*sin(d*x+c)
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c
),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)-
1)/cos(d*x+c)^2/sin(d*x+c)^5/(b/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 86, normalized size = 1.26

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x +
c))*sin(d*x + c))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(3/2)), x)
```

3.115 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2*
d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int \sqrt{b\sec(c+dx)}\,dx}{b^2}$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.00

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^2*
d)
```

Maple [C] Result contains complex when optimal does not.
time = 35.84, size = 98, normalized size = 2.39

method	result	size
default	$-\frac{2i\left(\frac{1}{\cos(dx+c)+1}\right)^{\frac{5}{2}}(\cos(dx+c)-1)\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)(\cos(dx+c)+1)^2}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}\sin(dx+c)^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*(1/(cos(d*x+c)+1))^(5/2)*(cos(d*x+c)-1)*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(3/2)/(cos(d*x+c)/(cos(d*x+c)+
1))^(3/2)/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 60, normalized size = 1.46

$$\frac{-i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2{(c+dx)}}{(b\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\cos{(c+d\,x)}^2\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(3/2)), x)
```

3.116 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3856, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 34.57, size = 311, normalized size = 7.59

method	result
risch	$-\frac{i\sqrt{2}}{db\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}} + \frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i}}}{}\right.}{}$
default	$\frac{2i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-2i\cos(dx+c)\ \mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*sin(d*x+c)*cos(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
cos(d*x+c)-1)/sin(d*x+c),I)+I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x
+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*sin(d*x
```

```
+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))/cos(d*x+c)^2/(b/cos(d
*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.15, size = 66, normalized size = 1.61

$$\frac{i\sqrt{2}\,\sqrt{b}\;\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,\sqrt{b}\;\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(3/2)), x)
```

3.117 $\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*
sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
2*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx = \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} + \frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^2}$$
$$= \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d} + \frac{2\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 0.82

$$\frac{\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
+ d*x)]))/(3*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 35.14, size = 131, normalized size = 1.82

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-(\cos^2(dx+c))+\mathrm{co}\right.}{3d\sin(dx+c)^3\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(I*sin(d*x+c)*EllipticF(I*(cos(d*x+c
)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)-cos(d*x+c)^2+cos(d*x+c))/sin(d*x+c)^3/(b/cos(d*x+c))^(3/2)/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c))^(-3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 87, normalized size = 1.21

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(b^2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*sec(d*x+c))**(3/2),x)

[Out] Integral((b*sec(c + d*x))**(-3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c))^(-3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b/cos(c + d*x))^(3/2),x)

[Out] int(1/(b/cos(c + d*x))^(3/2), x)

3.118 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*sin(d*x+c)/d/(b*sec(d*x+c))^(3/2)+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(
b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx \\
&= \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b} \\
&= \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}} + \frac{3\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} \\
&= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2\sin(c+dx)}{5d(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.87

$$\frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b^2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 35.95, size = 323, normalized size = 4.68

method	result
default	$\frac{\frac{6i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)}{5} - \frac{6i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{}{\mathrm{co}}}}{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/d*(3*I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*El
lipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*
```

```
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)
/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3
*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^4-2*cos(d*x+c)^2+3*cos(d*
x+c))/sin(d*x+c)/(b/cos(d*x+c))^(3/2)/cos(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 95, normalized size = 1.38

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{2}\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)/(b/cos(c + d*x))^(3/2), x)
```

3.119 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*b*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/b/d/(b*sec(d*x+c))
^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^2*d) + (2*b*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{5}{7}\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^2}\\ &= \frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21bd\sqrt{b\sec(c+dx)}}+\frac{\left(5\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right.}{21b^2}\\ &= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^2d}+\frac{2b\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{1}{21bd}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 66, normalized size = 0.67

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 33.45, size = 153, normalized size = 1.56

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-3(\cos^4(dx+c)\right.}{21d\cos(dx+c)^2\sin(dx+c)^3\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d
*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/cos(d*x+c)
^2/sin(d*x+c)^3/(b/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 100, normalized size = 1.02

$$\frac{2\left(3\cos\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))}{21\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2/(b/cos(c + d*x))^(3/2), x)
```

3.120 $\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/9*b^2*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*sin(d*x+c)/d/(b*sec(d*x+c))
^(3/2)+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3854, 3856, 2719}

$$\frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*b^2*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*Sin[c + d*x]
)/(45*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = b^3\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{1}{9}(7b)\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{15b}$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}} + \frac{7\int \sqrt{\cos(c+dx)}\,dx}{15b\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b^2\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45d(b\sec(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.75

$$\frac{84E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos^{\frac{3}{2}}(c+dx)(33\sin(c+dx)+5\sin(3(c+dx)))}{90d\cos^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Sec[c + d*x])^(3/2),x]
[Out] (84*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^(3/2)*(33*Sin[c + d*x] + 5*Sin
[3*(c + d*x)]))/(90*d*Cos[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 36.14, size = 333, normalized size = 3.43

method	result
default	$-\frac{2\left(5\left(\cos^6(dx+c)\right)+21i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-21i\sin(dx+c)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(5*cos(d*x+c)^6-21*I*cos(d*x+c)*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-
1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+21*I*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*I*sin(d*x+c)*Ellipt
icF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+21*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c)^4+
14*cos(d*x+c)^2-21*cos(d*x+c))/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 108, normalized size = 1.11

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+7\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{45\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
+ c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3/(b/cos(c + d*x))^(3/2), x)
```

3.121 $\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d}+\frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d}+\frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d}$$

```
[Out] 10/21*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*(b*sec(d*x+c))^(7/2)*sin(d*
x+c)/b^6/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{7/2}}{7b^6d}+\frac{10\sin(c+dx)(b\sec(c+dx))^{3/2}}{21b^4d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^3*d) + (10*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*b^4*d) + (2*(b*Sec[c
+ d*x])^(7/2)*Sin[c + d*x])/(7*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int (b\sec(c+dx))^{9/2}\,dx}{b^7}$$
$$= \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d} + \frac{5\int (b\sec(c+dx))^{5/2}\,dx}{7b^5}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d} + \frac{5\int \sqrt{b\sec(c}{21b}$$
$$= \frac{10(b\sec(c+dx))^{3/2}\sin(c+dx)}{21b^4d} + \frac{2(b\sec(c+dx))^{7/2}\sin(c+dx)}{7b^6d} + \frac{\left(5\sqrt{\cos(c+}\right.}{}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{21b^3d} + \frac{10(b\sec(c+dx))^{3/2}\sin(c+}{21b^4d}$$

Mathematica [A]
time = 0.22, size = 64, normalized size = 0.64

$$\frac{(b\sec(c+dx))^{5/2}\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*b^5*d)
```

Maple [C] Result contains complex when optimal does not.
time = 52.51, size = 152, normalized size = 1.52

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\right.}{21d\sin(dx+c)^3\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d
```

```
*x+c)^3*sin(d*x+c)-5*cos(d*x+c)^3+5*cos(d*x+c)^2-3*cos(d*x+c)+3)/sin(d*x+c)
^3/(b/cos(d*x+c))^(5/2)/cos(d*x+c)^6
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 117, normalized size = 1.17

$$\frac{-5i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\left(5\cos\left(dx+c\right)^{2}+3\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{21\,b^{3}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*cos(d*x + c)^2 + 3)*sqr
t(b/cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{7}\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(b*sec(d*x + c))^(5/2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^7\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^7*(b/cos(c + d*x))^(5/2)), x)
```

3.122 $\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^3d}+\frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d}$$

```
[Out] 2/5*(b*sec(d*x+c))^(5/2)*sin(d*x+c)/b^5/d-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(
1/2)/(b*sec(d*x+c))^(1/2)+6/5*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{5/2}}{5b^5d}+\frac{6\sin(c+dx)\sqrt{b\sec(c+dx)}}{5b^3d}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (6*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*b^3*d) + (2*(b*Sec[c + d*x
])^(5/2)*Sin[c + d*x])/(5*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\int (b\sec(c+dx))^{7/2}\,dx}{b^6}\\
&= \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d} + \frac{3\int (b\sec(c+dx))^{3/2}\,dx}{5b^4}\\
&= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^3d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d} - \frac{3\int \frac{1}{\sqrt{b\sec(c+}}}{5b^2}\\
&= \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^3d} + \frac{2(b\sec(c+dx))^{5/2}\sin(c+dx)}{5b^5d} - \frac{3\int \sqrt{\mathrm{co}}}{5b^2\sqrt{\cos(c+d}}\\
&= -\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{6\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5b^3d} + \frac{2(b\sec(}{}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 64, normalized size = 0.64

$$\frac{-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2(3+\sec^2(c+dx))\tan(c+dx)}{5b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-6*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(3 + Sec[c + d*x]^2)
*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 51.20, size = 351, normalized size = 3.51

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(3i\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(
d*x+c)^3*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)^3*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+3*
I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(c
os(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticE(I*(cos(d*x+c)
-1)/sin(d*x+c),I)*sin(d*x+c)+3*cos(d*x+c)^3-2*cos(d*x+c)^2-1)*(b/cos(d*x+c)
)^(5/2)/b^5/sin(d*x+c)^5
```

Maximа [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 123, normalized size = 1.23

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3\cos(dx+c)^2+1\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{5b^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x +
c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^6/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^6\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^6*(b/cos(c + d*x))^(5/2)), x)
```

3.123 $\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d}$$

```
[Out] 2/3*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(
b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2720}

$$\frac{2\sin(c+dx)(b\sec(c+dx))^{3/2}}{3b^4d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
3*d) + (2*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

EqQ[n^2, 1/4]

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\int (b\sec(c+dx))^{5/2}\,dx}{b^5}\\ &= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d}+\frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^3}\\ &= \frac{2(b\sec(c+dx))^{3/2}\sin(c+dx)}{3b^4d}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{3b^3}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2(b\sec(c+dx))^{3/2}\sin(c+d}{3b^4d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.71

$$\frac{2\sqrt{b\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\tan(c+dx)\right)}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[b*Sec[c + d*x]]*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan
[c + d*x]))/(3*b^3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 52.88, size = 130, normalized size = 1.81

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-\cos(dx+c)+1\right)(\cos(}{3d\sin(dx+c)^3\cos(dx+c)^4\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)-1)*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-cos
(d*x+c)+1)*(cos(d*x+c)+1)^2/sin(d*x+c)^3/cos(d*x+c)^4/(b/cos(d*x+c))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.21, size = 101, normalized size = 1.40

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{3b^3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b/cos(d*x + c))*sin(d*x + c))/
(b^3*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^5\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^5*(b/cos(c + d*x))^(5/2)), x)
```

3.124 $\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^3d}$$

```
[Out] -2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*sin(d*x+c)*(b*s
ec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 3853, 3856, 2719}

$$\frac{2\sin(c+dx)\sqrt{b\sec(c+dx)}}{b^3d}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\int (b\sec(c+dx))^{3/2}\,dx}{b^4}\\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^3 d} - \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b^2}\\
&= \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^3 d} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}\\
&= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2 d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{b^3 d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.75

$$
\frac{-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}} + 2\tan(c+dx)}{b^2 d\sqrt{b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-2*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*Tan[c + d*x])/(b^2*d
*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 53.60, size = 322, normalized size = 4.74

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2\left(i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-i\,\mathrm{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/
sin(d*x+c),I)-I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
```

```
(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+I*sin(d*x+c)
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c
),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)-
1)/cos(d*x+c)^3/sin(d*x+c)^5/(b/cos(d*x+c))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 86, normalized size = 1.26

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b/cos(d*x +
c))*sin(d*x + c))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^4*(b/cos(c + d*x))^(5/2)), x)
```

3.125 $\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3*
d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int \sqrt{b\sec(c+dx)}\,dx}{b^3}$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^3}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.00

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\sec(c+dx)}}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(b^3*
d)
```

Maple [C] Result contains complex when optimal does not.
time = 33.81, size = 98, normalized size = 2.39

method	result	size
default	$-\frac{2i\left(\frac{1}{\cos(dx+c)+1}\right)^{\frac{7}{2}}(\cos(dx+c)-1)\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)(\cos(dx+c)+1)^2}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{5}{2}}\sin(dx+c)^2}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/d*(1/(cos(d*x+c)+1))^(7/2)*(cos(d*x+c)-1)*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)*(cos(d*x+c)+1)^2/(b/cos(d*x+c))^(5/2)/(cos(d*x+c)/(cos(d*x+c)+
1))^(5/2)/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 60, normalized size = 1.46

$$\frac{-i\sqrt{2}\,\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+d\,x)^3\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(b/cos(c + d*x))^(5/2)), x)
```

3.126 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3856, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b^2}$$
$$= \frac{\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$
$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2 d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.93

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\cos^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(d*Cos[c + d*x]^(5/2)*(b*Sec[c + d*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 33.75, size = 311, normalized size = 7.59

method	result
risch	$-\frac{i\sqrt{2}}{d\,b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}} + \frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie}}{}\right.}{}$
default	$\frac{2i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-2i\cos(dx+c)\ \mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*sin(d*x+c)*cos(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
cos(d*x+c)-1)/sin(d*x+c),I)+I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x
+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*sin(d*x
```

```
+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+cos(d*x+c))/cos(d*x+c)^3/(b/cos(d
*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 66, normalized size = 1.61

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(5/2)), x)
```

3.127 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(
b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]

time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3854, 3856, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*b^
3*d) + (2*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b}\\ &= \frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{\int \sqrt{b\sec(c+dx)}\,dx}{3b^3}\\ &= \frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^3}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.86

$$\frac{\sec^2(c+dx)\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)}{3bd(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^2*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c
+ d*x)]))/(3*b*d*(b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 34.96, size = 131, normalized size = 1.82

method	result
default	$-\frac{2(\cos(dx+c)-1)\left(i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-\left(\cos^2(dx+c)\right)+\cos(dx+c)\right)(\cos(d}{3d\sin(dx+c)^3\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(cos(d*x+c)-1)*(I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)^2+c
os(d*x+c))*(cos(d*x+c)+1)^2/sin(d*x+c)^3/(b/cos(d*x+c))^(5/2)/cos(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 87, normalized size = 1.21

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{3\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*sqrt(b)*w
eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(5/2)), x)
```

3.128 $\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*sin(d*x+c)/b/d/(b*sec(d*x+c))^(3/2)+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2719}

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*
x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2}\\
&= \frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{3\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}\\
&= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.83

$$
\frac{\sqrt{b\sec(c+dx)}\left(12\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)+\sin(3(c+dx))\right)}{10b^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Si
n[c + d*x] + Sin[3*(c + d*x)]))/(10*b^3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 33.39, size = 321, normalized size = 4.46

method	result
default	$-\frac{2\left(-3i\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+3i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)\cdots}{\sin(dx+c)}\right.\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*(3*I*sin(d*x+c)*cos(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*c
os(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1
)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-
3*I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d
*x+c))/sin(d*x+c)/(b/cos(d*x+c))^(5/2)/cos(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 95, normalized size = 1.32

$$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^2\sin(dx+c) + 3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^2*sin(d*x + c) + 3*I*sqrt(2)*sqrt(
b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(c + d*x))^(5/2),x)
[Out] int(1/(b/cos(c + d*x))^(5/2), x)
```

3.129 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d}+\frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*sin(d*x+c)/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/b^2/d/(b*sec(d*x+c))
^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 3854, 3856, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d}+\frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^3*d) + (2*Sin[c + d*x])/(7*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x])/
(21*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx \\
&= \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b} \\
&= \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^3} \\
&= \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^2d\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right.}{21b^3} \\
&= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^3d} + \frac{2\sin(c+dx)}{7d(b\sec(c+dx))^{5/2}} + \frac{10}{21b^2d}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.68

$$\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 35.13, size = 153, normalized size = 1.58

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-3(\cos^4(dx+c))+\right.}{21d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)^3\sin(dx+c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d
*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/(b/cos(d*x
+c))^(5/2)/cos(d*x+c)^3/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.16, size = 100, normalized size = 1.03

$$\frac{2\left(3\cos\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)}{21\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)/(b/cos(c + d*x))^(5/2), x)
```

3.130 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}$$

```
[Out] 2/9*b*sin(d*x+c)/d/(b*sec(d*x+c))^(7/2)+14/45*sin(d*x+c)/b/d/(b*sec(d*x+c))
^(3/2)+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3854, 3856, 2719}

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}}+\frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*b*Sin[c + d*x])/(9*d*(b*Sec[c + d*x])^(7/2)) + (14*Sin[c + d*x]
)/(45*b*d*(b*Sec[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = b^2\int \frac{1}{(b\sec(c+dx))^{9/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{7}{9}\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{7\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{15b^2}$$
$$= \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{7\int \sqrt{\cos(c+dx)}\,dx}{15b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx}}$$
$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2b\sin(c+dx)}{9d(b\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{45bd(b\sec(c+dx))}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.74

$$\frac{\frac{336E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}} + 4\cos(c+dx)(33\sin(c+dx) + 5\sin(3(c+dx)))}{360b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((336*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*Cos[c + d*x]*(33*Si
n[c + d*x] + 5*Sin[3*(c + d*x)]))/(360*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 33.62, size = 333, normalized size = 3.40

method	result
default	$-\frac{2\left(5\left(\cos^6(dx+c)\right)+21i\cos(dx+c)\,\mathrm{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-21i\sin(dx+c)\cos(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/d*(5*cos(d*x+c)^6-21*I*cos(d*x+c)*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-
1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+21*I*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*I*sin(d*x+c)*Ellipt
icF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+21*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c)^4+
14*cos(d*x+c)^2-21*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.09, size = 108, normalized size = 1.10

$$\frac{2\left(5\cos\left(dx+c\right)^4+7\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{45\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^4 + 7*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
+ c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(c+dx\right)}{\left(b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(b/cos(c + d*x))^(5/2), x)
```

3.131 $\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^4d}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/7*sin(d*x+c)/b/d/(b*sec(d*x+c))^(5/2)+10/21*sin(d*x+c)/b^3/d/(b*sec(d*x+c
))^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^4/d
```

Rubi [A]

time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3854, 3856, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^4d}+\frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}}+\frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*
b^4*d) + (2*Sin[c + d*x])/(7*b*d*(b*Sec[c + d*x])^(5/2)) + (10*Sin[c + d*x]
)/(21*b^3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(b\sec(c+dx))^{7/2}}\,dx &= \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{5\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{7b^2} \\
&= \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}} + \frac{5\int \sqrt{b\sec(c+dx)}\,dx}{21b^4} \\
&= \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{21b^3d\sqrt{b\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right.}{21b^4} \\
&= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{21b^4d} + \frac{2\sin(c+dx)}{7bd(b\sec(c+dx))^{5/2}} + \frac{}{21b^3}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.66

$$
\frac{\sqrt{b\sec(c+dx)}\left(40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))\right)}{84b^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26
*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(84*b^4*d)
```

Maple [C] Result contains complex when optimal does not.
time = 33.98, size = 153, normalized size = 1.53

method	result
default	$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)\left(5i\,\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-3(\cos^4(dx+c))+\right.}{21d\sin(dx+c)^3\left(\frac{b}{\cos(dx+c)}\right)^{\frac{7}{2}}\cos(dx+c)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d
*x+c)-3*cos(d*x+c)^4+3*cos(d*x+c)^3-5*cos(d*x+c)^2+5*cos(d*x+c))/sin(d*x+c)
^3/(b/cos(d*x+c))^(7/2)/cos(d*x+c)^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 100, normalized size = 1.00

$$\frac{2\left(3\cos\left(dx+c\right)^{3}+5\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))}{21\,b^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(d*x + c)^3 + 5*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x +
c) - 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 5*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)))/(b^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(c + d*x))^(7/2),x)
```

```
[Out] int(1/(b/cos(c + d*x))^(7/2), x)
```

3.132 $\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d}+\frac{\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+}}{4d}$$

```
[Out] 3/8*sec(d*x+c)^(3/2)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+1/4*sec(d*x+c)^(7/2)
*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+3/8*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(
1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{4d}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{8d}+\frac{3\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(8*d*Sqrt[Sec[c + d*x]]) + (
3*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(8*d) + (Sec[c + d*
x]^(7/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\int \sec^5(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3\sqrt{b\sec(c+dx)}\right)\int \sec}{4\sqrt{\sec(c+dx)}}$$

$$= \frac{3\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{8d} + \frac{\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+}}{4d}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx}}{8d}$$

Mathematica [A]
time = 0.16, size = 64, normalized size = 0.60

$$\frac{\sqrt{b\sec(c+dx)}\left(3\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(3+2\sec^2(c+dx)\right)\tan(c+dx)\right)}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)*Sqrt[b*Sec[c + d*x]],x]
[Out] (Sqrt[b*Sec[c + d*x]]*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[c
+ d*x]^2)*Tan[c + d*x]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 32.08, size = 131, normalized size = 1.22

method	result
default	$\frac{\left(3\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+c))-3\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+c))+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx+c}{8d}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{4\left(e^{2i(dx+c)}+1\right)^3 d}+\frac{3\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln(e^{i(d}}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(3*ln(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-3*ln(-(cos(
d*x+c)-1+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+3*cos(d*x+c)^2*sin(d*x+c)+2*s
in(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(b/cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(89) = 178$.
time = 0.72, size = 1656, normalized size = 15.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x
+ 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2
 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d
*x + 2*c) + 1)*d)
```

Fricas [A]
time = 2.82, size = 229, normalized size = 2.14

$$\left[\frac{3\,\sqrt{b}\,\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{2}-2\,\sqrt{b}\,\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)+\frac{2\left(3\,\cos(dx+c)^{2}+2\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\,d\cos\left(dx+c\right)^{3}},\,-\frac{3\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{b}\right)\cos\left(dx+c\right)^{3}-\frac{\left(3\,\cos(dx+c)^{2}+2\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\,d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*(3*
cos(d*x + c)^2 + 2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
d*cos(d*x + c)^3), -1/8*(3*sqrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c)^3 - (3*cos(d*x + c)^2 + 2)*sq
rt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(9/2),x)
[Out] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(9/2), x)
```

3.133 $\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*sec(d*x+c)^(5/2)*sin(d*x+c)^3*(b*sec(d*x+c))^(1/2)/d+sin(d*x+c)*sec(d*x
+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {17, 3852}

$$\frac{\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (Sec[c + d*x]^(5
/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx&=\frac{\sqrt{b\sec(c+dx)}\,\int \sec^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\&=-\frac{\sqrt{b\sec(c+dx)}\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\&=\frac{\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+}}{3d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.64

$$\frac{\sqrt{b\sec(c+dx)}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[Sec[c + d*
x]])
```

Maple [A]
time = 33.39, size = 52, normalized size = 0.74

method	result	size
default	$\frac{\left(2\left(\cos^2(dx+c)\right)+1\right)\cos(dx+c)\sin(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\sqrt{\frac{b}{\cos(dx+c)}}}{3d}$	52
risch	$\frac{4i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(4\cos(dx+c)+2i\sin(dx+c)\right)}{3\left(e^{2i(dx+c)}+1\right)^2 d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(7/2)*(b/cos(
d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(60) = 120.
time = 0.60, size = 294, normalized size = 4.20

$$\frac{4\left(\left(3\cos\left(2dx+2c\right)+1\right)\sin\left(6dx+6c\right)+3\left(3\cos\left(2dx+2c\right)+1\right)\sin\left(4dx+4c\right)-3\cos\left(6dx+6c\right)\sin\left(2dx+2c\right)-9\cos\left(4dx+4c\right)\sin\left(2dx+2c\right)\right)\sqrt{b}}{3\left(2\left(3\cos\left(4dx+4c\right)+3\cos\left(2dx+2c\right)+1\right)\cos\left(6dx+6c\right)+\cos\left(6dx+6c\right)^2+6\left(3\cos\left(2dx+2c\right)+1\right)\cos\left(4dx+4c\right)+9\cos\left(4dx+4c\right)^2+9\cos\left(2dx+2c\right)^2+6\left(\sin\left(4dx+4c\right)+\sin\left(2dx+2c\right)\right)\sin\left(6dx+6c\right)+\sin\left(6dx+6c\right)^2+9\sin\left(4dx+4c\right)^2+18\sin\left(4dx+4c\right)\sin\left(2dx+2c\right)+9\sin\left(2dx+2c\right)^2+6\cos\left(2dx+2c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*
x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 +
6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 2.76, size = 43, normalized size = 0.61

$$\frac{\left(2 \cos \left(dx+c\right)^{2}+1\right) \sqrt{\frac{b}{\cos \left(dx+c\right)}}\, \sin \left(dx+c\right)}{3\, d \cos \left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^2 + 1)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c
)^(5/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 2.55, size = 126, normalized size = 1.80

$$\frac{2 \cos \left(c+d\,x\right) \sqrt{\frac{b}{\cos \left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos \left(c+d\,x\right)}}\,\left(4 \sin \left(c+d\,x\right)+5 \sin \left(3\,c+3\,d\,x\right)+\sin \left(5\,c+5\,d\,x\right)+\cos \left(c+d\,x\right)\,10\mathrm{i}+\cos \left(3\,c+3\,d\,x\right)\,5\mathrm{i}+\cos \left(5\,c+5\,d\,x\right)\,1\mathrm{i}\right)}{3\,d\,\left(10 \cos \left(c+d\,x\right)+5 \cos \left(3\,c+3\,d\,x\right)+\cos \left(5\,c+5\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (2*cos(c + d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)*(cos(c + d*x)
*10i + 4*sin(c + d*x) + cos(3*c + 3*d*x)*5i + cos(5*c + 5*d*x)*1i + 5*sin(3
*c + 3*d*x) + sin(5*c + 5*d*x)))/(3*d*(10*cos(c + d*x) + 5*cos(3*c + 3*d*x)
 + cos(5*c + 5*d*x)))
```

3.134 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/2*sec(d*x+c)^(3/2)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+1/2*arctanh(sin(d*x+
c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) + (Se
c[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{\sqrt{b\sec(c+dx)}\int \sec(c+}{2\sqrt{\sec(c+dx)}}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\ \text{s}}{2d}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.69

$$\frac{\sqrt{b\sec(c+dx)}\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/
(2*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 33.33, size = 112, normalized size = 1.56

method	result
default	$\frac{\left(\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)-\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)+\sin(dx+c)\right)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{}}{2d}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}-i\right)\cos(dx+c)}{d}+\sqrt{\frac{e}{e^{2i}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(ln(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d*x
+c))^(5/2)*(b/cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(60) = 120.
time = 0.61, size = 661, normalized size = 9.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [A]
time = 3.69, size = 199, normalized size = 2.76

$$\left[\frac{\sqrt{b}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,d\cos(dx+c)},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)\cos(dx+c)-\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2\,d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqrt(
-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)
*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*co
s(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2), x)
```

3.135 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\,\int \sec^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\
&= -\frac{\sqrt{b\sec(c+dx)}\,\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\
&= \frac{\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$\frac{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]],x]
[Out] (Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Maple [A]
time = 32.12, size = 39, normalized size = 1.22

method	result	size
default	$\frac{\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{d}$	39
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(b/cos(d*x+c))^(1/2)*sin(d*x+c)
```

Maxima [A]
time = 0.56, size = 54, normalized size = 1.69

$$\frac{2\sqrt{b}\,\sin\left(2\,dx+2\,c\right)}{\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 3.63, size = 30, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec{(c+dx)}}\sec^{\frac{3}{2}}{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sec(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sec(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 0.29, size = 46, normalized size = 1.44

$$\frac{(\cos{(d\,x)}-\sin{(d\,x)}\ 1\mathrm{i})\ (\sin{(c)}+\cos{(c)}\ 1\mathrm{i})\ \sqrt{\frac{b}{\cos{(c+d\,x)}}}\ \sqrt{\frac{1}{\cos{(c+d\,x)}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] ((cos(d*x) - sin(d*x)*1i)*(cos(c)*1i + sin(c))*(b/cos(c + d*x))^(1/2)*(1/co
s(c + d*x))^(1/2))/d
```

3.136 $\int \sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 33.72, size = 52, normalized size = 1.58

method	result
default	$-\frac{2\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{b}{\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}\right)}{d}$
risch	$\frac{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}+i\right)\cos(dx+c)}{d}-\frac{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}-i\right)\cos(dx+c)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*(b/cos(d*x+c))^(1/2)*arctanh((cos(d*x+
c)-1)/sin(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.62, size = 65, normalized size = 1.97

$$\frac{\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 2.89, size = 111, normalized size = 3.36

$$\left[\frac{\sqrt{b}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2d},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*arctan(sqrt(-
b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec(c+dx)}\,\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2), x)
```

3.137 $\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

```
[Out] x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int 1\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Maple [A]
time = 34.55, size = 32, normalized size = 1.33

method	result	size
default	$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,(dx+c)}{d\sqrt{\frac{1}{\cos(dx+c)}}}$	32
risch	$\frac{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)*(d*x+c)
```

Maxima [A]
time = 0.58, size = 26, normalized size = 1.08

$$\frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A]
time = 2.14, size = 98, normalized size = 4.08

$$\left[\frac{\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^{2}-b\right)}{2\,d}, \frac{\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/d, sqrt(b)*arctan(sqrt(b/cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A]
time = 4.77, size = 22, normalized size = 0.92

$$\frac{x\sqrt{b\sec\left(c+dx\right)}}{\sqrt{\sec\left(c+dx\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] x*sqrt(b*sec(c + d*x))/sqrt(sec(c + d*x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sqrt(sec(d*x + c)), x)
```

Mupad [B]
time = 0.20, size = 24, normalized size = 1.00

$$\frac{x\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] (x*(b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2)
```

3.138 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\sec(c+dx)}\int\cos(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 34.86, size = 41, normalized size = 1.28

method	result	size
default	$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)}$	41
risch	$-\frac{i\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/cos(d*x+c))^(1/2)*sin(d*x+c)/(1/cos(d*x+c))^(3/2)/cos(d*x+c)
```

Maxima [A]
time = 0.69, size = 13, normalized size = 0.41

$$\frac{\sqrt{b}\,\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(b)*sin(d*x + c)/d
```

Fricas [A]
time = 2.11, size = 30, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [A]
time = 9.63, size = 46, normalized size = 1.44

$$\begin{cases} \frac{\sqrt{b \sec{\left(c + d x \right)}} \tan{\left(c + d x \right)}}{d \sec^{\frac{3}{2}}{\left(c + d x \right)}} & \text{for}\: d \neq 0 \\ \frac{x \sqrt{b \sec{\left(c \right)}}}{\sec^{\frac{3}{2}}{\left(c \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((sqrt(b*sec(c + d*x))*tan(c + d*x)/(d*sec(c + d*x)**(3/2)), Ne(d,
 0)), (x*sqrt(b*sec(c))/sec(c)**(3/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.22, size = 32, normalized size = 1.00

$$\frac{\sin\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(d*(1/cos(c + d*x))^(1/2))
```

3.139 $\int \frac{\sqrt{b \sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/2*x*(b*sec(d*x+c))
^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (Sqrt[b*Sec[c + d*x]]*Sin
[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\int\cos^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b\sec(c+dx)}\int 1\,dx}{2\sqrt{\sec(c+dx)}}$$
$$= \frac{x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [**A**]
time = 0.07, size = 45, normalized size = 0.71

$$\frac{\sqrt{b\sec(c+dx)}\,(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[Sec[c + d
*x]])
```

Maple [**A**]
time = 33.36, size = 54, normalized size = 0.86

method	result	size
default	$\frac{(\sin(dx+c)\cos(dx+c)+dx+c)\sqrt{\frac{b}{\cos(dx+c)}}}{2d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)^{2}}$	54
risch	$\frac{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	188

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)*(b/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(5/
2)/cos(d*x+c)^2
```

Maxima [**A**]
time = 0.58, size = 25, normalized size = 0.40

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A]
time = 3.14, size = 158, normalized size = 2.51

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{4d}, \frac{\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)*log
(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos
(d*x + c)^2 - b))/d, 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x +
 c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*
x + c)))))/d]
```

Sympy [A]
time = 17.60, size = 107, normalized size = 1.70

$$\begin{cases} \frac{x\sqrt{b\sec(c+dx)}\tan^2(c+dx)}{2\sec^{\frac{5}{2}}(c+dx)} + \frac{x\sqrt{b\sec(c+dx)}}{2\sec^{\frac{5}{2}}(c+dx)} + \frac{\sqrt{b\sec(c+dx)}\tan(c+dx)}{2d\sec^{\frac{5}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x\sqrt{b\sec(c)}}{\sec^{\frac{5}{2}}(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((x*sqrt(b*sec(c + d*x))*tan(c + d*x)**2/(2*sec(c + d*x)**(5/2)) +
 x*sqrt(b*sec(c + d*x))/(2*sec(c + d*x)**(5/2)) + sqrt(b*sec(c + d*x))*tan(
c + d*x)/(2*d*sec(c + d*x)**(5/2)), Ne(d, 0)), (x*sqrt(b*sec(c))/sec(c)**(5
/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 0.44, size = 41, normalized size = 0.65

$$\frac{\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{4\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] ((sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*d*(1/cos(c + d*x))^(
1/2))
```

3.140 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{\sqrt{b\sec(c+dx)}\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)-1/3*sin(d*x+c)^3*(b*sec(
d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (Sqrt[b*Sec[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\,\int\cos^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= -\frac{\sqrt{b\sec(c+dx)}\,\mathrm{Subst}\left(\int(1-x^2)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$
$$= \frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}} - \frac{\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [**A**]
time = 0.12, size = 45, normalized size = 0.64

$$\frac{(5+\cos(2(c+dx)))\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{6d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(6*d*Sqrt[Sec[c
+ d*x]])
```

Maple [**A**]
time = 32.30, size = 52, normalized size = 0.74

method	result	size
default	$\frac{(2+\cos^2(dx+c))\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{3d\cos(dx+c)^3\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}}$	52
risch	$-\frac{3i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{3i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sin(3dx+3c)}{12\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	199

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*(b/cos(d*x+c))^(1/2)*sin(d*x+c)/cos(d*x+c)^3/(1/cos(
d*x+c))^(7/2)
```

Maxima [**A**]
time = 0.65, size = 42, normalized size = 0.60

$$\frac{\sqrt{b}\,\left(\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))/d
```

Fricas [**A**]
time = 3.67, size = 48, normalized size = 0.69

$$\frac{\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*
sqrt(cos(d*x + c)))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 0.52, size = 45, normalized size = 0.64

$$\frac{\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{12\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] ((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*d*(1/cos(c
 + d*x))^(1/2))
```

3.141 $\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)}+\frac{3\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/4*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)+3/8*sin(d*x+c)*(b*se
c(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+3/8*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{3\sin(c+dx)\sqrt{b\sec(c+dx)}}{8d\sec^{\frac{3}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\sec(c+dx)}}{4d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(9/2),x]
```

```
[Out] (3*x*Sqrt[b*Sec[c + d*x]])/(8*Sqrt[Sec[c + d*x]]) + (Sqrt[b*Sec[c + d*x]]*S
in[c + d*x])/(4*d*Sec[c + d*x]^(7/2)) + (3*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x
])/(8*d*Sec[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(c+dx)}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\sec(c+dx)}\int\cos^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(3\sqrt{b\sec(c+dx)}\right)\int\cos^2(c+dx)\,dx}{4\sqrt{\sec(c+dx)}}$$

$$= \frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3\sqrt{b\sec(c+dx)}\right.}{8\sqrt{\sec(c+d}}$$

$$= \frac{3x\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3\sqrt{b\sec(c+dx)}\,\sin(c+dx}{8d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.56

$$\frac{\sqrt{b\sec(c+dx)}\,(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(32*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 36.39, size = 74, normalized size = 0.76

method	result	size
default	$\frac{(2(\cos^3(dx+c))\sin(dx+c)+3\sin(dx+c)\cos(dx+c)+3dx+3c)\sqrt{\frac{b}{\cos(dx+c)}}}{8d\cos(dx+c)^4\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}}$	74
risch	$\frac{3\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{i\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sin(4dx+4c)}{32\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	253

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)+3*d*x+3*c)*(b/cos(
d*x+c))^(1/2)/cos(d*x+c)^4/(1/cos(d*x+c))^(9/2)
```

Maxima [A]
time = 0.63, size = 49, normalized size = 0.50

$$\frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A]
time = 4.05, size = 202, normalized size = 2.06

$$\left[\frac{\frac{2\left(2\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+3\sqrt{-b}\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2b\cos\left(dx+c\right)^2-b\right)}{16\,d}, \frac{\frac{\left(2\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^2\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+3\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{b}\sqrt{\cos\left(dx+c\right)}}\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) + 3*sqrt(-b)*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*
cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/d, 1/8*((2*cos(d
*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(
cos(d*x + c)))))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c))/sec(d*x + c)^(9/2), x)
```

Mupad [B]
time = 0.71, size = 52, normalized size = 0.53

$$\frac{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] ((b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))/(
32*d*(1/cos(c + d*x))^(1/2))
```

3.142 $\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{3b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}}+\frac{3b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d}+\frac{b\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(}}{4d}$$

```
[Out] 3/8*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+1/4*b*sec(d*x+c)^(
7/2)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+3/8*b*arctanh(sin(d*x+c))*(b*sec(d*x
+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{b\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{4d}+\frac{3b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{8d}+\frac{3b\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(8*d*Sqrt[Sec[c + d*x]]) +
 (3*b*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(8*d) + (b*Sec[
c + d*x]^(7/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^5(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{b\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(3b\sqrt{b\sec(c+dx)}\right)\int}{4\sqrt{\sec(c+}}$$

$$= \frac{3b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d} + \frac{b\sec^{\frac{7}{2}}(c+dx)\sqrt{b\sec(}}{4d}$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{8d\sqrt{\sec(c+dx)}} + \frac{3b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+}}{8d}$$

Mathematica [**A**]
time = 0.12, size = 64, normalized size = 0.58

$$\frac{(b\sec(c+dx))^{3/2}\left(3\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(3+2\sec^2(c+dx)\right)\tan(c+dx)\right)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(3*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3 + 2*Sec[
c + d*x]^2)*Tan[c + d*x]))/(8*d*Sec[c + d*x]^(3/2))
```

Maple [**A**]
time = 35.19, size = 131, normalized size = 1.19

method	result
default	$-\frac{\left(3\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+c))-3\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^4(dx+c))-3(\cos^2(dx+c))\sin(dx+c)-2\sin(dx}{8d}$
risch	$-\frac{ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{4\left(e^{2i(dx+c)}+1\right)^3 d} - \frac{3b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln(e^{i}}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(3*ln(-(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-3*ln(-(cos
(d*x+c)-1-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-3*cos(d*x+c)^2*sin(d*x+c)-2*
sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)*(b/cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(92) = 184$.
time = 0.69, size = 1742, normalized size = 15.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
 + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4
*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3
*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
+ 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
+ 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*s
in(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*
d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2
*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*
(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 +
 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 +
 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*si
n(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d
*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x +
 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*
c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*
b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*
(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4
4*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*c
os(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

```
 + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4
*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1)*d)
```

Fricas [A]
time = 3.30, size = 236, normalized size = 2.15

$$\left[\frac{3\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{2}-2\,\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2\,b}{\cos(dx+c)^{2}}\right)+\frac{2\left(3\,b\cos(dx+c)^{2}+2\,b\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\,d\cos\left(dx+c\right)^{3}},-\frac{3\,\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{b}\right)\cos\left(dx+c\right)^{3}-\frac{\left(3\,b\cos(dx+c)^{2}+2\,b\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\,d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*(3*
b*cos(d*x + c)^2 + 2*b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c)^3), -1/8*(3*sqrt(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)*cos(d*x + c)^3 - (3*b*cos(d*x + c)^2
 + 2*b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(7/2), x)
```

3.143 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{b\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*b*sec(d*x+c)^(5/2)*sin(d*x+c)^3*(b*sec(d*x+c))^(1/2)/d+b*sin(d*x+c)*sec
(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3852}

$$\frac{b\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (b*Sec[c + d*x
]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b\sqrt{b\sec(c+dx)}\right)\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{b\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec}}{3}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.62

$$\frac{(b\sec(c+dx))^{3/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sec[c + d*x]^
(3/2))
```

Maple [A]
time = 34.25, size = 52, normalized size = 0.72

method	result	size
default	$\frac{(2(\cos^2(dx+c))+1)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)}{3d}$	52
risch	$\frac{4ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(4\cos(dx+c)+2i\sin(dx+c))}{3\left(e^{2i(dx+c)}+1\right)^2 d}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(b/cos(d*x+c))^(3/
2)*sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(62) = 124.
time = 0.66, size = 299, normalized size = 4.15

$$-\frac{4\left(3\,b\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)-\left(3\,b\cos\left(2\,dx+2\,c\right)+b\right)\sin\left(6\,dx+6\,c\right)-3\left(3\,b\cos\left(2\,dx+2\,c\right)+b\right)\sin\left(4\,dx+4\,c\right)\right)\sqrt{b}}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^2+6\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\,\cos\left(4\,dx+4\,c\right)^2+9\,\cos\left(2\,dx+2\,c\right)^2+6\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^2+9\,\sin\left(4\,dx+4\,c\right)^2+18\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,\sin\left(2\,dx+2\,c\right)^2+6\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d*
x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x +
 2*c) + b)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin
(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9
*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x +
2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 2.85, size = 44, normalized size = 0.61

$$\frac{\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*(2*b*cos(d*x + c)^2 + b)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c)^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 1.36, size = 127, normalized size = 1.76

$$\frac{2\,b\,\cos\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(4\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\,x\right)\,10{}\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,5{}\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,1{}\mathrm{i}\right)}{3\,d\,\left(10\,\cos\left(c+d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),x)
[Out] (2*b*cos(c + d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)*(cos(c + d*
x)*10i + 4*sin(c + d*x) + cos(3*c + 3*d*x)*5i + cos(5*c + 5*d*x)*1i + 5*sin
(3*c + 3*d*x) + sin(5*c + 5*d*x)))/(3*d*(10*cos(c + d*x) + 5*cos(3*c + 3*d*
x) + cos(5*c + 5*d*x)))
```

3.144 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/2*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+1/2*b*arctanh(sin(
d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {17, 3853, 3855}

$$\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{b\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) + (
b*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \mathrm{s}\ldots}{2\sqrt{\sec(c+d\ldots}}$$

$$= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+d\ldots}}{2d}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.68

$$\frac{(b\sec(c+dx))^{3/2}\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])
)/(2*d*Sec[c + d*x]^(3/2))
```

Maple [A]
time = 32.75, size = 112, normalized size = 1.51

method	result
default	$\frac{\left(\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)-\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)\left(\cos^2(dx+c)\right)+\sin(dx+c)\right)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{\ldots}{\cos\ldots}\right.}{2d}$
risch	$-\frac{ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}-\frac{b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}-i\right)\cos(dx+c)}{d}+\frac{b\sqrt{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(ln(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d*x
+c))^(3/2)*(b/cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(62) = 124$.
time = 0.61, size = 691, normalized size = 9.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*
c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*c
os(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d
*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*co
s(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*s
qrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]

time = 2.51, size = 202, normalized size = 2.73

$$\left[\frac{b^{\frac{3}{2}}\cos(dx+c)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4d\cos(dx+c)},-\frac{\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)\cos(dx+c)-\frac{b\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b*sqrt(b
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(sqr
t(-b)*b*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/b)*cos(d*x + c) - b*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2), x)
```

3.145 $\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=33

$$\frac{b\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] b*sin(d*x+c)*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= -\frac{\left(b\sqrt{b\sec(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$
$$= \frac{b\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.97

$$\frac{(b\sec(c+dx))^{3/2}\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2),x]
[Out] ((b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 33.76, size = 39, normalized size = 1.18

method	result	size
default	$\frac{\sin(dx+c)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{d}$	39
risch	$\frac{2ib\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/d*sin(d*x+c)*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*(b/cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.59, size = 54, normalized size = 1.64

$$\frac{2\,b^{\frac{3}{2}}\sin\left(2\,dx+2\,c\right)}{\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 3.15, size = 31, normalized size = 0.94

$$\frac{b\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] b*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)*sqrt(sec(d*x + c)), x)
```

Mupad [B]
time = 0.22, size = 47, normalized size = 1.42

$$\frac{b\left(\cos(d\,x)-\sin(d\,x)\,1\mathrm{i}\right)\left(\sin(c)+\cos(c)\,1\mathrm{i}\right)\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] (b*(cos(d*x) - sin(d*x)*1i)*(cos(c)*1i + sin(c))*(b/cos(c + d*x))^(1/2)*(1/
cos(c + d*x))^(1/2))/d
```

3.146 $\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=34

$$\frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] b*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{b\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.97

$$\frac{\tanh^{-1}(\sin(c+dx))(b\sec(c+dx))^{3/2}}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
[Out] (ArcTanh[Sin[c + d*x]]*(b*Sec[c + d*x])^(3/2))/(d*Sec[c + d*x]^(3/2))
```

Maple [A]
time = 33.52, size = 52, normalized size = 1.53

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}\right)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)}{d\sqrt{\frac{1}{\cos(dx+c)}}}$	52
risch	$-\frac{b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*arctanh((cos(d*x+c)-1)/sin(d*x+c))*(b/cos(d*x+c))^(3/2)*cos(d*x+c)/(1/
cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.
time = 0.62, size = 68, normalized size = 2.00

$$\frac{\left(b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A]

time = 3.01, size = 112, normalized size = 3.29

$$\left[\frac{b^{\frac{3}{2}}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2d}, -\frac{\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] [1/2*b^(3/2)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*b*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\sec(c+dx))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
[Out] Integral((b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(3/2)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)
[Out] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)
```

3.147 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

```
[Out] b*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (b*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{bx\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.96

$$\frac{x(b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (x*(b*Sec[c + d*x])^(3/2))/Sec[c + d*x]^(3/2)
```

Maple [A]
time = 33.20, size = 32, normalized size = 1.28

method	result	size
default	$\frac{\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}(dx+c)}{d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}}$	32
risch	$\frac{b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/cos(d*x+c))^(3/2)/(1/cos(d*x+c))^(3/2)*(d*x+c)
```

Maxima [A]
time = 0.56, size = 26, normalized size = 1.04

$$\frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A]
time = 3.04, size = 99, normalized size = 3.96

$$\left[\frac{\sqrt{-b}\,b\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,d},\ \frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin
(d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, b^(3/2)*arctan(sqrt(b/cos(d*x + c))*
sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [A]
time = 7.14, size = 22, normalized size = 0.88

$$\frac{x(b\sec{(c+dx)})^{\frac{3}{2}}}{\sec^{\frac{3}{2}}{(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] x*(b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.20, size = 25, normalized size = 1.00

$$\frac{b\,x\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (b*x*(b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2)
```

3.148 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

```
[Out] b*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.97

$$\frac{(b\sec(c+dx))^{3/2}\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
[Out] ((b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2))
```

Maple [A]
time = 34.12, size = 41, normalized size = 1.24

method	result	size
default	$\frac{\sin(dx+c)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)}$	41
risch	$-\frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/d*sin(d*x+c)*(b/cos(d*x+c))^(3/2)/(1/cos(d*x+c))^(5/2)/cos(d*x+c)
```

Maxima [A]
time = 0.64, size = 13, normalized size = 0.39

$$\frac{b^{\frac{3}{2}}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] b^(3/2)*sin(d*x + c)/d
```

Fricas [A]
time = 3.17, size = 31, normalized size = 0.94

$$\frac{b\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] b*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [A]
time = 29.16, size = 46, normalized size = 1.39

$$\begin{cases} \frac{(b\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)} & \text{for } d\neq 0 \\ \frac{x(b\sec(c))^{\frac{3}{2}}}{\sec^{\frac{5}{2}}(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Piecewise(((b*sec(c + d*x))**(3/2)*tan(c + d*x)/(d*sec(c + d*x)**(5/2)), Ne
(d, 0)), (x*(b*sec(c))**(3/2)/sec(c)**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(5/2), x)
```

Mupad [B]
time = 0.25, size = 33, normalized size = 1.00

$$\frac{b\,\sin(c+d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (b*sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(d*(1/cos(c + d*x))^(1/2))
```

3.149 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*b*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/2*b*x*(b*sec(d*x
+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {17, 2715, 8}

$$\frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (b*x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (b*Sqrt[b*Sec[c + d*x]]
*Sin[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{2\sqrt{\sec(c+dx)}}$$
$$= \frac{bx\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.69

$$\frac{(b\sec(c+dx))^{3/2}(2(c+dx)+\sin(2(c+dx)))}{4d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x]
^(3/2))
```

Maple [A]
time = 36.63, size = 54, normalized size = 0.83

method	result	size
default	$\frac{(\sin(dx+c)\cos(dx+c)+dx+c)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{2d\cos(dx+c)^{2}\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}}$	54
risch	$\frac{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	191

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)*(b/cos(d*x+c))^(3/2)/cos(d*x+c)^2/(1/co
s(d*x+c))^(7/2)
```

Maxima [A]
time = 0.65, size = 28, normalized size = 0.43

$$\frac{(2\,(dx+c)b + b\sin\,(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A]
time = 3.59, size = 161, normalized size = 2.48

$$\left[\frac{2\,b\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+\sqrt{-b}\,b\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-b\right)}{4\,d},\ \frac{b\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{b}\sqrt{\cos\left(dx+c\right)}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)*b
*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b
*cos(d*x + c)^2 - b))/d, 1/2*(b*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin
(d*x + c) + b^(3/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(
cos(d*x + c)))))/d]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 0.43, size = 42, normalized size = 0.65

$$\frac{b\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{4\,d\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] (b*(sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*d*(1/cos(c + d*x))
^(1/2))
```

3.150 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{b\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] b*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)-1/3*b*sin(d*x+c)^3*(b*
sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.087$, Rules used = {17, 2713}

$$\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{b\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (b*Sqrt[b*Se
c[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}} \\
&= -\frac{\left(b\sqrt{b\sec(c+dx)}\right)\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}} \\
&= \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}} - \frac{b\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 45, normalized size = 0.62

$$\frac{(5+\cos(2(c+dx)))(b\sec(c+dx))^{3/2}\sin(c+dx)}{6d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
[Out] ((5 + Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(6*d*Sec[c + d
*x]^(3/2))
```

Maple [A]
time = 36.28, size = 52, normalized size = 0.72

method	result	size
default	$\frac{(2+\cos^2(dx+c))\sin(dx+c)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{3d\cos(dx+c)^3\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}}$	52
risch	$-\frac{3ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\;e^{i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\;d}+\frac{3ib\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\;e^{-i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\;d}+\frac{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\;\sin(3dx+3c)}{12\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\;d}$	202

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b/cos(d*x+c))^(3/2)/cos(d*x+c)^3/(1/cos(
d*x+c))^(9/2)
```

Maxima [A]
time = 0.60, size = 45, normalized size = 0.62

$$\frac{\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))*sqrt(b)/d
```

Fricas [A]
time = 3.85, size = 51, normalized size = 0.71

$$\frac{\left(b\cos\left(dx+c\right)^{3}+2\,b\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*cos(d*x + c)^3 + 2*b*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)
/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(9/2), x)
```

Mupad [B]
time = 0.44, size = 46, normalized size = 0.64

$$\frac{b\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{12\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] (b*(9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*d*(1/cos
(c + d*x))^(1/2))
```

3.151 $\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)}+\frac{3b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/4*b*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)+3/8*b*sin(d*x+c)*(
b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+3/8*b*x*(b*sec(d*x+c))^(1/2)/sec(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}}+\frac{3b\sin(c+dx)\sqrt{b\sec(c+dx)}}{8d\sec^{\frac{3}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\sec(c+dx)}}{4d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (3*b*x*Sqrt[b*Sec[c + d*x]])/(8*Sqrt[Sec[c + d*x]]) + (b*Sqrt[b*Sec[c + d*x
]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(7/2)) + (3*b*Sqrt[b*Sec[c + d*x]]*Sin[c
 + d*x])/(8*d*Sec[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^{3/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\sec(c+dx)}\right)\int \cos^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$

$$= \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{\sec(c+dx)}}$$

$$= \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{8d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\sec(c+\right.}{8\sqrt{\sec(c}}$$

$$= \frac{3bx\sqrt{b\sec(c+dx)}}{8\sqrt{\sec(c+dx)}} + \frac{b\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{4d\sec^{\frac{7}{2}}(c+dx)} + \frac{3b\sqrt{b\sec(c+dx)}\,\sin(c+}{8d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.17, size = 55, normalized size = 0.54

$$\frac{(b\sec(c+dx))^{3/2}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(11/2),x]
[Out] ((b*Sec[c + d*x])^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(32*d*Sec[c + d*x]^(3/2))
```

Maple [A]
time = 37.97, size = 74, normalized size = 0.73

method	result	si
default	$\frac{\left(2\left(\cos^{3}(dx+c)\right)\sin(dx+c)+3\sin(dx+c)\cos(dx+c)+3dx+3c\right)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}{8d\cos(dx+c)^{4}\left(\frac{1}{\cos(dx+c)}\right)^{\frac{11}{2}}}$	7
risch	$\frac{3b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,x}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}} - \frac{ib\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{2i(dx+c)}}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d} + \frac{ib\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-2i(dx+c)}}{8\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d} + \frac{b\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\sin(4dx+4c)}{32\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)+3*d*x+3*c)*(b/cos(
d*x+c))^(3/2)/cos(d*x+c)^4/(1/cos(d*x+c))^(11/2)
```

Maxima [A]
time = 0.62, size = 53, normalized size = 0.52

$$\frac{\left(12\,(dx+c)b+b\sin\left(4\,dx+4\,c\right)+8\,b\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A]
time = 3.69, size = 207, normalized size = 2.05

$$\left[\frac{3\,\sqrt{-b}\,b\log\left(-2\,\sqrt{-b}\,\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^{2}-b\right)+\frac{2\,\left(2\,b\cos\left(dx+c\right)^{4}+3\,b\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{16\,d},\ \frac{3\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}}\right)+\frac{\left(2\,b\cos\left(dx+c\right)^{4}+3\,b\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(-b)*b*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)
*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b) + 2*(2*b*cos(d*x + c)^4 + 3*b*cos(d
*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d, 1/8*(3*
b^(3/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c)
))) + (2*b*cos(d*x + c)^4 + 3*b*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(3/2)/sec(d*x + c)^(11/2), x)
```

Mupad [B]
time = 0.59, size = 53, normalized size = 0.52

$$\frac{b\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] (b*(b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))
/(32*d*(1/cos(c + d*x))^(1/2))
```

3.152 $\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d}+\frac{b^2\sec^{\frac{9}{2}}(c+dx)}{}$$

```
[Out] 2/3*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)^3*(b*sec(d*x+c))^(1/2)/d+1/5*b^2*sec(d*
x+c)^(9/2)*sin(d*x+c)^5*(b*sec(d*x+c))^(1/2)/d+b^2*sin(d*x+c)*sec(d*x+c)^(1
/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3852}

$$\frac{b^2\sin^5(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2b^2\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c
 + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d) + (b^2*Sec[c + d*x
]^(9/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^5)/(5*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{7}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^6(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec}}{3}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 57, normalized size = 0.49

$$\frac{(b\sec(c+dx))^{5/2}\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]
^5/5))/(d*Sec[c + d*x]^(5/2))
```

Maple [A]
time = 36.38, size = 62, normalized size = 0.53

method	result	size
default	$\frac{\left(8\left(\cos^4(dx+c)\right)+4\left(\cos^2(dx+c)\right)+3\right)\cos(dx+c)\sin(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{15d}$	62
risch	$\frac{16ib^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(10\,e^{3i(dx+c)}+6\cos(dx+c)+4i\sin(dx+c)\right)}{15\left(e^{2i(dx+c)}+1\right)^4 d}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d*(8*cos(d*x+c)^4+4*cos(d*x+c)^2+3)*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c
))^(7/2)*(b/cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(100) = 200$.
time = 0.73, size = 705, normalized size = 6.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -16/15*(5*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(10*d*x + 10*c
) + 25*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 5
0*(2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - (10*b^
2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(10*d*x + 10*c) - 5*(
10*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(8*d*x + 8*c) -
10*(10*b^2*cos(4*d*x + 4*c) + 5*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c
))*sqrt(b)/((2*(5*cos(8*d*x + 8*c) + 10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4
*c) + 5*cos(2*d*x + 2*c) + 1)*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 1
```

```
0*(10*cos(6*d*x + 6*c) + 10*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + 25*cos(8*d*x + 8*c)^2 + 20*(10*cos(4*d*x + 4*c) + 5*cos(2*d*
x + 2*c) + 1)*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 20*(5*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 100*cos(4*d*x + 4*c)^2 + 25*cos(2*d*x + 2*c)^
2 + 10*(sin(8*d*x + 8*c) + 2*sin(6*d*x + 6*c) + 2*sin(4*d*x + 4*c) + sin(2*
d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 + 50*(2*sin(6*d*x + 6
*c) + 2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 25*sin(8*d*
x + 8*c)^2 + 100*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
 100*sin(6*d*x + 6*c)^2 + 100*sin(4*d*x + 4*c)^2 + 100*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 25*sin(2*d*x + 2*c)^2 + 10*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 3.17, size = 63, normalized size = 0.54

$$\frac{\left(8\,b^{2}\cos\left(dx+c\right)^{4}+4\,b^{2}\cos\left(dx+c\right)^{2}+3\,b^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\,d\cos\left(dx+c\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*cos(d*x + c)^4 + 4*b^2*cos(d*x + c)^2 + 3*b^2)*sqrt(b/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^(9/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(7/2), x)
```

Mupad [B]
time = 4.70, size = 205, normalized size = 1.77

$$-\frac{\sqrt{-\frac{b}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\,\left(\frac{b^2\,\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\,8{}\mathrm{i}}{15\,d}+\frac{b^2\,\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\,\left(-2\sin\left(2\,c+2\,d\,x\right)^2+\sin\left(4\,c+4\,d\,x\right)\,1{}\mathrm{i}+1\right)\,16{}\mathrm{i}}{3\,d}+\frac{b^2\,\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\,\left(-2\sin\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+1\right)\,8{}\mathrm{i}}{3\,d}\right)\,\left(2\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)^2+\sin\left(5\,c+5\,d\,x\right)\,1{}\mathrm{i}-1\right)}{16\,\left(\sin\left(c+d\,x\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(7/2),x)
```

```
[Out] -((-b/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*((b^2*(-1/(2*sin(c/2 + (d*x)/2)^2
 - 1))^(1/2)*8i)/(15*d) + (b^2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin
(4*c + 4*d*x)*1i - 2*sin(2*c + 2*d*x)^2 + 1)*16i)/(3*d) + (b^2*(-1/(2*sin(c
/2 + (d*x)/2)^2 - 1))^(1/2)*(sin(2*c + 2*d*x)*1i - 2*sin(c + d*x)^2 + 1)*8i
)/(3*d))*(sin(5*c + 5*d*x)*1i + 2*sin((5*c)/2 + (5*d*x)/2)^2 - 1))/(16*(sin
(c + d*x)^2 - 1)^2)
```

3.153 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d}$$

```
[Out] 1/3*b^2*sec(d*x+c)^(5/2)*sin(d*x+c)^3*(b*sec(d*x+c))^(1/2)/d+b^2*sin(d*x+c)
*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3852}

$$\frac{b^2\sin^3(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{3d}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (b^2*Sec[c +
 d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec}}{3}\end{aligned}$$

Mathematica [**A**]
time = 0.09, size = 45, normalized size = 0.59

$$\frac{(b\sec(c+dx))^{5/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sec[c + d*x]^
(5/2))
```

Maple [**A**]
time = 32.87, size = 52, normalized size = 0.68

method	result	size
default	$\frac{\left(2\left(\cos^2(dx+c)\right)+1\right)\cos(dx+c)\sin(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{3d}$	52
risch	$\frac{4ib^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(4\cos(dx+c)+2i\sin(dx+c))}{3\left(e^{2i(dx+c)}+1\right)^2 d}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*cos(d*x+c)*sin(d*x+c)*(1/cos(d*x+c))^(3/2)*(b/cos(
d*x+c))^(5/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(66) = 132$.
time = 0.60, size = 311, normalized size = 4.09

$$-\frac{4\,\left(3\,b^2\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b^2\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)-\left(3\,b^2\cos\left(2\,dx+2\,c\right)+b^2\right)\sin\left(6\,dx+6\,c\right)-3\,\left(3\,b^2\cos\left(2\,dx+2\,c\right)+b^2\right)\sin\left(4\,dx+4\,c\right)\right)\sqrt{b}}{3\,\left(2\,\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^2+6\,\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\,\cos\left(4\,dx+4\,c\right)^2+9\,\cos\left(2\,dx+2\,c\right)^2+6\,\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^2+9\,\sin\left(4\,dx+4\,c\right)^2+18\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,\sin\left(2\,dx+2\,c\right)^2+6\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*c
os(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) +
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 3.34, size = 48, normalized size = 0.63

$$\frac{\left(2\,b^2\cos\left(dx+c\right)^2+b^2\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*cos(d*x + c)^2 + b^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(3/2), x)
```

Mupad [B]
time = 1.37, size = 129, normalized size = 1.70

$$\frac{2\,b^2\,\cos\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\left(4\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+\cos\left(c+d\,x\right)\,10\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,5\mathrm{i}+\cos\left(5\,c+5\,d\,x\right)\,1\mathrm{i}\right)}{3\,d\,\left(10\,\cos\left(c+d\,x\right)+5\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(5\,c+5\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] (2*b^2*cos(c + d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)*(cos(c +
d*x)*10i + 4*sin(c + d*x) + cos(3*c + 3*d*x)*5i + cos(5*c + 5*d*x)*1i + 5*s
in(3*c + 3*d*x) + sin(5*c + 5*d*x)))/(3*d*(10*cos(c + d*x) + 5*cos(3*c + 3*
d*x) + cos(5*c + 5*d*x)))
```

3.154 $\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}}+\frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+1/2*b^2*arctanh(
sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}{2d}+\frac{b^2\sqrt{b\sec(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(2*d*Sqrt[Sec[c + d*x]]) +
 (b^2*Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{5/2}\,dx = \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d} + \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)}{2\sqrt{\sec(c+}}$$
$$= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{2d\sqrt{\sec(c+dx)}} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+}}{2d}$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.64

$$\frac{(b\sec(c+dx))^{5/2}\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{2d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])
)/(2*d*Sec[c + d*x]^(5/2))
```

Maple [A]
time = 36.99, size = 112, normalized size = 1.44

method	result
default	$\frac{\left(\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))+\sin(dx+c)\right)\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}\left(\frac{}{\cos}\right.}{2d}$
risch	$-\frac{ib^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)d}-\frac{b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}-i\right)\cos(dx+c)}{d}+\frac{b^2\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(ln(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(cos(d*x+
c)-1+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+sin(d*x+c))*cos(d*x+c)*(1/cos(d*x
+c))^(1/2)*(b/cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(66) = 132.
time = 0.63, size = 747, normalized size = 9.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(
4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*
d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2
 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2
*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2
*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x
 + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 3.68, size = 208, normalized size = 2.67

$$\left[\frac{b^{\frac{5}{2}}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2b^2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4d\cos\left(dx+c\right)}, -\frac{\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{b}\right)\cos\left(dx+c\right)-\frac{b^2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{2d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*b^2*sqrt
(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)), -1/2*(s
qrt(-b)*b^2*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/b)*cos(d*x + c) - b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2), x)
```

3.155 $\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

<u>**Optimal**</u>. Leaf size=35

$$\frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

```
[Out] b^2*sin(d*x+c)*sec(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

<u>**Rubi**</u> [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b^2*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 17</u>

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

<u>Rule 3852</u>

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

<u>Rubi steps</u>

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\sec(c+dx)}}$$
$$= \frac{b^2\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.91

$$\frac{(b\sec(c+dx))^{5/2}\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(3/2))
```

Maple [A]
time = 35.53, size = 39, normalized size = 1.11

method	result	size
default	$\frac{\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)\sin(dx+c)}{d\sqrt{\frac{1}{\cos(dx+c)}}}$	39
risch	$\frac{2ib^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/cos(d*x+c))^(5/2)*cos(d*x+c)*sin(d*x+c)/(1/cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.62, size = 54, normalized size = 1.54

$$\frac{2\,b^{\frac{5}{2}}\sin\left(2\,dx+2\,c\right)}{\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 3.30, size = 33, normalized size = 0.94

$$\frac{b^2\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] b^2*sqrt(b/cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sqrt(sec(d*x + c)), x)
```

Mupad [**B**]
time = 0.72, size = 66, normalized size = 1.89

$$\frac{b^2\sqrt{\frac{b}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1\mathrm{i}\right)}{d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] (b^2*(b/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/
(d*(cos(2*c + 2*d*x) + 1)*(1/cos(c + d*x))^(1/2))
```

3.156 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

[Out] b^2*arctanh(sin(d*x+c))*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{b^2\sqrt{b\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]

[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Sec[c + d*x]])/(d*Sqrt[Sec[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.92

$$\frac{\tanh^{-1}(\sin(c+dx))(b\sec(c+dx))^{5/2}}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Sec[c + d*x])^(5/2))/(d*Sec[c + d*x]^(5/2))
```

Maple [A]
time = 35.34, size = 52, normalized size = 1.44

method	result	size
default	$-\frac{2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}\right)\cos(dx+c)}{d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}}$	52
risch	$\frac{b^{2}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}-\frac{b^{2}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(b/cos(d*x+c))^(5/2)*arctanh((cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)/(1/
cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(32) = 64$.
time = 0.63, size = 72, normalized size = 2.00

$$\frac{\left(b^{2}\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-b^{2}\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A]

time = 3.54, size = 114, normalized size = 3.17

$$\left[\frac{b^{\frac{5}{2}}\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2d}, -\frac{\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(5/2)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/d, -sqrt(-b)*b^2*arctan(sq
rt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/d]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

3.157 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

```
[Out] b^2*x*(b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{17, 8\}$

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (b^2*x*Sqrt[b*Sec[c + d*x]])/Sqrt[Sec[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b^2x\sqrt{b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.89

$$\frac{x(b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
[Out] (x*(b*Sec[c + d*x])^(5/2))/Sec[c + d*x]^(5/2)
```

Maple [A]
time = 36.03, size = 32, normalized size = 1.19

method	result	size
default	$\frac{\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}(dx+c)}{d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}}$	32
risch	$\frac{b^2\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b/cos(d*x+c))^(5/2)/(1/cos(d*x+c))^(5/2)*(d*x+c)
```

Maxima [A]
time = 0.60, size = 26, normalized size = 0.96

$$\frac{2\,b^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
[Out] 2*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A]
time = 3.35, size = 101, normalized size = 3.74

$$\left[\frac{\sqrt{-b}\,b^2\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,d},\ \frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b^2*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*s
in(d*x + c) + 2*b*cos(d*x + c)^2 - b)/d, b^(5/2)*arctan(sqrt(b/cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/d]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 0.12, size = 27, normalized size = 1.00

$$\frac{b^2\,x\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] (b^2*x*(b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2)
```

3.158 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}$$

```
[Out] b^2*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int\cos(c+dx)\,dx}{\sqrt{\sec(c+dx)}}\\ &= \frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 0.91

$$\frac{(b\sec(c+dx))^{5/2}\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*Sec[c + d*x]^(5/2))
```

Maple [A]
time = 35.50, size = 41, normalized size = 1.17

method	result	size
default	$\frac{\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\sin(dx+c)}{d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\cos(dx+c)}$	41
risch	$-\frac{ib^{2}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{i(dx+c)}}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{ib^{2}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/cos(d*x+c))^(5/2)*sin(d*x+c)/(1/cos(d*x+c))^(7/2)/cos(d*x+c)
```

Maxima [A]
time = 0.59, size = 13, normalized size = 0.37

$$\frac{b^{\frac{5}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] b^(5/2)*sin(d*x + c)/d
```

Fricas [A]
time = 3.04, size = 33, normalized size = 0.94

$$\frac{b^{2}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
[Out] b^2*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 0.33, size = 35, normalized size = 1.00

$$\frac{b^2\,\sin\left(c+d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(7/2),x)
[Out] (b^2*sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(d*(1/cos(c + d*x))^(1/2))
```

3.159 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] 1/2*b^2*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/2*b^2*x*(b*sec
(d*x+c))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (b^2*x*Sqrt[b*Sec[c + d*x]])/(2*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[b*Sec[c + d
*x]]*Sin[c + d*x])/(2*d*Sec[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= \frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int 1\,dx}{2\sqrt{\sec(c+dx)}}$$
$$= \frac{b^2x\sqrt{b\sec(c+dx)}}{2\sqrt{\sec(c+dx)}} + \frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.11, size = 45, normalized size = 0.65

$$\frac{(b\sec(c+dx))^{5/2}(2(c+dx)+\sin(2(c+dx)))}{4d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sec[c + d*x]
^(5/2))
```

Maple [A]
time = 35.36, size = 54, normalized size = 0.78

method	result	size
default	$\frac{(\sin(dx+c)\cos(dx+c)+dx+c)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{2d\cos(dx+c)^2\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}}$	54
risch	$\frac{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ib^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{ib^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	197

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)*(b/cos(d*x+c))^(5/2)/cos(d*x+c)^2/(1/co
s(d*x+c))^(9/2)
```

Maxima [A]
time = 0.62, size = 32, normalized size = 0.46

$$\frac{(2\,(dx+c)b^2+b^2\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
[Out] 1/4*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A]
time = 3.30, size = 167, normalized size = 2.42

$$\left[\frac{2\,b^2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+\sqrt{-b}\,b^2\log\left(-2\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2\,b\cos\left(dx+c\right)^2-b\right)}{4\,d},\ \frac{b^2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{b}\sqrt{\cos\left(dx+c\right)}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
[Out] [1/4*(2*b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + sqrt(-b)
*b^2*log(-2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) +
 2*b*cos(d*x + c)^2 - b))/d, 1/2*(b^2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/
2)*sin(d*x + c) + b^(5/2)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)
*sqrt(cos(d*x + c)))))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(9/2), x)
```

Mupad [B]
time = 0.37, size = 44, normalized size = 0.64

$$\frac{b^2\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{4\,d\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] (b^2*(sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*d*(1/cos(c + d*x
))^(1/2))
```

3.160 $\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}}-\frac{b^2\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] b^2*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)-1/3*b^2*sin(d*x+c)^3
*(b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{b^2\sin(c+dx)\sqrt{b\sec(c+dx)}}{d\sqrt{\sec(c+dx)}}-\frac{b^2\sin^3(c+dx)\sqrt{b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (b^2*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]) - (b^2*Sqrt[
b*Sec[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\int\cos^3(c+dx)\,dx}{\sqrt{\sec(c+dx)}}$$
$$= -\frac{\left(b^2\sqrt{b\sec(c+dx)}\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\sec(c+dx)}}$$
$$= \frac{b^2\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sec(c+dx)}} - \frac{b^2\sqrt{b\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [A]
time = 0.18, size = 45, normalized size = 0.59

$$\frac{(5+\cos(2(c+dx)))(b\sec(c+dx))^{5/2}\sin(c+dx)}{6d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*(b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(6*d*Sec[c + d
*x]^(5/2))
```

Maple [A]
time = 38.96, size = 52, normalized size = 0.68

method	result	size
default	$\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}{3d\cos(dx+c)^3\left(\frac{1}{\cos(dx+c)}\right)^{\frac{11}{2}}}$	52
risch	$-\frac{3ib^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{3ib^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sin(3dx+3c)}{12\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	208

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b/cos(d*x+c))^(5/2)/cos(d*x+c)^3/(1/cos(
d*x+c))^(11/2)
```

Maxima [A]
time = 0.71, size = 49, normalized size = 0.64

$$\frac{\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))*sqrt(b)/d
```

Fricas [**A**]
time = 3.09, size = 55, normalized size = 0.72

$$\frac{\left(b^{2} \cos \left(dx+c\right)^{3}+2\,b^{2} \cos \left(dx+c\right)\right)\sqrt{\frac{b}{\cos \left(dx+c\right)}}\,\sin \left(dx+c\right)}{3\,d\sqrt{\cos \left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(d*x + c)^3 + 2*b^2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x
+ c)/(d*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(5/2)/sec(d*x + c)^(11/2), x)
```

Mupad [**B**]
time = 0.50, size = 48, normalized size = 0.63

$$\frac{b^2\,\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{12\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(11/2),x)
```

```
[Out] (b^2*(9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*d*(1/c
os(c + d*x))^(1/2))
```

3.161 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2d\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/2*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)+1/2*arctanh(sin(d*x+
c))*sec(d*x+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 3853, 3855}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*d*Sqrt[b*Sec[c + d*x]]) + (Se
c[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{2\sqrt{b\sec(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2d\sqrt{b\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 37.20, size = 114, normalized size = 1.58

method	result
default	$-\frac{\left(\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\sin(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\cos(dx+c)}{2d\sqrt{\frac{b}{\cos(dx+c)}}}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{3i(dx+c)}-e^{i(dx+c)})}{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d(e^{2i(dx+c)}+1)^2}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln(e^{i(dx+c)}+i)}{2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln(e^{i(dx+c)}-i)}{2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(ln(-(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-ln(-(cos(d*x
+c)-1-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-sin(d*x+c))*(1/cos(d*x+c))^(7/2)
*cos(d*x+c)/(b/cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(60) = 120.
time = 0.63, size = 661, normalized size = 9.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
1)*sqrt(b)*d)
```

Fricas [A]
time = 2.98, size = 205, normalized size = 2.85

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)+\frac{2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\,bd\cos\left(dx+c\right)},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{b}\right)\cos\left(dx+c\right)-\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{2\,bd\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*d*cos(d*x + c)), -1/2*(sqr
t(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b*
d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(1/2), x)
```

3.162 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

[Out] sec(d*x+c)^(3/2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{17, 3852, 8\}$

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(5/2)/Sqrt[b*Sec[c + d*x]],x]

[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}$$
$$= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[b*Sec[c + d*x]],x]
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 35.92, size = 39, normalized size = 1.22

method	result	size
default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)\sin(dx+c)}{d\sqrt{\frac{b}{\cos(dx+c)}}}$	39
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)*sin(d*x+c)/(b/cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.
time = 0.59, size = 59, normalized size = 1.84

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b\cos(2\,dx+2\,c)^2+b\sin(2\,dx+2\,c)^2+2\,b\cos(2\,dx+2\,c)+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 +
2*b*cos(2*d*x + 2*c) + b)*d)
```

Fricas [A]
time = 3.17, size = 33, normalized size = 1.03

$$\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c)), x)
```

Mupad [B]
time = 0.27, size = 51, normalized size = 1.59

$$\frac{\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1{}\mathrm{i}\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,1{}\mathrm{i}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] ((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*(b/cos(c + d*x))^(1/2)*(1/co
s(c + d*x))^(1/2)*1i)/(b*d)
```

3.163 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[b*Sec[c + d*x]],x]
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 34.89, size = 52, normalized size = 1.58

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}\right)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}}{d\sqrt{\frac{b}{\cos(dx+c)}}}$	52
risch	$\frac{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}-\frac{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	139

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*arctanh((cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(b/
cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.
time = 0.57, size = 65, normalized size = 1.97

$$\frac{\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)}{2\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A]

time = 3.42, size = 114, normalized size = 3.45

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2\sqrt{b}\,d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(sqrt(b)*d), -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(1/2),x)
[Out] int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(1/2), x)
```

3.164 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

```
[Out] x*sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{x\sqrt{\sec(c+dx)}}{\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[b*Sec[c + d*x]],x]
[Out] (x*Sqrt[Sec[c + d*x]])/Sqrt[b*Sec[c + d*x]]
```

Maple [A]
time = 34.76, size = 32, normalized size = 1.33

method	result	size
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,(dx+c)}{d\sqrt{\frac{b}{\cos(dx+c)}}}$	32
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(1/2)*(d*x+c)
```

Maxima [A]
time = 0.57, size = 26, normalized size = 1.08

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A]
time = 3.41, size = 101, normalized size = 4.21

$$\left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^2-b\right)}{2\,bd},\ \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{\sqrt{b}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/(b*d), arctan(sqrt(b/cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(sqrt(b)*d)]
```

Sympy [A]
time = 11.35, size = 22, normalized size = 0.92

$$\frac{x\sqrt{\sec{\left(c+dx\right)}}}{\sqrt{b\sec{\left(c+dx\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] x*sqrt(sec(c + d*x))/sqrt(b*sec(c + d*x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c)), x)
```

Mupad [B]
time = 0.30, size = 27, normalized size = 1.12

$$\frac{x\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{b\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(b/cos(c + d*x))^(1/2),x)
```

```
[Out] (x*(b/cos(c + d*x))^(1/2))/(b*(1/cos(c + d*x))^(1/2))
```

3.165 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {18, 2717}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 35.23, size = 41, normalized size = 1.28

method	result	size
default	$\frac{\sin(dx+c)}{d\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{b}{\cos(dx+c)}}\,\cos(dx+c)}$	41
risch	$-\frac{i\,\mathrm{e}^{2i(dx+c)}}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{i}{2\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*sin(d*x+c)/(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [A]
time = 0.59, size = 13, normalized size = 0.41

$$\frac{\sin(dx+c)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(sqrt(b)*d)
```

Fricas [A]
time = 3.35, size = 33, normalized size = 1.03

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d)
```

Sympy [A]
time = 20.37, size = 46, normalized size = 1.44

$$\begin{cases} \frac{\tan{(c+dx)}}{d\sqrt{b\sec{(c+dx)}}\sqrt{\sec{(c+dx)}}} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{b\sec{(c)}}\sqrt{\sec{(c)}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((tan(c + d*x)/(d*sqrt(b*sec(c + d*x))*sqrt(sec(c + d*x))), Ne(d,
0)), (x/(sqrt(b*sec(c))*sqrt(sec(c))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sqrt(sec(d*x + c))), x)
```

Mupad [B]
time = 0.31, size = 35, normalized size = 1.09

$$\frac{\sin{(c+d\,x)}\,\sqrt{\frac{b}{\cos{(c+d\,x)}}}}{b\,d\,\sqrt{\frac{1}{\cos{(c+d\,x)}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] (sin(c + d*x)*(b/cos(c + d*x))^(1/2))/(b*d*(1/cos(c + d*x))^(1/2))
```

3.166 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+1/2*x*sec(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{18, 2715, 8\}$

$$\frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*d*Sqrt[Se
c[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\,\int \cos^2(c+dx)\,dx}{\sqrt{b\sec(c+dx)}} \\
&= \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\,\int 1\,dx}{2\sqrt{b\sec(c+dx)}} \\
&= \frac{x\sqrt{\sec(c+dx)}}{2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 45, normalized size = 0.71

$$
\frac{\sqrt{\sec(c+dx)}\,(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Sec[c + d
*x]])
```

Maple [**A**]
time = 37.02, size = 54, normalized size = 0.86

method	result
default	$\frac{\sin(dx+c)\cos(dx+c)+dx+c}{2d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{b}{\cos(dx+c)}}\,\cos(dx+c)^2}$
risch	$\frac{e^{i(dx+c)}x}{2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right)} - \frac{i e^{3i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right)d} + \frac{}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)/(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(1/
2)/cos(d*x+c)^2
```

Maxima [**A**]
time = 0.59, size = 25, normalized size = 0.40

$$
\frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,\sqrt{b}\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(sqrt(b)*d)
```

Fricas [A]
time = 4.14, size = 165, normalized size = 2.62

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)-\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{4bd}, \frac{\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{2bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*log
(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(
d*x + c)^2 - b))/(b*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*
x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(cos
(d*x + c)))))/(b*d)]
```

Sympy [A]
time = 21.85, size = 107, normalized size = 1.70

$$\begin{cases} \frac{x\tan^2(c+dx)}{2\sqrt{b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)}+\frac{x}{2\sqrt{b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)}+\frac{\tan(c+dx)}{2d\sqrt{b\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{b\sec(c)}\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*tan(c + d*x)**2/(2*sqrt(b*sec(c + d*x))*sec(c + d*x)**(3/2)) +
 x/(2*sqrt(b*sec(c + d*x))*sec(c + d*x)**(3/2)) + tan(c + d*x)/(2*d*sqrt(b*
sec(c + d*x))*sec(c + d*x)**(3/2)), Ne(d, 0)), (x/(sqrt(b*sec(c))*sec(c)**(
3/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sec(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 0.43, size = 44, normalized size = 0.70

$$\frac{\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{4\,b\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] ((sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*b*d*(1/cos(c + d*x))
^(1/2))
```

3.167 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*sec(d*x
+c)^(1/2)/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {18, 2713}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[c +
d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int\cos^3(c+dx)\,dx}{\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.64

$$\frac{(5+\cos(2(c+dx)))\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*Sqrt[b*Sec[c
+ d*x]])
```

Maple [A]
time = 35.18, size = 52, normalized size = 0.74

method	result
default	$\frac{\sin(dx+c)(2+\cos^2(dx+c))}{3d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^3}$
risch	$-\frac{i e^{4i(dx+c)}}{24\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}-\frac{3i e^{2i(dx+c)}}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{}{8\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(cos(d*x+c)^2+2)/(1/cos(d*x+c))^(5/2)/(b/cos(d*x+c))^(1/2)
/cos(d*x+c)^3
```

Maxima [A]
time = 0.58, size = 42, normalized size = 0.60

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)}{12\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(sqrt(b)*d)
```

Fricas [A]
time = 3.89, size = 51, normalized size = 0.73

$$\frac{\left(\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b*
d*sqrt(cos(d*x + c)))
```

Sympy [A]
time = 43.46, size = 82, normalized size = 1.17

$$\begin{cases} \frac{2\tan^{3}{\left(c+dx \right)}}{3d\sqrt{b\sec{\left(c+dx \right)}}\sec^{\frac{5}{2}}{\left(c+dx \right)}} + \frac{\tan{\left(c+dx \right)}}{d\sqrt{b\sec{\left(c+dx \right)}}\sec^{\frac{5}{2}}{\left(c+dx \right)}} & \text{for}\: d \neq 0 \\ \frac{x}{\sqrt{b\sec{\left(c \right)}}\sec^{\frac{5}{2}}{\left(c \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3/(3*d*sqrt(b*sec(c + d*x))*sec(c + d*x)**(5/2))
 + tan(c + d*x)/(d*sqrt(b*sec(c + d*x))*sec(c + d*x)**(5/2)), Ne(d, 0)), (x
/(sqrt(b*sec(c))*sec(c)**(5/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c))*sec(d*x + c)^(5/2)), x)
```

Mupad [B]
time = 0.52, size = 48, normalized size = 0.69

$$\frac{\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{12\,b\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] ((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*b*d*(1/cos
(c + d*x))^(1/2))
```

3.168 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2bd\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/2*sec(d*x+c)^(5/2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)+1/2*arctanh(sin(d*
x+c))*sec(d*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*b*d*Sqrt[b*Sec[c + d*x]]) + (
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{2b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2bd\sqrt{b\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.64

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{2d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*d*(b*Sec[c + d*x])^(3/2))
```

Maple [A]
time = 34.30, size = 114, normalized size = 1.46

method	result
default	$-\frac{\left(\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\sin(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}\cos(dx+c)}{2d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{3i(dx+c)}-e^{i(dx+c)})}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln(e^{i(dx+c)}-i)}{2b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,\ln(e^{i(dx+c)}+i)}{2b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^2
*ln(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(1/cos(d*x+c))^(9/2)
*cos(d*x+c)/(b/cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(66) = 132.
time = 0.64, size = 670, normalized size = 8.59

$$-\frac{4\,(\sin(4\,dx+4\,c)+2\,\sin(2\,dx+2\,c))\cos\left(\frac{3}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)-4\,(\sin(4\,dx+4\,c)+2\,\sin(2\,dx+2\,c))\cos\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)-(2\,(2\cos(2\,dx+2\,c)+1)\cos(4\,dx+4\,c)+\cos(4\,dx+4\,c)^2+4\cos(2\,dx+2\,c)^2+\sin(4\,dx+4\,c)^2+4\sin(4\,dx+4\,c)\sin(2\,dx+2\,c)+4\sin(2\,dx+2\,c)^2+4\cos(2\,dx+2\,c)+1)\log\left(\cos\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)^2+\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)^2+2\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)+1\right)+(2\,(2\cos(2\,dx+2\,c)+1)\cos(4\,dx+4\,c)+\cos(4\,dx+4\,c)^2+4\cos(2\,dx+2\,c)^2+\sin(4\,dx+4\,c)^2+4\sin(4\,dx+4\,c)\sin(2\,dx+2\,c)+4\sin(2\,dx+2\,c)^2+4\cos(2\,dx+2\,c)+1)\log\left(\cos\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)^2+\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)^2-2\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)+1\right)-4\,(\cos(4\,dx+4\,c)+2\cos(2\,dx+2\,c)+1)\sin\left(\frac{3}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)+4\,(\cos(4\,dx+4\,c)+2\cos(2\,dx+2\,c)+1)\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c))\right)}{4\,(b\cos(4\,dx+4\,c)^2+4\,b\cos(2\,dx+2\,c)^2+b\sin(4\,dx+4\,c)^2+4\,b\sin(4\,dx+4\,c)\sin(2\,dx+2\,c)+4\,b\sin(2\,dx+2\,c)^2+2\,(2\,b\cos(2\,dx+2\,c)+b)\cos(4\,dx+4\,c)+4\,b\cos(2\,dx+2\,c)+b)\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)
^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin
(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2
*d*x + 2*c) + b)*sqrt(b)*d)
```

Fricas [A]
time = 3.61, size = 205, normalized size = 2.63

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos\left(dx+c\right)^{2}-2\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2b}{\cos\left(dx+c\right)^{2}}\right)+\frac{2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{4b^{2}d\cos\left(dx+c\right)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b}\right)\cos\left(dx+c\right)-\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{2b^{2}d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^2*d*cos(d*x + c)), -1/2*(s
qrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(9/2)/(b/cos(c + d*x))^(3/2), x)
```

3.169 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

```
[Out] sec(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}$$
$$= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.91

$$\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(3/2))
```

Maple [A]
time = 34.02, size = 39, normalized size = 1.11

method	result	size
default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}\cos(dx+c)\sin(dx+c)}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$	39
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/cos(d*x+c))^(7/2)*cos(d*x+c)*sin(d*x+c)/(b/cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(31) = 62$.
time = 0.58, size = 67, normalized size = 1.91

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^
2 + 2*b^2*cos(2*d*x + 2*c) + b^2)*d)
```

Fricas [**A**]
time = 3.38, size = 33, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{b^2d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [**B**]
time = 0.27, size = 51, normalized size = 1.46

$$\frac{\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1{}\mathrm{i}\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,1{}\mathrm{i}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] ((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*(b/cos(c + d*x))^(1/2)*(1/co
s(c + d*x))^(1/2)*1i)/(b^2*d)
```

3.170 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.92

$$\frac{\tanh^{-1}(\sin(c+dx))\sec^{\frac{3}{2}}(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sec[c + d*x]^(3/2))/(d*(b*Sec[c + d*x])^(3/2))
```

Maple [A]
time = 37.47, size = 52, normalized size = 1.44

method	result	size
default	$-\frac{2\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}\right)}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$	52
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}+i\right)}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}-i\right)}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*arctanh((cos(d*x+c)-1)/sin(d*x+c))/(b/
cos(d*x+c))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.
time = 0.62, size = 65, normalized size = 1.81

$$\frac{\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)}{2\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A]

time = 3.32, size = 114, normalized size = 3.17

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2b^{\frac{3}{2}}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(b^(3/2)*d), -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/(b^2*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(3/2), x)
```

3.171 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}$$

[Out] x*sec(d*x+c)^(1/2)/b/(b*sec(d*x+c))^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(3/2),x]

[Out] (x*Sqrt[Sec[c + d*x]])/(b*Sqrt[b*Sec[c + d*x]])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{b\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]

time = 0.02, size = 24, normalized size = 0.89

$$\frac{x\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (x*Sec[c + d*x]^(3/2))/(b*Sec[c + d*x])^(3/2)
```

Maple [A]
time = 34.40, size = 32, normalized size = 1.19

method	result	size
default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}(dx+c)}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}}$	32
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{b\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(3/2)*(d*x+c)
```

Maxima [A]
time = 0.53, size = 26, normalized size = 0.96

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A]
time = 4.26, size = 101, normalized size = 3.74

$$\left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2\,b\cos(dx+c)^{2}-b\right)}{2\,b^{2}d},\ \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{b^{\frac{3}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/(b^2*d), arctan(sqrt(b/cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(b^(3/2)*d)]
```

Sympy [A]
time = 13.45, size = 22, normalized size = 0.81

$$\frac{x\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] x*sec(c + d*x)**(3/2)/(b*sec(c + d*x))**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.25, size = 27, normalized size = 1.00

$$\frac{x\sqrt{\frac{b}{\cos(c+d\,x)}}}{b^2\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] (x*(b/cos(c + d*x))^(1/2))/(b^2*(1/cos(c + d*x))^(1/2))
```

3.172 $\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(3/2),x]
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.91

$$\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(3/2))
```

Maple [A]
time = 35.29, size = 41, normalized size = 1.17

method	result	size
default	$\frac{\sin(dx+c)\sqrt{\frac{1}{\cos(dx+c)}}}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)}$	41
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, e^{i(dx+c)}}{2b\sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}+\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, e^{-i(dx+c)}}{2b\sqrt{\frac{b\, e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\, d}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*sin(d*x+c)*(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(3/2)/cos(d*x+c)
```

Maxima [A]
time = 0.61, size = 13, normalized size = 0.37

$$\frac{\sin(dx+c)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(3/2)*d)
```

Fricas [A]
time = 4.14, size = 33, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d)
```

Sympy [A]
time = 11.70, size = 46, normalized size = 1.31

$$\begin{cases} \frac{\tan(c+dx)\sqrt{\sec(c+dx)}}{d(b\sec(c+dx))^{\frac{3}{2}}} & \text{for } d \neq 0 \\ \frac{x\sqrt{\sec(c)}}{(b\sec(c))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((tan(c + d*x)*sqrt(sec(c + d*x))/(d*(b*sec(c + d*x))**(3/2)), Ne(
d, 0)), (x*sqrt(sec(c))/(b*sec(c))**(3/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.42, size = 39, normalized size = 1.11

$$\frac{\sin(2c+2dx)\sqrt{\frac{b}{\cos(c+dx)}}\sqrt{\frac{1}{\cos(c+dx)}}}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(b/cos(c + d*x))^(3/2),x)
```

```
[Out] (sin(2*c + 2*d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2))/(2*b^2*d)
```

3.173 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+1/2*x*sec(d*x+c)^(
1/2)/b/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 2715, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*b*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*b*d*Sqr
t[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int\cos^2(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{2b\sqrt{b\sec(c+dx)}}$$
$$= \frac{x\sqrt{\sec(c+dx)}}{2b\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 45, normalized size = 0.65

$$\frac{\sec^{\frac{3}{2}}(c+dx)(2(c+dx)+\sin(2(c+dx)))}{4d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(3/2)),x]
[Out] (Sec[c + d*x]^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*(b*Sec[c + d*x])
^(3/2))
```

Maple [A]
time = 35.50, size = 54, normalized size = 0.78

method	result
default	$\frac{\sin(dx+c)\cos(dx+c)+dx+c}{2d\sqrt{\frac{1}{\cos(dx+c)}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)^2}$
risch	$\frac{e^{i(dx+c)}x}{2b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ie^{3i(dx+c)}}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)/(1/cos(d*x+c))^(1/2)/(b/cos(d*x+c))^(3/
2)/cos(d*x+c)^2
```

Maxima [A]
time = 0.60, size = 25, normalized size = 0.36

$$\frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(3/2)*d)
```

Fricas [A]
time = 4.21, size = 165, normalized size = 2.39

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)-\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^{2}-b\right)}{4b^{2}d},\ \frac{\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{2b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*log
(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(
d*x + c)^2 - b))/(b^2*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(c
os(d*x + c)))))/(b^2*d)]
```

Sympy [A]
time = 21.92, size = 107, normalized size = 1.55

$$\begin{cases} \frac{x\tan^{2}(c+dx)}{2(b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}+\frac{x}{2(b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}+\frac{\tan(c+dx)}{2d(b\sec(c+dx))^{\frac{3}{2}}\sqrt{\sec(c+dx)}} & \text{for}\ d\neq 0\\ \frac{x}{(b\sec(c))^{\frac{3}{2}}\sqrt{\sec(c)}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((x*tan(c + d*x)**2/(2*(b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x)))
 + x/(2*(b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))) + tan(c + d*x)/(2*d*(b*
sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), Ne(d, 0)), (x/((b*sec(c))**(3/2)*
sqrt(sec(c))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sqrt(sec(d*x + c))), x)
```

Mupad [B]
time = 0.37, size = 44, normalized size = 0.64

$$\frac{\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{4\,b^2\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] ((sin(2*c + 2*d*x) + 2*d*x)*(b/cos(c + d*x))^(1/2))/(4*b^2*d*(1/cos(c + d*x
))^(1/2))
```

3.174 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\,\sin^3(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*sec(d
*x+c)^(1/2)/b/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {18, 2713}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3bd\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[c
+ d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\,\int \cos^3(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}\\
&= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int (1-x^2)\,dx,x,-\sin(c+dx)\right)}{bd\sqrt{b\sec(c+dx)}}\\
&= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\,\sin^3(c+dx)}{3bd\sqrt{b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 45, normalized size = 0.59

$$\frac{(5+\cos(2(c+dx)))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(b*Sec[c + d*
x])^(3/2))
```

Maple [A]
time = 35.54, size = 52, normalized size = 0.68

method	result
default	$\frac{\sin(dx+c)\left(2+\cos^2(dx+c)\right)}{3d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)^3}$
risch	$-\frac{i\mathrm{e}^{4i(dx+c)}}{24b\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}-\frac{3i\mathrm{e}^{2i(dx+c)}}{8b\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{}{8b\sqrt{\frac{\mathrm{e}^{i(dx+}}{\mathrm{e}^{2i(dx+}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(cos(d*x+c)^2+2)/(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(3/2)
/cos(d*x+c)^3
```

Maxima [A]
time = 0.59, size = 42, normalized size = 0.55

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)}{12\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(3/2)*d)
```

Fricas [A]
time = 3.77, size = 51, normalized size = 0.67

$$\frac{\left(\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,b^2d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^
2*d*sqrt(cos(d*x + c)))
```

Sympy [A]
time = 30.24, size = 82, normalized size = 1.08

$$\begin{cases} \frac{2\tan^{3}(c+dx)}{3d(b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)} + \frac{\tan(c+dx)}{d(b\sec(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x}{(b\sec(c))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3/(3*d*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/
2)) + tan(c + d*x)/(d*(b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3/2)), Ne(d, 0
)), (x/((b*sec(c))**(3/2)*sec(c)**(3/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 0.33, size = 48, normalized size = 0.63

$$\frac{\left(9\,\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}{12\,b^2\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] ((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*b^2*d*(1/c
os(c + d*x))^(1/2))
```

3.175 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/4*sin(d*x+c)/b/d/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2)+3/8*sin(d*x+c)/b/d
/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+3/8*x*sec(d*x+c)^(1/2)/b/(b*sec(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {18, 2715, 8}

$$\frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (3*x*Sqrt[Sec[c + d*x]])/(8*b*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(4*b*d*S
ec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]) + (3*Sin[c + d*x])/(8*b*d*Sqrt[Sec[
c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \cos^4(c+dx)\,dx}{b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{\left(3\sqrt{\sec(c+dx)}\right)\int \cos^2(c+d}{4b\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8bd\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx}}$$
$$= \frac{3x\sqrt{\sec(c+dx)}}{8b\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{4bd\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{}{8bd\sqrt{\sec(}}$$

Mathematica [A]
time = 0.13, size = 55, normalized size = 0.51

$$\frac{\sec^{\frac{3}{2}}(c+dx)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^(3/2)),x]
[Out] (Sec[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*d*(b*Sec[c + d*x])^(3/2))
```

Maple [A]
time = 32.79, size = 74, normalized size = 0.69

method	result
default	$\frac{2(\cos^3(dx+c))\sin(dx+c)+3\sin(dx+c)\cos(dx+c)+3dx+3c}{8d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)^4}$
risch	$\frac{3\,e^{i(dx+c)}x}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ie^{5i(dx+c)}}{64b\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{}{8b\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)+3*d*x+3*c)/(1/cos(
d*x+c))^(5/2)/(b/cos(d*x+c))^(3/2)/cos(d*x+c)^4
```

Maxima [A]
time = 0.65, size = 49, normalized size = 0.46

$$\frac{12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)}{32\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(3/2)*d)
```

Fricas [A]
time = 3.76, size = 208, normalized size = 1.94

$$\left[\frac{\frac{2\left(2\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}-3\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2b\cos\left(dx+c\right)^{2}-b\right)}{16b^{2}d},\ \frac{\frac{\left(2\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+3\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{b}\sqrt{\cos\left(dx+c\right)}}\right)}{8b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) - 3*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*c
os(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/(b^2*d), 1/8*((2*
cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
sqrt(cos(d*x + c)))))/(b^2*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(3/2)*sec(d*x + c)^(5/2)), x)
```

Mupad [B]
time = 0.62, size = 55, normalized size = 0.51

$$\frac{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,b^2\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)),x)
[Out] ((b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))/(
32*b^2*d*(1/cos(c + d*x))^(1/2))
```

3.176 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/2*sec(d*x+c)^(5/2)*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)+1/2*arctanh(sin(
d*x+c))*sec(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 3853, 3855}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}+\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*b^2*d*Sqrt[b*Sec[c + d*x]]) +
 (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{2b^2\sqrt{b\sec(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{2b^2d\sqrt{b\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{2b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(11/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(2
*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 36.76, size = 114, normalized size = 1.46

method	result
default	$-\frac{\left(\ln\left(-\frac{\cos(dx+c)-1+\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\ln\left(-\frac{\cos(dx+c)-1-\sin(dx+c)}{\sin(dx+c)}\right)(\cos^2(dx+c))-\sin(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{11}{2}}\cos(dx+c)}{2d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{3i(dx+c)}-e^{i(dx+c)})}{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}-i\right)}{2b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\ln\left(e^{i(dx+c)}+i\right)}{2b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(cos(d*x+c)-1+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^2
*ln(-(cos(d*x+c)-1-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(1/cos(d*x+c))^(11/2
)*cos(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(66) = 132.
time = 0.60, size = 688, normalized size = 8.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x +
2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d
*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)*d)
```

Fricas [A]
time = 2.69, size = 205, normalized size = 2.63

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)\log\left(-\frac{b\cos\left(dx+c\right)^2-2\sqrt{b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2b}{\cos\left(dx+c\right)^2}\right)+\frac{2\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{4b^3d\cos\left(dx+c\right)},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b}\right)\cos\left(dx+c\right)-\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{2b^3d\cos\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2) + 2*sqrt(b/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(b^3*d*cos(d*x + c)), -1/2*(s
qrt(-b)*arctan(sqrt(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/b)*cos(d*x + c) - sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(
b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(11/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(11/2)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(11/2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(11/2)/(b/cos(c + d*x))^(5/2), x)
```

3.177 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \sec^2(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}$$
$$= -\frac{\sqrt{\sec(c+dx)}\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{b^2 d\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b^2 d\sqrt{b\sec(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.91

$$\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(b*Sec[c + d*x])^(5/2),x]
[Out] (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(5/2))
```

Maple [A]
time = 36.86, size = 39, normalized size = 1.11

method	result	size
default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{9}{2}}\cos(dx+c)\sin(dx+c)}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$	39
risch	$\frac{2i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d\left(e^{2i(dx+c)}+1\right)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/cos(d*x+c))^(9/2)*cos(d*x+c)*sin(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.
time = 0.61, size = 67, normalized size = 1.91

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^
2 + 2*b^3*cos(2*d*x + 2*c) + b^3)*d)
```

Fricas [A]
time = 2.87, size = 33, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{b^3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.26, size = 51, normalized size = 1.46

$$\frac{(\cos(d\,x)-\sin(d\,x)\ 1\mathrm{i})\ (\cos(c)-\sin(c)\ 1\mathrm{i})\ \sqrt{\frac{b}{\cos(c+d\,x)}}\ \sqrt{\frac{1}{\cos(c+d\,x)}}\ 1\mathrm{i}}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(9/2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] ((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*(b/cos(c + d*x))^(1/2)*(1/co
s(c + d*x))^(1/2)*1i)/(b^3*d)
```

3.178 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*sec(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{\sec(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \sec(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.92

$$\frac{\tanh^{-1}(\sin(c+dx))\sec^{\frac{5}{2}}(c+dx)}{d(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sec[c + d*x]^(5/2))/(d*(b*Sec[c + d*x])^(5/2))
```

Maple [A]
time = 35.56, size = 52, normalized size = 1.44

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\cos(dx+c)-1}{\sin(dx+c)}\right)\cos(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}}}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$	52
risch	$-\frac{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b^{2}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}+\frac{\sqrt{\frac{\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b^{2}\sqrt{\frac{b\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*arctanh((cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(7/2)/(b/
cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.
time = 0.63, size = 65, normalized size = 1.81

$$\frac{\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)}{2\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A]

time = 3.05, size = 114, normalized size = 3.17

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2b}{\cos(dx+c)^2}\right)}{2b^{\frac{5}{2}}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{b}\right)}{b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b)/cos(d*x + c)^2)/(b^(5/2)*d), -sqrt(-b)*arctan(sqrt
(-b)*sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/b)/(b^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)/(b/cos(c + d*x))^(5/2), x)
```

3.179 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}$$

```
[Out] x*sec(d*x+c)^(1/2)/b^2/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(b^2*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{x\sqrt{\sec(c+dx)}}{b^2\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 0.89

$$\frac{x\sec^{\frac{5}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(b*Sec[c + d*x])^(5/2),x]
[Out] (x*Sec[c + d*x]^(5/2))/(b*Sec[c + d*x])^(5/2)
```

Maple [A]
time = 35.24, size = 32, normalized size = 1.19

method	result	size
default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}(dx+c)}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/cos(d*x+c))^(5/2)/(b/cos(d*x+c))^(5/2)*(d*x+c)
```

Maxima [A]
time = 0.59, size = 26, normalized size = 0.96

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A]
time = 4.26, size = 101, normalized size = 3.74

$$\left[-\frac{\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)+2b\cos(dx+c)^2-b\right)}{2b^3d},\ \frac{\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{b^{\frac{5}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d
*x + c) + 2*b*cos(d*x + c)^2 - b)/(b^3*d), arctan(sqrt(b/cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*sqrt(cos(d*x + c))))/(b^(5/2)*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [**B**]
time = 0.32, size = 27, normalized size = 1.00

$$\frac{x\sqrt{\frac{b}{\cos(c+d\,x)}}}{b^3\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(5/2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] (x*(b/cos(c + d*x))^(1/2))/(b^3*(1/cos(c + d*x))^(1/2))
```

3.180 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\,\int \cos(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 35, normalized size = 1.00

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 36.15, size = 41, normalized size = 1.17

method	result	size
default	$\frac{\sin(dx+c)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}}{d\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)}$	41
risch	$-\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{i(dx+c)}}{2b^{2}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}+\frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-i(dx+c)}}{2b^{2}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	140

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*sin(d*x+c)*(1/cos(d*x+c))^(3/2)/(b/cos(d*x+c))^(5/2)/cos(d*x+c)
```

Maxima [A]
time = 0.65, size = 13, normalized size = 0.37

$$\frac{\sin\left(dx+c\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(5/2)*d)
```

Fricas [A]
time = 2.45, size = 33, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d)
```

Sympy [A]
time = 30.44, size = 46, normalized size = 1.31

$$\begin{cases} \frac{\tan(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(b\sec(c+dx))^{\frac{5}{2}}} & \text{for } d \neq 0 \\ \frac{x\sec^{\frac{3}{2}}(c)}{(b\sec(c))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)
[Out] Piecewise((tan(c + d*x)*sec(c + d*x)**(3/2)/(d*(b*sec(c + d*x))**(5/2)), Ne
(d, 0)), (x*sec(c)**(3/2)/(b*sec(c))**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.39, size = 39, normalized size = 1.11

$$\frac{\sin(2\,c+2\,d\,x)\,\sqrt{\frac{b}{\cos(c+d\,x)}}\,\sqrt{\frac{1}{\cos(c+d\,x)}}}{2\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(3/2)/(b/cos(c + d*x))^(5/2),x)
[Out] (sin(2*c + 2*d*x)*(b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2))/(2*b^3*d)
```

3.181 $\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)/b^2/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+1/2*x*sec(d*x+c)
^(1/2)/b^2/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {17, 2715, 8}

$$\frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\,\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Sec[c + d*x]])/(2*b^2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(2*b^2*d
*Sqrt[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\int 1\,dx}{2b^2\sqrt{b\sec(c+dx)}}$$
$$= \frac{x\sqrt{\sec(c+dx)}}{2b^2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{2b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Mathematica [**A**]
time = 0.06, size = 48, normalized size = 0.70

$$\frac{\sqrt{\sec(c+dx)}\,(2(c+dx)+\sin(2(c+dx)))}{4b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Sec[c
 + d*x]])
```

Maple [**A**]
time = 32.28, size = 54, normalized size = 0.78

method	result	size
default	$\frac{(\sin(dx+c)\cos(dx+c)+dx+c)\sqrt{\frac{1}{\cos(dx+c)}}}{2d\cos(dx+c)^2\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$	54
risch	$\frac{\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,x}{2b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{8b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{i\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{-2i(dx+c)}}{8b^2\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	197

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)*(1/cos(d*x+c))^(1/2)/cos(d*x+c)^2/(b/co
s(d*x+c))^(5/2)
```

Maxima [**A**]
time = 0.58, size = 25, normalized size = 0.36

$$\frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(5/2)*d)
```

Fricas [A]
time = 3.57, size = 165, normalized size = 2.39

$$\left[\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) - \sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) + 2b\cos(dx+c)^2 - b\right)}{4b^3d}, \frac{\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)^{\frac{3}{2}}\sin(dx+c) + \sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{b}\sqrt{\cos(dx+c)}}\right)}{2b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) - sqrt(-b)*log
(2*sqrt(-b)*sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(
d*x + c)^2 - b))/(b^3*d), 1/2*(sqrt(b/cos(d*x + c))*cos(d*x + c)^(3/2)*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*sqrt(c
os(d*x + c)))))/(b^3*d)]
```

Sympy [A]
time = 19.41, size = 107, normalized size = 1.55

$$\begin{cases} \frac{x\tan^2(c+dx)\sqrt{\sec(c+dx)}}{2(b\sec(c+dx))^{\frac{5}{2}}} + \frac{x\sqrt{\sec(c+dx)}}{2(b\sec(c+dx))^{\frac{5}{2}}} + \frac{\tan(c+dx)\sqrt{\sec(c+dx)}}{2d(b\sec(c+dx))^{\frac{5}{2}}} & \text{for}\ d \neq 0 \\ \frac{x\sqrt{\sec(c)}}{(b\sec(c))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((x*tan(c + d*x)**2*sqrt(sec(c + d*x))/(2*(b*sec(c + d*x))**(5/2))
 + x*sqrt(sec(c + d*x))/(2*(b*sec(c + d*x))**(5/2)) + tan(c + d*x)*sqrt(sec
(c + d*x))/(2*d*(b*sec(c + d*x))**(5/2)), Ne(d, 0)), (x*sqrt(sec(c))/(b*sec
(c))**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.63, size = 64, normalized size = 0.93

$$\frac{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{8\,b^3\,d\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^(1/2)/(b/cos(c + d*x))^(5/2),x)
```

```
[Out] ((b/cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x) + 4*d*x*cos(c + d*
x)))/(8*b^3*d*cos(c + d*x)*(1/cos(c + d*x))^(1/2))
```

3.182 $\int \frac{1}{\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\,\sin^3(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

```
[Out] sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*sec
(d*x+c)^(1/2)/b^2/d/(b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {18, 2713}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\sqrt{b\sec(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Sec[c + d*x]]) - (Sqrt[Sec[
c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\sec(c+dx)}\int \cos^3(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}\\ &= -\frac{\sqrt{\sec(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{b^2d\sqrt{b\sec(c+dx)}}\\ &= \frac{\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\sec(c+dx)}}-\frac{\sqrt{\sec(c+dx)}\,\sin^3(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 48, normalized size = 0.63

$$\frac{(5+\cos(2(c+dx)))\sqrt{\sec(c+dx)}\,\sin(c+dx)}{6b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((5 + Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*b^2*d*Sqrt[b*Se
c[c + d*x]])
```

Maple [A]
time = 34.75, size = 52, normalized size = 0.68

method	result
default	$\frac{\sin(dx+c)(2+\cos^2(dx+c))}{3d\cos(dx+c)^3\sqrt{\frac{1}{\cos(dx+c)}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}}$
risch	$-\frac{i e^{4i(dx+c)}}{24b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)d}-\frac{3i e^{2i(dx+c)}}{8b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\left(e^{2i(dx+c)}+1\right)d}+\frac{}{8b^2\sqrt{\frac{}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(cos(d*x+c)^2+2)/cos(d*x+c)^3/(1/cos(d*x+c))^(1/2)/(b/cos(
d*x+c))^(5/2)
```

Maxima [A]
time = 0.61, size = 42, normalized size = 0.55

$$\frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c)\right)\right)}{12\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(5/2)*d)
```

Fricas [A]
time = 3.18, size = 51, normalized size = 0.67

$$\frac{\left(\cos(dx+c)^3+2\,\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{3\,b^3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 + 2*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(d*x + c)/(b^
3*d*sqrt(cos(d*x + c)))
```

Sympy [A]
time = 41.97, size = 82, normalized size = 1.08

$$\begin{cases} \frac{2\tan^3(c+dx)}{3d(b\sec(c+dx))^{\frac{5}{2}}\sqrt{\sec(c+dx)}} + \frac{\tan(c+dx)}{d(b\sec(c+dx))^{\frac{5}{2}}\sqrt{\sec(c+dx)}} & \text{for } d \neq 0 \\ \frac{x}{(b\sec(c))^{\frac{5}{2}}\sqrt{\sec(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3/(3*d*(b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x)
)) + tan(c + d*x)/(d*(b*sec(c + d*x))**(5/2)*sqrt(sec(c + d*x))), Ne(d, 0))
, (x/((b*sec(c))**(5/2)*sqrt(sec(c))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(5/2)*sqrt(sec(d*x + c))), x)
```

Mupad [B]
time = 0.42, size = 48, normalized size = 0.63

$$\frac{(9\sin(c+d\,x)+\sin(3\,c+3\,d\,x))\,\sqrt{\frac{b}{\cos(c+d\,x)}}}{12\,b^3\,d\,\sqrt{\frac{1}{\cos(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] ((9*sin(c + d*x) + sin(3*c + 3*d*x))*(b/cos(c + d*x))^(1/2))/(12*b^3*d*(1/c
os(c + d*x))^(1/2))
```

3.183 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

```
[Out] 1/4*sin(d*x+c)/b^2/d/sec(d*x+c)^(5/2)/(b*sec(d*x+c))^(1/2)+3/8*sin(d*x+c)/b
^2/d/sec(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+3/8*x*sec(d*x+c)^(1/2)/b^2/(b*se
c(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 2715, 8}

$$\frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}}+\frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (3*x*Sqrt[Sec[c + d*x]])/(8*b^2*Sqrt[b*Sec[c + d*x]]) + Sin[c + d*x]/(4*b^2
*d*Sec[c + d*x]^(5/2)*Sqrt[b*Sec[c + d*x]]) + (3*Sin[c + d*x])/(8*b^2*d*Sqr
t[Sec[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\sec(c+dx)}\int \cos^4(c+dx)\,dx}{b^2\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{\left(3\sqrt{\sec(c+dx)}\right)\int \cos^2(c+dx}{4b^2\sqrt{b\sec(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{3\sin(c+dx)}{8b^2d\sqrt{\sec(c+dx)}\sqrt{b\sec(c+d}}$$
$$= \frac{3x\sqrt{\sec(c+dx)}}{8b^2\sqrt{b\sec(c+dx)}} + \frac{\sin(c+dx)}{4b^2d\sec^{\frac{5}{2}}(c+dx)\sqrt{b\sec(c+dx)}} + \frac{}{8b^2d\sqrt{\sec}}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.54

$$\frac{\sqrt{\sec(c+dx)}\,(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Maple [A]
time = 35.69, size = 74, normalized size = 0.69

method	result
default	$\frac{2(\cos^3(dx+c))\sin(dx+c)+3\sin(dx+c)\cos(dx+c)+3dx+3c}{8d\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(\frac{b}{\cos(dx+c)}\right)^{\frac{5}{2}}\cos(dx+c)^4}$
risch	$\frac{3\,e^{i(dx+c)}x}{8b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+1)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{ie^{5i(dx+c)}}{64b^2\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+1)\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d} + \frac{}{8b^2\sqrt{\frac{e^{i(dx+}}{e^{2i(dx+c}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(2*cos(d*x+c)^3*sin(d*x+c)+3*sin(d*x+c)*cos(d*x+c)+3*d*x+3*c)/(1/cos(
d*x+c))^(3/2)/(b/cos(d*x+c))^(5/2)/cos(d*x+c)^4
```

Maxima [A]
time = 0.66, size = 49, normalized size = 0.46

$$\frac{12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)}{32\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(5/2)*d)
```

Fricas [A]
time = 3.62, size = 208, normalized size = 1.94

$$\left[\frac{\frac{2\left(2\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}-3\sqrt{-b}\log\left(2\sqrt{-b}\sqrt{\frac{b}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)^{\frac{3}{2}}\sin\left(dx+c\right)+2b\cos\left(dx+c\right)^{2}-b\right)}{16b^{3}d}, \frac{\frac{\left(2\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+3\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{b}\sqrt{\cos\left(dx+c\right)}}\right)}{8b^{3}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)) - 3*sqrt(-b)*log(2*sqrt(-b)*sqrt(b/cos(d*x + c))*c
os(d*x + c)^(3/2)*sin(d*x + c) + 2*b*cos(d*x + c)^2 - b))/(b^3*d), 1/8*((2*
cos(d*x + c)^4 + 3*cos(d*x + c)^2)*sqrt(b/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 3*sqrt(b)*arctan(sqrt(b/cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
sqrt(cos(d*x + c)))))/(b^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c))^(5/2)*sec(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 0.46, size = 55, normalized size = 0.51

$$\frac{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,\left(8\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)+12\,d\,x\right)}{32\,b^3\,d\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)),x)
[Out] ((b/cos(c + d*x))^(1/2)*(8*sin(2*c + 2*d*x) + sin(4*c + 4*d*x) + 12*d*x))/(
32*b^3*d*(1/cos(c + d*x))^(1/2))
```

3.184 $\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)
*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{7/3}\,dx}{b^2}$$
$$= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}\,dx}{b^2}$$
$$= \frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 1.03

$$\frac{3\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sec^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Csc[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(7*b*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec\left(c+dx\right)}\,\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(1/3),x)
[Out] Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/3)/cos(c + d*x)^2,x)
[Out] int((b/cos(c + d*x))^(1/3)/cos(c + d*x)^2, x)
```

3.185 $\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+c)
/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{\int (b\sec(c+dx))^{4/3}\,dx}{b}$$
$$= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx}{b}$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.13

$$\frac{3\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec(c+dx)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(1/3),x)
[Out] Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/3)/cos(c + d*x),x)
[Out] int((b/cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

3.186 $\int \sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=56

$$-\frac{3b\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*b*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3b\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\sec(c+dx)}\,dx &= \sqrt[3]{\frac{\cos(c+dx)}{b}}\,\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx\\ &= -\frac{3\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.98

$$\frac{3\cot(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sqrt{-\tan^2(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(1/3),x]
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/d
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/3),x)
[Out] int((b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/3),x)
```

```
[Out] int((b/cos(c + d*x))^(1/3), x)
```

3.187 $\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\sec(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

[Out] -3/5*b^2*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3),x]

[Out] (-3*b^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = b\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx$$
$$= \left(b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}dx$$
$$= -\frac{3\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.00

$$-\frac{3b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{2d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2]*Sqrt[-
Tan[c + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \sec (c+dx)} \cos (c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(1/3),x)
[Out] Integral((b*sec(c + d*x))**(1/3)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos (c+d\,x)\,\left(\frac{b}{\cos (c+d\,x)}\right)^{1/3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b/cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)*(b/cos(c + d*x))^(1/3), x)
```

3.188 $\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\sec(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/8*b^3*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c
))^(8/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d
*(b*Sec[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = b^2\int \frac{1}{(b\sec(c+dx))^{5/3}}\,dx$$
$$= \left(b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{5/3}dx$$
$$= -\frac{3\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{8d\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.16, size = 59, normalized size = 1.02

$$\frac{3\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\sec^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(2(c+dx))}{10d\sqrt{-\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(1/3),x]
[Out] (3*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)
*Sin[2*(c + d*x)])/(10*d*Sqrt[-Tan[c + d*x]^2])
```

Maple [**F**]
time = 0.43, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec (c+dx)}\,\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(1/3),x)
[Out] Integral((b*sec(c + d*x))**(1/3)*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3), x)
```

3.189 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(7/3)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3
)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{\int (b\sec(c+dx))^{10/3}\,dx}{b^2}$$
$$= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10/3}}\,dx}{b^2}$$
$$= \frac{3b\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sec(c+dx)\sqrt[3]{b\sec(c+dx)}\,\tan(c+d}{7d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 1.03

$$\frac{3\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\sec^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sqrt{-\tan^2(c+dx)}}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
[Out] (3*Csc[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(10*b*d)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^{\frac{4}{3}}\sec^2 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3),x)
[Out] Integral((b*sec(c + d*x))**(4/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(4/3)/cos(c + d*x)^2,x)
[Out] int((b/cos(c + d*x))^(4/3)/cos(c + d*x)^2, x)
```

3.190 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=55

$$\frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*x+
c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)
*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{\int (b\sec(c+dx))^{7/3}\,dx}{b}$$
$$= \frac{\left(\sqrt[3]{\frac{\cos(c+dx)}{b}}\,\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}\,dx}{b}$$
$$= \frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.09

$$\frac{3\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sec^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sqrt{-\tan^2(c+dx)}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(7*b*d)
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{4}{3}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3),x)
[Out] Integral((b*sec(c + d*x))**(4/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(4/3)/cos(c + d*x),x)
[Out] int((b/cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

3.191 $\int (b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=54

$$\frac{3b\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*b*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+
c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\sec(c+dx))^{4/3}\,dx &= \sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.06

$$\frac{3\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(4/3),x]
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(4/3),x)
[Out] int((b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(c+dx\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(4/3),x)
```

```
[Out] int((b/cos(c + d*x))^(4/3), x)
```

3.192 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*b^2*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c
))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}\,dx = b\int \sqrt[3]{b\sec(c+dx)}\,dx$$
$$= \left(b\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx$$
$$= -\frac{3b\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.01, size = 56, normalized size = 0.97

$$\frac{3b\cot(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3),x]
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2]*(b*Sec[c
 + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/d
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b/cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)*(b/cos(c + d*x))^(4/3), x)
```

3.193 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\sec(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*b^3*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3857, 2722}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}\,dx = b^2\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx$$
$$= \left(b^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}dx$$
$$= -\frac{3b\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+}{5d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.03

$$-\frac{3b^2\cot(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{2d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c + d*x]^2]*Sqrt
[-Tan[c + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*b*cos(d*x + c)^2*sec(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3), x)
```

3.194 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(2/3)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)
*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{5/3}\,dx}{b^2}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{5/3}}\,dx}{b^2}$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 60, normalized size = 1.03

$$\frac{3\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sec^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Csc[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2]*(b*Sec[c
+ d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(5*b*d)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(1/3)), x)
```

3.195 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\sec(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/
3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{3\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Se
c[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\,dx}{b}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b}$$
$$= -\frac{3\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 1.13

$$\frac{3\cot(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*b*d)
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/3)),x)
[Out] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(1/3)), x)
```

3.196 $\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{3b\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*b*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3b\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (-3*b*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx &= \left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx\\ &= -\frac{3\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.98

$$-\frac{3\cot(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{d\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-1/3),x]
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2])/(d*(b*Sec[c + d*x])^(1/3))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(1/3),x)
[Out] int(1/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)/(b*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(c + d*x))^(1/3),x)
```

```
[Out] int(1/(b/cos(c + d*x))^(1/3), x)
```

3.197 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\sec(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*b^2*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = b\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx$$
$$= \left(b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{4/3}dx$$
$$= -\frac{3\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 58, normalized size = 1.00

$$-\frac{3b\cot(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{4d(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2]*Sqrt[-
Tan[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)/(b*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{(c+dx)}}{\sqrt[3]{b\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(1/3),x)
[Out] Integral(cos(c + d*x)/(b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos{(c+d\,x)}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b/cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)/(b/cos(c + d*x))^(1/3), x)
```

3.198 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\sec(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/10*b^3*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(10/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*
d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx &= b^2\int \frac{1}{(b\sec(c+dx))^{7/3}}\,dx\\ &= \left(b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{7/3}\,dx\\ &= -\frac{3\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{10bd\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 60, normalized size = 1.03

$$-\frac{3b^2\cot(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{7d(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d*x]^2]*Sqr
t[-Tan[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Maple [**F**]
time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(dx+c)}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)^2/(b*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(1/3),x)
[Out] Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^2}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)^2/(b/cos(c + d*x))^(1/3), x)
```

3.199 $\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*sec(d*x+c))^(
1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$-\frac{3\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*
Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b^2}$$
$$= -\frac{3\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.07

$$\frac{3\cot(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2]*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*b^2*d)
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*sec(d*x+c))**(4/3),x)
[Out] Integral(sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(c+d\,x)^2\,\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)),x)
[Out] int(1/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)
```

3.200 $\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=55

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(
4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*
Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx}{b}$$
$$= -\frac{3\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.02, size = 58, normalized size = 1.05

$$-\frac{3\cot(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2])/(b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [**F**]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{\left(c + dx \right)}}{\left(b \sec{\left(c + dx \right)}\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))**(4/3),x)
[Out] Integral(sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)),x)
[Out] int(1/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)
```

3.201 $\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3b\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\sec(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*b*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c)
)^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3857, 2722}

$$-\frac{3b\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(-4/3),x]
```

```
[Out] (-3*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*
(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx &= \left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\int \left(\frac{\cos(c+dx)}{b}\right)^{4/3}\,dx\\ &= -\frac{3\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 57, normalized size = 1.02

$$-\frac{3 \cot(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{4d(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*sec(c + d*x))**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(c + d*x))^(4/3),x)
```

```
[Out] int(1/(b/cos(c + d*x))^(4/3), x)
```

3.202 $\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^2\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\sec(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/10*b^2*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(10/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{3b^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*
d*(b*Sec[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx &= b\int \frac{1}{(b\sec(c+dx))^{7/3}}\,dx\\ &= \left(b\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{7/3}dx\\ &= -\frac{3\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 58, normalized size = 1.00

$$-\frac{3b\cot(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{7d(b\sec(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Sec[c + d*x]^2]*Sqrt[
-Tan[c + d*x]^2])/(7*d*(b*Sec[c + d*x])^(7/3))
```

Maple [**F**]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (c + dx)}{(b \sec (c + dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))**(4/3),x)
[Out] Integral(cos(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos (c + d\,x)}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b/cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)/(b/cos(c + d*x))^(4/3), x)
```

3.203 $\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3b^3\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\sin(c+dx)}{13d(b\sec(c+dx))^{13/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/13*b^3*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*
x+c))^(13/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 3857, 2722}

$$-\frac{3b^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*b^3*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(1
3*d*(b*Sec[c + d*x])^(13/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = b^2\int \frac{1}{(b\sec(c+dx))^{10/3}}\,dx$$
$$= \left(b^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{10/3}dx$$
$$= -\frac{3\cos^5(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{13b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 60, normalized size = 1.03

$$-\frac{3b^2\cot(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{10d(b\sec(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Sec[c + d*x])^(4/3),x]
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[-5/3, 1/2, -2/3, Sec[c + d*x]^2]*Sqr
t[-Tan[c + d*x]^2])/(10*d*(b*Sec[c + d*x])^(10/3))
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
[Out] int(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*sec(d*x+c))**(4/3),x)
[Out] Integral(cos(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^2}{\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b/cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)^2/(b/cos(c + d*x))^(4/3), x)
```

3.204 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=81

$$\frac{3b\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1-3m);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d(1+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*b*hypergeom([1/2, -1/6-1/2*m],[5/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*(b*s
ec(d*x+c))^(1/3)*sin(d*x+c)/d/(1+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$\frac{3b\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c + d
*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}\,dx = \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}}$$
$$= \left(b\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\right)\int \cos^{-\frac{4}{3}-m}(c+dx)$$
$$= \frac{3b\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1-3m);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)\sec^m(c+dx)\sqrt[3]{b\sec(}}{d(1+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.10, size = 83, normalized size = 1.02

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{4}{3}+m\right);\frac{1}{2}\left(\frac{10}{3}+m\right);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{d\left(\frac{4}{3}+m\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(d
*(4/3 + m))
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)\left(b\sec(dx+c)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*b*sec(d*x + c)^m*sec(d*x + c), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m,x)
[Out] int((b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m, x)
```

3.205 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=82

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c+dx)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*hypergeom([1/2, 1/6-1/2*m],[7/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*(
b*sec(d*x+c))^(2/3)*sin(d*x+c)/d/(1-3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{3\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
+ d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}\,dx = \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)\,dx}{\sec^{\frac{2}{3}}(c+dx)}$$
$$= \left(\cos^{\frac{2}{3}+m}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^{2/3}\right)\int \cos^{-\frac{2}{3}-m}(c+dx$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\,\mathrm{se}}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.15, size = 83, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{3}+m\right);\frac{1}{2}\left(\frac{8}{3}+m\right);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{d\left(\frac{2}{3}+m\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (8/3 + m)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(d*
(2/3 + m))
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int (\sec^m(dx+c))\,(b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^{\frac{2}{3}}\sec^m (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3),x)
[Out] Integral((b*sec(c + d*x))**(2/3)*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos \left(c+d\,x\right)}\right)}^{2/3}\,{\left(\frac{1}{\cos \left(c+d\,x\right)}\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m,x)
[Out] int((b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m, x)
```

3.206 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*hypergeom([1/2, 1/3-1/2*m],[4/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*(
b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(2-3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
+ d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,dx = \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}}$$

$$= \left(\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\right)\int \cos^{-\frac{1}{3}-m}(c+dx)\,dx$$

$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec}}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 83, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{3}+m\right);\frac{1}{2}\left(\frac{7}{3}+m\right);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{d\left(\frac{1}{3}+m\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (7/3 + m)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(d*
(1/3 + m))
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)\left(b\sec(dx+c)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec{(c+dx)}}\,\sec^{m}{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3),x)
[Out] Integral((b*sec(c + d*x))**(1/3)*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m,x)
[Out] int((b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m, x)
```

3.207 $\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*hypergeom([1/2, 2/3-1/2*m],[5/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*s
in(d*x+c)/d/(4-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3857, 2722}

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{1}{3}-m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}}$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 83, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{3}+m\right);\frac{1}{2}\left(\frac{5}{3}+m\right);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt{-\tan^2(c+dx)}}{d\left(-\frac{1}{3}+m\right)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2, (5/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1/3 + m)*(b*Sec[c +
d*x])^(1/3))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sec^m(dx+c)}{\left(b\sec(dx+c)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{m}(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(1/3),x)
[Out] Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(c+d\,x)}\right)^m}{\left(\frac{b}{\cos(c+d\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(1/3),x)
[Out] int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(1/3), x)
```

3.208 $\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

[Out] -3*hypergeom([1/2, 5/6-1/2*m],[11/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(5-3*m)/(b*sec(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A]

time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{3\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3),x]

[Out] (-3*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}}$$
$$= \frac{\left(\cos^{\frac{1}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{2}{3}-m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}}$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 83, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-\frac{2}{3}+m\right);\frac{1}{2}\left(\frac{4}{3}+m\right);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt{-\tan^2(c+dx)}}{d\left(-\frac{2}{3}+m\right)(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(2/3),x]
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-2/3 + m)/2, (4/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-2/3 + m)*(b*Sec[c +
d*x])^(2/3))
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sec^m(dx+c)}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{m}\left(c + dx\right)}{\left(b \sec\left(c + dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(2/3),x)
[Out] Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(2/3),x)
[Out] int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(2/3), x)
```

3.209 $\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*hypergeom([1/2, 7/6-1/2*m],[13/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-2+m)*
sin(d*x+c)/b/d/(7-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{3\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{4}{3}-m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= -\frac{3\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.18, size = 83, normalized size = 0.98

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-\frac{4}{3}+m\right);\frac{1}{2}\left(\frac{2}{3}+m\right);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt{-\tan^2(c+dx)}}{d\left(-\frac{4}{3}+m\right)(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m/(b*Sec[c + d*x])^(4/3),x]
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-4/3 + m)/2, (2/3 + m)/2, Sec[c + d*x
]^2]*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-4/3 + m)*(b*Sec[c +
d*x])^(4/3))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\sec^m(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*sec(d*x + c)^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^m (c + dx)}{(b \sec (c + dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**m/(b*sec(d*x+c))**(4/3),x)

[Out] Integral(sec(c + d*x)**m/(b*sec(c + d*x))**(4/3), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^m/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\cos(c+d\,x)}\right)}^m}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(4/3),x)

[Out] int((1/cos(c + d*x))^m/(b/cos(c + d*x))^(4/3), x)

3.210 $\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=89

$$-\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-m-n);\frac{1}{2}(3-m-n);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(1-m-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -hypergeom([1/2, 1/2-1/2*m-1/2*n],[3/2-1/2*m-1/2*n],cos(d*x+c)^2)*sec(d*x+c
)^(-1+m)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(1-m-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {20, 3857, 2722}

$$-\frac{\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n+1);\frac{1}{2}(-m-n+3);\cos^2(c+dx)\right)}{d(-m-n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[Sin
[c + d*x]^2]))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{m+n}(c+dx)\,dx$$
$$= \left(\cos^{m+n}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\right)\int \cos^{-m-n}(c+dx)\,dx$$
$$= -\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-m-n);\frac{1}{2}(3-m-n);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b}{d(1-m-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.07, size = 76, normalized size = 0.85

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+n}{2};\frac{1}{2}(2+m+n);\sec^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n,x]
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2, (2 + m + n)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(m +
 n))
```

Maple [**F**]
time = 0.48, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^n \sec^m (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n,x)
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos \left(c+d\,x\right)}\right)}^n\,{\left(\frac{1}{\cos \left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^m,x)
[Out] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^m, x)
```

3.211 $\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=72

$$\frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{1+n}\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] hypergeom([1/2, -1/2-1/2*n],[1/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(1+n)*
sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$\frac{\sin(c+dx)(b\sec(c+dx))^{n+1}{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d
*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^n\,dx = \frac{\int (b\sec(c+dx))^{2+n}\,dx}{b^2}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-2-n}dx}{b^2}$$
$$= \frac{{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{1+n}\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 71, normalized size = 0.99

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sec^2(c+dx)\right)\sec(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2]*
Sec[c + d*x]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(2 + n))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^n \sec^2 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n/cos(c + d*x)^2,x)
```

```
[Out] int((b/cos(c + d*x))^n/cos(c + d*x)^2, x)
```

3.212 $\int \sec(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=61

$$\frac{{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] hypergeom([1/2, -1/2*n],[1-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)
/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {16, 3857, 2722}

$$\frac{\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])
^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^n\,dx = \frac{\int (b\sec(c+dx))^{1+n}\,dx}{b}$$
$$= \frac{\left(\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-1-n}\,dx}{b}$$
$$= \frac{{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.04, size = 65, normalized size = 1.07

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n,x]
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c + d*x]^2]*
(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1 + n))
```

Maple [**F**]
time = 0.31, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(b\sec(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n \sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n,x)
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n/cos(c + d*x),x)
[Out] int((b/cos(c + d*x))^n/cos(c + d*x), x)
```

3.213 $\int (b\sec(c+dx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-1+n}\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-1+
n)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3857, 2722}

$$-\frac{b\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c
+ d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\sec(c+dx))^n\,dx &= \left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n \int \left(\frac{\cos(c+dx)}{b}\right)^{-n} dx\\ &= -\frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 0.84

$$\frac{\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n,x]
```

```
[Out] (Cot[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*n)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n,x)
```

```
[Out] int((b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n,x)
```

```
[Out] Integral((b*sec(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n,x)
```

```
[Out] int((b/cos(c + d*x))^n, x)
```

3.214 $\int \cos(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b^2\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-2+n}\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^2*hypergeom([1/2, 1-1/2*n],[2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-2+n)
*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {16, 3857, 2722}

$$-\frac{b^2\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)(b\sec(c+dx))^n\,dx &= b\int (b\sec(c+dx))^{-1+n}\,dx \\
&= \left(b\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{1-n}dx \\
&= -\frac{\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.91

$$
\frac{b\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\sec^2(c+dx)\right)(b\sec(c+dx))^{-1+n}\sqrt{-\tan^2(c+dx)}}{d(-1+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (b*Cot[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Sec[c + d*x]^
2]*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + n))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$
\int \cos(dx+c)\,(b\sec(dx+c))^n\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n \cos(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**n,x)
[Out] Integral((b*sec(c + d*x))**n*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b/cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)*(b/cos(c + d*x))^n, x)
```

3.215 $\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b^3\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-3+n}\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^3*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-
3+n)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{b^3\sin(c+dx)(b\sec(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^n\,dx = b^2\int (b\sec(c+dx))^{-2+n}\,dx$$
$$= \left(b^2\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2-n} dx$$
$$= -\frac{\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n \sin(c+d}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 0.95

$$\frac{\cos^2(c+dx)\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n,x]
[Out] (Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Sec[c
+ d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-2 + n))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n \cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n,x)
[Out] Integral((b*sec(c + d*x))**n*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\left(\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b/cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^2*(b/cos(c + d*x))^n, x)
```

3.216 $\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=75

$$-\frac{b^4\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-4+n}\sin(c+dx)}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -b^4*hypergeom([1/2, 2-1/2*n],[3-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-4+n)
*sin(d*x+c)/d/(4-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 3857, 2722}

$$-\frac{b^4\sin(c+dx)(b\sec(c+dx))^{n-4}\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Sec[c + d*x])^n,x]
```

```
[Out] -((b^4*Hypergeometric2F1[1/2, (4 - n)/2, (6 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-4 + n)*Sin[c + d*x])/(d*(4 - n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \cos^3(c+dx)(b\sec(c+dx))^n\,dx = b^3\int (b\sec(c+dx))^{-3+n}\,dx$$
$$= \left(b^3\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{3-n} dx$$
$$= -\frac{\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-n}{2};\frac{6-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx}{d(4-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 73, normalized size = 0.97

$$\frac{\cos^3(c+dx)\cot(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(-3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]^3*Cot[c + d*x]*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2,
 Sec[c + d*x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + n))
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int \left(\cos^3(dx+c)\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n*cos(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b/cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^3*(b/cos(c + d*x))^n, x)
```

3.217 $\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=80

$$\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3-2n);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*hypergeom([1/2, -3/4-1/2*n],[1/4-1/2*n],cos(d*x+c)^2)*sec(d*x+c)^(3/2)*(b
*sec(d*x+c))^n*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{20, 3857, 2722\}$

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-3);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (2*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*x]^2]*Sec[c
+ d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{5}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{-\frac{5}{2}-n}(c+dx)\,d$$
$$= \frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3-2n);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{5}{2}+n\right);\frac{1}{2}\left(\frac{9}{2}+n\right);\sec^2(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(\frac{5}{2}+n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (5/2 + n)/2, (9/2 + n)/2, Sec[c + d*x]
^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(5/2 +
n))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{5}{2}}(dx+c)\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(b*sec(d*x+c))**n,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(5/2),x)
[Out] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(5/2), x)
```

3.218 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=80

$$\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*hypergeom([1/2, -1/4-1/2*n],[3/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3857, 2722}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
```

```
[Out] (2*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt[S
ec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{3}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{-\frac{3}{2}-n}(c+dx)\,dx$$
$$= \frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,(b\sec(c+}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.10, size = 81, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{3}{2}+n\right);\frac{1}{2}\left(\frac{7}{2}+n\right);\sec^2(c+dx)\right)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(\frac{3}{2}+n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n,x]
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (3/2 + n)/2, (7/2 + n)/2, Sec[c + d*x]
^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(3/2 +
n))
```

Maple [**F**]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{3}{2}}(dx+c)\right)(b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^n \sec^{\frac{3}{2}} (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n,x)
[Out] Integral((b*sec(c + d*x))**n*sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^n\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2),x)
[Out] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2), x)
```

3.219 $\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*hypergeom([1/2, 1/4-1/2*n],[5/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*si
n(d*x+c)/d/(1-2*n)/sec(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{-\frac{1}{2}-n}(c+dx)$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c}{d(1-2n)\sqrt{\sec(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.10, size = 81, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{2}+n\right);\frac{1}{2}\left(\frac{5}{2}+n\right);\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(\frac{1}{2}+n\right)\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n,x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (1/2 + n)/2, (5/2 + n)/2, Sec[c + d*x]
^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(1/2 + n)*Sqrt[Sec[c + d*x
]])
```

Maple [**F**]
time = 0.35, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c))^n\left(\sqrt{\sec}\,(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n \sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*sec(d*x+c)**(1/2),x)
[Out] Integral((b*sec(c + d*x))**n*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*sec(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^n\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2),x)
[Out] int((b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2), x)
```

3.220 $\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*hypergeom([1/2, 3/4-1/2*n],[7/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*si
n(d*x+c)/d/(3-2*n)/sec(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{\frac{1}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{2}+n\right);\frac{1}{2}\left(\frac{3}{2}+n\right);\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(-\frac{1}{2}+n\right)\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sqrt[Sec[c + d*x]],x]
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3/2 + n)/2, Sec[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1/2 + n)*Sec[c + d*x]^(
3/2))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c))^n}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(c+dx))^n}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(1/2), x)
```

3.221 $\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*hypergeom([1/2, 5/4-1/2*n],[9/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*si
n(d*x+c)/d/(5-2*n)/sec(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3857, 2722}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n/Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*Sec
[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{\frac{3}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.14, size = 81, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-\frac{3}{2}+n\right);\frac{1}{2}\left(\frac{1}{2}+n\right);\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(-\frac{3}{2}+n\right)\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1/2 + n)/2, Sec[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-3/2 + n)*Sec[c + d*x]^(
5/2))
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

```
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(3/2),x)

[Out] Integral((b*sec(c + d*x))**n/sec(c + d*x)**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(3/2),x)

[Out] int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(3/2), x)

3.222 $\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(7-2n)\sec^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*hypergeom([1/2, 7/4-1/2*n],[11/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*s
in(d*x+c)/d/(7-2*n)/sec(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {20, 3857, 2722}

$$-\frac{2\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\,\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[1/2, (7 - 2*n)/4, (11 - 2*n)/4, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^n*Sin[c + d*x])/(d*(7 - 2*n)*Sec[c + d*x]^(7/2)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{5}{2}+n}(c+dx)\,dx$$
$$= \left(\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{\frac{5}{2}-n}(c+dx)\,dx$$
$$= -\frac{2\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7-2n);\frac{1}{4}(11-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(7-2n)\sec^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.18, size = 81, normalized size = 1.01

$$\frac{\csc(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(-\frac{5}{2}+n\right);\frac{1}{2}\left(-\frac{1}{2}+n\right);\sec^2(c+dx)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d\left(-\frac{5}{2}+n\right)\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Csc[c + d*x]*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-1/2 + n)/2, Sec[c + d*
x]^2]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-5/2 + n)*Sec[c + d*x]^
(7/2))
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c))^n}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

```
[Out] int((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(c+dx))^n}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n/sec(d*x+c)**(5/2),x)
[Out] Integral((b*sec(c + d*x))**n/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n/sec(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c))^n/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(5/2),x)
[Out] int((b/cos(c + d*x))^n/(1/cos(c + d*x))^(5/2), x)
```

3.223 $\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx$

Optimal. Leaf size=20

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

[Out] 2/5*d*(d*sec(b*x+a))^(5/2)/b

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2702, 30}

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(d*Sec[a + b*x])^(7/2)*Sin[a + b*x],x]

[Out] (2*d*(d*Sec[a + b*x])^(5/2))/(5*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{7/2}\sin(a+bx)\,dx &= \frac{d\text{Subst}\left(\int x^{3/2}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= \frac{2d(d\sec(a+bx))^{5/2}}{5b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 20, normalized size = 1.00

$$\frac{2d(d\sec(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(7/2)*Sin[a + b*x],x]
```

```
[Out] (2*d*(d*Sec[a + b*x])^(5/2))/(5*b)
```

Maple [**A**]
time = 0.41, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2d(d\sec(bx+a))^{\frac{5}{2}}}{5b}$	17
default	$\frac{2d(d\sec(bx+a))^{\frac{5}{2}}}{5b}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(7/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*d*(d*sec(b*x+a))^(5/2)/b
```

Maxima [**A**]
time = 0.29, size = 23, normalized size = 1.15

$$\frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{7}{2}}\cos\left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/5*(d/cos(b*x + a))^(7/2)*cos(b*x + a)/b
```

Fricas [**A**]
time = 2.19, size = 28, normalized size = 1.40

$$\frac{2\,d^3\sqrt{\frac{d}{\cos\left(bx+a\right)}}}{5\,b\cos\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/5*d^3*sqrt(d/cos(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(7/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.
time = 0.45, size = 33, normalized size = 1.65

$$\frac{2\, d^4 \mathrm{sgn}(\cos{(bx+a)})}{5\, \sqrt{d \cos{(bx+a)}}\, b \cos{(bx+a)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(7/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2/5*d^4*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b*cos(b*x + a)^2)
```

Mupad [B]
time = 1.59, size = 77, normalized size = 3.85

$$\frac{8\, d^3 \sqrt{\frac{d}{\cos{(a+b\,x)}}}\, \left(4\, \cos{(2\,a+2\,b\,x)}+\cos{(4\,a+4\,b\,x)}+3\right)}{5\,b\, \left(15\, \cos{(2\,a+2\,b\,x)}+6\, \cos{(4\,a+4\,b\,x)}+\cos{(6\,a+6\,b\,x)}+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d/cos(a + b*x))^(7/2),x)
```

```
[Out] (8*d^3*(d/cos(a + b*x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))/
(5*b*(15*cos(2*a + 2*b*x) + 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

3.224 $\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx$

Optimal. Leaf size=20

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*d*(d*sec(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2702, 30}

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{5/2}\sin(a+bx)\,dx &= \frac{d\text{Subst}\left(\int \sqrt{x}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= \frac{2d(d\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 1.00

$$\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Maple [A]
time = 0.40, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$\frac{2d(d\sec(bx+a))^{\frac{3}{2}}}{3b}$	17
default	$\frac{2d(d\sec(bx+a))^{\frac{3}{2}}}{3b}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(5/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d*(d*sec(b*x+a))^(3/2)/b
```

Maxima [A]
time = 0.32, size = 23, normalized size = 1.15

$$\frac{2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{5}{2}}\cos(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/3*(d/cos(b*x + a))^(5/2)*cos(b*x + a)/b
```

Fricas [A]
time = 2.07, size = 28, normalized size = 1.40

$$\frac{2\,d^2\sqrt{\frac{d}{\cos(bx+a)}}}{3\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 2/3*d^2*sqrt(d/cos(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.
time = 0.43, size = 33, normalized size = 1.65

$$\frac{2\,d^3\mathrm{sgn}(\cos{(bx+a)})}{3\,\sqrt{d\cos{(bx+a)}}\,b\cos{(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2/3*d^3*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b*cos(b*x + a))
```

Mupad [B]
time = 0.25, size = 39, normalized size = 1.95

$$\frac{4\,d^2\,\cos{(a+b\,x)}\,\sqrt{\frac{d}{\cos{(a+b\,x)}}}}{3\,b\,\left(\cos{(2\,a+2\,b\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d/cos(a + b*x))^(5/2),x)
```

```
[Out] (4*d^2*cos(a + b*x)*(d/cos(a + b*x))^(1/2))/(3*b*(cos(2*a + 2*b*x) + 1))
```

3.225 $\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal. Leaf size=18

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

[Out] 2*d*(d*sec(b*x+a))^(1/2)/b

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2702, 30}

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

[In] Int[(d*Sec[a + b*x])^(3/2)*Sin[a + b*x],x]

[Out] (2*d*Sqrt[d*Sec[a + b*x]])/b

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{3/2}\sin(a+bx)\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= \frac{2d\sqrt{d\sec(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$\frac{2d\sqrt{d\sec(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*d*Sqrt[d*Sec[a + b*x]])/b
```

Maple [A]
time = 0.40, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\frac{2d\sqrt{d\sec\left(bx+a\right)}}{b}$	17
default	$\frac{2d\sqrt{d\sec\left(bx+a\right)}}{b}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(3/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(d*sec(b*x+a))^(1/2)/b
```

Maxima [A]
time = 0.30, size = 23, normalized size = 1.28

$$\frac{2\left(\frac{d}{\cos\left(bx+a\right)}\right)^{\frac{3}{2}}\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*(d/cos(b*x + a))^(3/2)*cos(b*x + a)/b
```

Fricas [A]
time = 3.35, size = 18, normalized size = 1.00

$$\frac{2\,d\sqrt{\frac{d}{\cos\left(bx+a\right)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*d*sqrt(d/cos(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\sec\left(a+bx\right)\right)^{\frac{3}{2}}\sin\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(3/2)*sin(b*x+a),x)
```

```
[Out] Integral((d*sec(a + b*x))**(3/2)*sin(a + b*x), x)
```

Giac [A]
time = 0.43, size = 25, normalized size = 1.39

$$\frac{2\,d^2\mathrm{sgn}(\cos{(bx+a)})}{\sqrt{d\cos{(bx+a)}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2*d^2*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a))*b)
```

Mupad [B]
time = 0.10, size = 18, normalized size = 1.00

$$\frac{2\,d\,\sqrt{\frac{d}{\cos{(a+b\,x)}}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d/cos(a + b*x))^(3/2),x)
```

```
[Out] (2*d*(d/cos(a + b*x))^(1/2))/b
```

3.226 $\int \sqrt{d \sec(a+bx)} \sin(a+bx) \, dx$

Optimal. Leaf size=18

$$-\frac{2d}{b\sqrt{d \sec(a+bx)}}$$

```
[Out] -2*d/b/(d*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2702, 30}

$$-\frac{2d}{b\sqrt{d \sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d \sec(a+bx)} \sin(a+bx) \, dx &= \frac{d\text{Subst}\left(\int \frac{1}{x^{3/2}} \, dx, x, d \sec(a+bx)\right)}{b}\\ &= -\frac{2d}{b\sqrt{d \sec(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$-\frac{2d}{b\sqrt{d \sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Sec[a + b*x]])
```

Maple [A]
time = 0.46, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$-\frac{2d}{b\sqrt{d\sec\left(bx+a\right)}}$	17
default	$-\frac{2d}{b\sqrt{d\sec\left(bx+a\right)}}$	17
risch	$-\frac{2\sqrt{2}\,\sqrt{\frac{d\,e^{i(bx+a)}}{e^{2i(bx+a)}+1}}\,\cos(bx+a)}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -2*d/b/(d*sec(b*x+a))^(1/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.28

$$-\frac{2\sqrt{\frac{d}{\cos\left(bx+a\right)}}\,\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b
```

Fricas [A]
time = 2.22, size = 23, normalized size = 1.28

$$-\frac{2\sqrt{\frac{d}{\cos\left(bx+a\right)}}\,\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d/cos(b*x + a))*cos(b*x + a)/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \sec (a + bx)} \sin (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(1/2)*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(d*sec(a + b*x))*sin(a + b*x), x)
```

Giac [**A**]
time = 0.42, size = 22, normalized size = 1.22

$$-\frac{2 \sqrt{d \cos (bx + a)} \operatorname{sgn}(\cos (bx + a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -2*sqrt(d*cos(b*x + a))*sgn(cos(b*x + a))/b
```

Mupad [**B**]
time = 0.23, size = 23, normalized size = 1.28

$$-\frac{2 \cos (a + b\, x) \sqrt{\frac{d}{\cos (a + b\, x)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d/cos(a + b*x))^(1/2),x)
```

```
[Out] -(2*cos(a + b*x)*(d/cos(a + b*x))^(1/2))/b
```

3.227 $\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

```
[Out] -2/3*d/b/(d*sec(b*x+a))^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2702, 30}

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] (-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sqrt{d\sec(a+bx)}}\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\sec(a+bx)\right)}{b}\\ &= -\frac{2d}{3b(d\sec(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 20, normalized size = 1.00

$$-\frac{2d}{3b(d\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Sec[a + b*x]],x]
[Out] (-2*d)/(3*b*(d*Sec[a + b*x])^(3/2))
```

Maple [A]
time = 0.44, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\frac{2d}{3b(d\sec(bx+a))^{\frac{3}{2}}}$	17
default	$-\frac{2d}{3b(d\sec(bx+a))^{\frac{3}{2}}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*d/b/(d*sec(b*x+a))^(3/2)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 1.15

$$-\frac{2\cos\left(bx+a\right)}{3\,b\sqrt{\frac{d}{\cos\left(bx+a\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/3*cos(b*x + a)/(b*sqrt(d/cos(b*x + a)))
```

Fricas [A]
time = 2.40, size = 28, normalized size = 1.40

$$-\frac{2\sqrt{\frac{d}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)^{2}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(d/cos(b*x + a))*cos(b*x + a)^2/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(a+bx\right)}{\sqrt{d\sec\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)/sqrt(d*sec(a + b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.
time = 0.44, size = 33, normalized size = 1.65

$$-\frac{2\sqrt{d\cos\left(bx+a\right)}\cos\left(bx+a\right)}{3\,bd\mathrm{sgn}\left(\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/(b*d*sgn(cos(b*x + a)))
```

Mupad [B]
time = 0.24, size = 28, normalized size = 1.40

$$-\frac{2\cos\left(a+b\,x\right)^2\,\sqrt{\frac{d}{\cos\left(a+b\,x\right)}}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d/cos(a + b*x))^(1/2),x)
```

```
[Out] -(2*cos(a + b*x)^2*(d/cos(a + b*x))^(1/2))/(3*b*d)
```

3.228 $\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{2d^3}{b\sqrt{d\sec(a+bx)}}+\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*d*(d*sec(b*x+a))^(3/2)/b+2*d^3/b/(d*sec(b*x+a))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2702, 14}

$$\frac{2d^3}{b\sqrt{d\sec(a+bx)}}+\frac{2d(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (2*d^3)/(b*Sqrt[d*Sec[a + b*x]]) + (2*d*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{5/2}\sin^3(a+bx)\,dx&=\frac{d^3\text{Subst}\left(\int\frac{-1+\frac{x^2}{d^2}}{x^{3/2}}\,dx,x,d\sec(a+bx)\right)}{b}\\&=\frac{d^3\text{Subst}\left(\int\left(-\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{d^2}\right)\,dx,x,d\sec(a+bx)\right)}{b}\\&=\frac{2d^3}{b\sqrt{d\sec(a+bx)}}+\frac{2d(d\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 32, normalized size = 0.78

$$\frac{d(5+3\cos(2(a+bx)))(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*Sin[a + b*x]^3,x]
```

```
[Out] (d*(5 + 3*Cos[2*(a + b*x)])*(d*Sec[a + b*x])^(3/2))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(35) = 70$.
time = 88.99, size = 357, normalized size = 8.71

method	result
default	$-\frac{(-1+\cos(bx+a))\left(12\left(\cos^3(bx+a)\right)\sqrt{-\frac{\cos(bx+a)}{(\cos(bx+a)+1)^2}}-3\left(\cos^2(bx+a)\right)\ln\left(-\frac{2\left(\cos^2(bx+a)\right)\sqrt{-\frac{\cos(bx+a)}{(\cos(bx+a)+1)^2}}-\left(\cos^2(bx+\right.}{\sin(bx+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*(-1+cos(b*x+a))*(12*cos(b*x+a)^3*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)-3*cos(b*x+a)^2*ln(-(2*cos(b*x+a)^2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-c
os(b*x+a)^2+2*cos(b*x+a)-2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-1)/sin(b*x+
a)^2)+3*cos(b*x+a)^2*ln(-2*(2*cos(b*x+a)^2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(
1/2)-cos(b*x+a)^2+2*cos(b*x+a)-2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-1)/si
n(b*x+a)^2)+12*cos(b*x+a)^2*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+4*cos(b*x+
a)*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+4*(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2))*cos(b*x+a)*(d/cos(b*x+a))^(5/2)/(-cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/s
in(b*x+a)^2
```

Maxima [A]
time = 0.29, size = 36, normalized size = 0.88

$$\frac{2\left(\frac{3\,d^2}{\sqrt{\frac{d}{\cos\left(bx+a\right)}}}+\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}\right)d}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 2/3*(3*d^2/sqrt(d/cos(b*x + a)) + (d/cos(b*x + a))^(3/2))*d/b
```

Fricas [A]
time = 2.23, size = 42, normalized size = 1.02

$$\frac{2\left(3\,d^2\cos\left(bx+a\right)^2+d^2\right)\sqrt{\frac{d}{\cos\left(bx+a\right)}}}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 2/3*(3*d^2*cos(b*x + a)^2 + d^2)*sqrt(d/cos(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(5/2)*sin(b*x+a)**3,x)
[Out] Timed out
```

Giac [A]
time = 0.44, size = 49, normalized size = 1.20

$$\frac{2\left(3\,\sqrt{d\cos\left(bx+a\right)}\,d+\frac{d^2}{\sqrt{d\cos\left(bx+a\right)}\,\cos(bx+a)}\right)d\mathrm{sgn}(\cos\left(bx+a\right))}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*sin(b*x+a)^3,x, algorithm="giac")
[Out] 2/3*(3*sqrt(d*cos(b*x + a))*d + d^2/(sqrt(d*cos(b*x + a))*cos(b*x + a)))*d*
sgn(cos(b*x + a))/b
```

Mupad [B]
time = 0.59, size = 50, normalized size = 1.22

$$\frac{d^2\,\sqrt{\frac{d}{\cos\left(a+b\,x\right)}}\,\left(\frac{13\,\cos(a+b\,x)}{3}+\cos\left(3\,a+3\,b\,x\right)\right)}{b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d/cos(a + b*x))^(5/2),x)
[Out] (d^2*(d/cos(a + b*x))^(1/2)*((13*cos(a + b*x))/3 + cos(3*a + 3*b*x)))/(b*(c
os(2*a + 2*b*x) + 1))
```

3.229 $\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}+\frac{2d(d\sec(a+bx))^{7/2}}{7b}$$

```
[Out] -2/3*d^3*(d*sec(b*x+a))^(3/2)/b+2/7*d*(d*sec(b*x+a))^(7/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2702, 14}

$$\frac{2d(d\sec(a+bx))^{7/2}}{7b}-\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(9/2)*Sin[a + b*x]^3,x]
```

```
[Out] (-2*d^3*(d*Sec[a + b*x])^(3/2))/(3*b) + (2*d*(d*Sec[a + b*x])^(7/2))/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (d\sec(a+bx))^{9/2}\sin^3(a+bx)\,dx &= \frac{d^3\text{Subst}\left(\int \sqrt{x}\left(-1+\frac{x^2}{d^2}\right)\,dx,x,d\sec(a+bx)\right)}{b}\\ &= \frac{d^3\text{Subst}\left(\int \left(-\sqrt{x}+\frac{x^{5/2}}{d^2}\right)\,dx,x,d\sec(a+bx)\right)}{b}\\ &= -\frac{2d^3(d\sec(a+bx))^{3/2}}{3b}+\frac{2d(d\sec(a+bx))^{7/2}}{7b}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 42, normalized size = 0.98

$$-\frac{d^4(1+7\cos(2(a+bx)))\sec^3(a+bx)\sqrt{d\sec(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(9/2)*Sin[a + b*x]^3,x]
[Out] -1/21*(d^4*(1 + 7*Cos[2*(a + b*x)])*Sec[a + b*x]^3*Sqrt[d*Sec[a + b*x]])/b
```

Maple [A]
time = 82.17, size = 36, normalized size = 0.84

method	result	size
default	$-\frac{2\left(7\left(\cos^2(bx+a)\right)-3\right)\cos(bx+a)\left(\frac{d}{\cos(bx+a)}\right)^{\frac{9}{2}}}{21b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -2/21/b*(7*cos(b*x+a)^2-3)*cos(b*x+a)*(d/cos(b*x+a))^(9/2)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.88

$$-\frac{2\left(7\,d^2\left(\frac{d}{\cos(bx+a)}\right)^{\frac{3}{2}}-3\left(\frac{d}{\cos(bx+a)}\right)^{\frac{7}{2}}\right)d}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -2/21*(7*d^2*(d/cos(b*x + a))^(3/2) - 3*(d/cos(b*x + a))^(7/2))*d/b
```

Fricas [A]
time = 2.10, size = 44, normalized size = 1.02

$$-\frac{2\left(7\,d^4\cos\left(bx+a\right)^2-3\,d^4\right)\sqrt{\frac{d}{\cos\left(bx+a\right)}}}{21\,b\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -2/21*(7*d^4*cos(b*x + a)^2 - 3*d^4)*sqrt(d/cos(b*x + a))/(b*cos(b*x + a)^3
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(9/2)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 49, normalized size = 1.14

$$-\frac{2\left(7\,d^{5}\cos\left(bx+a\right)^{2}-3\,d^{5}\right)\mathrm{sgn}(\cos\left(bx+a\right))}{21\,\sqrt{d\cos\left(bx+a\right)}\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(9/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/21*(7*d^5*cos(b*x + a)^2 - 3*d^5)*sgn(cos(b*x + a))/(sqrt(d*cos(b*x + a)
)*b*cos(b*x + a)^3)
```

Mupad [**B**]
time = 4.34, size = 95, normalized size = 2.21

$$-\frac{4\,d^{4}\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{d}{\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}}}\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+7\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+7\right)}{21\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d/cos(a + b*x))^(9/2),x)
```

```
[Out] -(4*d^4*exp(a*1i + b*x*1i)*(d/(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/
2))^(1/2)*(2*exp(a*2i + b*x*2i) + 7*exp(a*4i + b*x*4i) + 7))/(21*b*(exp(a*2
i + b*x*2i) + 1)^3)
```

3.230 $\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=128

$$-\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}+\frac{4d^4\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin}}{7b}$$

[Out] -4/7*c*d^3*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(1/2)-2/7*c*d*(d*csc(b*x+a
))^(7/2)/b/(c*sec(b*x+a))^(1/2)-4/7*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))^(1/2)*(c*sec
(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b

Rubi [A]
time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2705, 2710, 2653, 2720}

$$\frac{4d^4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{7b}-\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[(d*Csc[a + b*x])^(9/2)*Sqrt[c*Sec[a + b*x]],x]

[Out] (-4*c*d^3*(d*Csc[a + b*x])^(3/2))/(7*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Cs
c[a + b*x])^(7/2))/(7*b*Sqrt[c*Sec[a + b*x]]) + (4*d^4*Sqrt[d*Csc[a + b*x]]
*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(7*b)

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{9/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}} + \frac{1}{7}(6d^2)\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+b$$

$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}} - \frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}} + \frac{1}{7}(4d^4)\int \sqrt{d\,c$$

$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}} - \frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}} + \frac{1}{7}\Big(4d^4\sqrt{c\cos(a$$

$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}} - \frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}} + \frac{1}{7}\Big(4d^4\sqrt{d\csc(a$$

$$= -\frac{4cd^3(d\csc(a+bx))^{3/2}}{7b\sqrt{c\sec(a+bx)}} - \frac{2cd(d\csc(a+bx))^{7/2}}{7b\sqrt{c\sec(a+bx)}} + \frac{4d^4\sqrt{d\csc(a+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 17.59, size = 122, normalized size = 0.95

$$\frac{2d^4\cos(2(a+bx))\cot(a+bx)\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\left((-2+\cos(2(a+bx)))\csc^4(a+bx)-2(-\cot^2(a+bx))^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\sec^2(a+bx)\right)}{7b\left(-2+\csc^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*d^4*Cos[2*(a + b*x)]*Cot[a + b*x]*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*
x]]*((-2 + Cos[2*(a + b*x)])*Csc[a + b*x]^4 - 2*(-Cot[a + b*x]^2)^(3/4)*Hyp
ergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2]*Sec[a + b*x]^2))/(7*b*(-2 + C
sc[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(133) = 266.
time = 43.71, size = 542, normalized size = 4.23

method	result
default	$\frac{\left(-4\left(\cos^3(bx+a)\right)\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/b*(-4*cos(b*x+a)^3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))-4*cos(b*x+a)^2*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
4*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*(-(c
os(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)+2*2^(1/2)*cos(b*x+a)^3-
3*2^(1/2)*cos(b*x+a))*(d/sin(b*x+a))^(9/2)*(c/cos(b*x+a))^(1/2)*sin(b*x+a)*
2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.71, size = 175, normalized size = 1.37

$$-\frac{2\left(\left(i\,d^4\cos\left(bx+a\right)^2-i\,d^4\right)\sqrt{-4i\,cd}\ \mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)\sin\left(bx+a\right)+\left(-i\,d^4\cos\left(bx+a\right)^2+i\,d^4\right)\sqrt{4i\,cd}\ \mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)\sin\left(bx+a\right)+\left(2\,d^4\cos\left(bx+a\right)^3-3\,d^4\cos\left(bx+a\right)\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}\right)}{7\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/7*((I*d^4*cos(b*x + a)^2 - I*d^4)*sqrt(-4*I*c*d)*ellipticF(cos(b*x + a)
+ I*sin(b*x + a), -1)*sin(b*x + a) + (-I*d^4*cos(b*x + a)^2 + I*d^4)*sqrt(4
*I*c*d)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1)*sin(b*x + a) + (2*d^4*
```

```
cos(b*x + a)^3 - 3*d^4*cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x +
a)))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(9/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{c}{\cos(a+b\,x)}}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(9/2),x)
[Out] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(9/2), x)
```

3.231 $\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=69

$$-\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

```
[Out] -2/5*c*d*(d*csc(b*x+a))^(5/2)/b/(c*sec(b*x+a))^(1/2)-8/5*c*d^3*(d*csc(b*x+a
))^(1/2)/b/(c*sec(b*x+a))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2705, 2699}

$$-\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-8*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*Sqrt[c*Sec[a + b*x]]) - (2*c*d*(d*Csc[
a + b*x])^(5/2))/(5*b*Sqrt[c*Sec[a + b*x]])
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx &= -\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}+\frac{1}{5}\left(4d^2\right)\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+\\ &= -\frac{8cd^3\sqrt{d\csc(a+bx)}}{5b\sqrt{c\sec(a+bx)}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b\sqrt{c\sec(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 56, normalized size = 0.81

$$-\frac{2d^3\sqrt{d\csc(a+bx)}\,(4\cos(a+bx)+\cot(a+bx)\csc(a+bx))\sqrt{c\sec(a+bx)}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*Sqrt[c*Sec[a + b*x]],x]
[Out] (-2*d^3*Sqrt[d*Csc[a + b*x]]*(4*Cos[a + b*x] + Cot[a + b*x]*Csc[a + b*x])*S
qrt[c*Sec[a + b*x]])/(5*b)
```

Maple [A]
time = 29.17, size = 54, normalized size = 0.78

method	result	size
default	$\frac{2\left(4\left(\cos^2(bx+a)\right)-5\right)\cos(bx+a)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{7}{2}}\sqrt{\frac{c}{\cos(bx+a)}}\,\sin(bx+a)}{5b}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5/b*(4*cos(b*x+a)^2-5)*cos(b*x+a)*(d/sin(b*x+a))^(7/2)*(c/cos(b*x+a))^(1/
2)*sin(b*x+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(7/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [A]
time = 2.61, size = 67, normalized size = 0.97

$$-\frac{2\left(4\,d^3\cos\left(bx+a\right)^3-5\,d^3\cos\left(bx+a\right)\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{5\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(4*d^3*cos(b*x + a)^3 - 5*d^3*cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(
d/sin(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(7/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [**B**]
time = 1.34, size = 85, normalized size = 1.23

$$-\frac{4\,d^3\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(3\,\cos\left(a+b\,x\right)-4\,\cos\left(3\,a+3\,b\,x\right)+\cos\left(5\,a+5\,b\,x\right)\right)}{5\,b\,\left(\cos\left(4\,a+4\,b\,x\right)-4\,\cos\left(2\,a+2\,b\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(7/2),x)
[Out] -(4*d^3*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(3*cos(a + b*x) - 4*c
os(3*a + 3*b*x) + cos(5*a + 5*b*x)))/(5*b*(cos(4*a + 4*b*x) - 4*cos(2*a + 2
*b*x) + 3))
```

3.232 $\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=93

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}+\frac{2d^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3b}$$

```
[Out] -2/3*c*d*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(1/2)-2/3*d^2*(sin(a+1/4*Pi+
b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc
(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2705, 2710, 2653, 2720}

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}-\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*d^2*Sqrt[d*
Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*
a + 2*b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
```

```
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{1}{3}(2d^2)\int \sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+$$

$$= -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{1}{3}\Big(2d^2\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+bx)}$$

$$= -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{1}{3}\Big(2d^2\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}$$

$$= -\frac{2cd(d\csc(a+bx))^{3/2}}{3b\sqrt{c\sec(a+bx)}} + \frac{2d^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.90, size = 109, normalized size = 1.17

$$-\frac{d(\cos(a+bx)+\cos(3(a+bx)))(d\csc(a+bx))^{3/2}\left(\cot^2(a+bx)+(-\cot^2(a+bx))^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)\sec^2(a+bx)\sqrt{c\sec(a+bx)}}{3b\,(-2+\csc^2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] -1/3*(d*(Cos[a + b*x] + Cos[3*(a + b*x)])*(d*Csc[a + b*x])^(3/2)*(Cot[a + b
*x]^2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*
x]^2])*Sec[a + b*x]^2*Sqrt[c*Sec[a + b*x]])/(b*(-2 + Csc[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(104) = 208$.
time = 33.08, size = 284, normalized size = 3.05

method	result
default	$-\frac{\left(-2\cos(bx+a)\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(-2*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))-2*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*
x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)+2^(1/2)*cos(b*
x+a))*(d/sin(b*x+a))^(5/2)*(c/cos(b*x+a))^(1/2)*sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.84, size = 117, normalized size = 1.26

$$\frac{-i\sqrt{-4i\,cd}\,d^2\mathrm{ellipticF}(\cos(bx+a)+i\,\sin(bx+a)\,,-1)\sin(bx+a)+i\sqrt{4i\,cd}\,d^2\mathrm{ellipticF}(\cos(bx+a)-i\,\sin(bx+a)\,,-1)\sin(bx+a)-2\,d^2\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)}{3\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(-4*I*c*d)*d^2*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1)*sin
(b*x + a) + I*sqrt(4*I*c*d)*d^2*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1
)*sin(b*x + a) - 2*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x +
a))/(b*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2),x)
[Out] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2), x)
```

3.233 $\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=31

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

```
[Out] -2*c*d*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2699}

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

Mathematica [A]
time = 0.07, size = 31, normalized size = 1.00

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*Sqrt[c*Sec[a + b*x]])
```

Maple [A]
time = 31.08, size = 42, normalized size = 1.35

method	result	size
default	$-\frac{2\sin(bx+a)\cos(bx+a)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{3}{2}}\sqrt{\frac{c}{\cos(bx+a)}}}{b}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/b*sin(b*x+a)*cos(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/cos(b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)
```

Fricas [A]
time = 1.78, size = 36, normalized size = 1.16

$$-\frac{2\,d\sqrt{\frac{c}{\cos(bx+a)}}\,\sqrt{\frac{d}{\sin(bx+a)}}\,\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2*d*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a)), x)
```

Mupad [B]
time = 0.37, size = 36, normalized size = 1.16

$$-\frac{2\,d\,\cos\left(a+b\,x\right)\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(3/2),x)
[Out] -(2*d*cos(a + b*x)*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2))/b
```

3.234 $\int \sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}{b}$$

[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x), 2^(1/2))*(d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2710, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (Sqrt[d*Csc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqr
t[Sin[2*a + 2*b*x]])/b
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx = \left(\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}\right.$$
$$= \left(\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+}}$$
$$= \frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.70, size = 68, normalized size = 1.28

$$\frac{(-\cot^2(a+bx))^{7/4}\sqrt{d\csc(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\sqrt{c\sec(a+bx)}\tan^3(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] ((-Cot[a + b*x]^2)^(7/4)*Sqrt[d*Csc[a + b*x]]*Hypergeometric2F1[1/2, 3/4, 3
/2, Csc[a + b*x]^2]*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^3)/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(73) = 146.
time = 31.77, size = 155, normalized size = 2.92

method	result
default	$-\frac{\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\,(\sin^2(bx+a))\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{Ell}}{b(-1+\cos(bx+a))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*(c/cos(b*x+a))^(1/2)*(d/sin(b*x+a))^(1/2)*sin(b*x+a)^2*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))/(-1+cos(b*x+a))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.61, size = 56, normalized size = 1.06

$$\frac{-i\sqrt{-4i\,cd}\ \text{ellipticF}(\cos{(bx+a)}+i\sin{(bx+a)},-1)+i\sqrt{4i\,cd}\ \text{ellipticF}(\cos{(bx+a)}-i\sin{(bx+a)},-1)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*sqrt(-4*I*c*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) + I*sqr
t(4*I*c*d)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c\sec{(a+bx)}}\,\sqrt{d\csc{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x))*sqrt(d*csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{\frac{c}{\cos{(a+b\,x)}}}\,\sqrt{\frac{d}{\sin{(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2),x)
```

```
[Out] int((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2), x)
```

3.235 $\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal. Leaf size=270

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\cdots$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/b*2^(1/2)/(d*c
sc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c
*sec(b*x+a))^(1/2)/b*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/4*ln(1
-2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(c*sec(b*x+a))^(1/2)/b*2^(1/2)/(d*csc
(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-1/4*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a
))*(c*sec(b*x+a))^(1/2)/b*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2709, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,b\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]]) + (Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a
+ b*x]])/(2*Sqrt[2]*b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + S
qrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*Sqrt[2]*
b*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

`IntegerQ[n]`

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx &= \frac{\sqrt{c\sec(a+bx)}\int\sqrt{\tan(a+bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= \frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int\frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(a+bx)\right)}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= \frac{\left(2\sqrt{c\sec(a+bx)}\right)\text{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(a+bx)}\right)}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(a+bx)}\right)}{b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int\right.}{b\sqrt{d\csc(a+b}}\\
&= \frac{\sqrt{c\sec(a+bx)}\,\text{Subst}\left(\int\frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt{\tan(a+bx)}\right)}{2b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\,\text{Sub}}{2b\sqrt{d}}\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a+bx)}}{2\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}-\frac{\log\left(1+\sqrt{2}\sqrt{\tan}\right.}{2\sqrt{2}b}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}b\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(a+b}\right.}{\sqrt{2}b\sqrt{d\csc(a+bx)}}
\end{aligned}$$

Mathematica [A]
time = 1.26, size = 121, normalized size = 0.45

$$-\frac{\left(\text{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\right)\cot(a+bx)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,b\sqrt[4]{\cot^2(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] -(((ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))] +
ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])])*Cot[a
 + b*x]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*(Cot[a + b*x]^2)^(1/4)*Sqrt[d*Csc[
a + b*x]]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.04, size = 275, normalized size = 1.02

method	result
default	$-\frac{\sqrt{\frac{c}{\cos(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a}{\sin(b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(c/cos(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2
*2^(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2+1/2*I,1/2*2^(1/2)))*sin(b*x+a)/(d/sin(b*x+a))^(1/2)/(-1+cos(b*x+a))*2^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x))/sqrt(d*csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/sqrt(d*csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}}{\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(1/2),x)
```

```
[Out] int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(1/2), x)
```

3.236 $\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{c}{bd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}+\frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}{2bd^2}$$

```
[Out] -c/b/d/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)-1/2*(sin(a+1/4*Pi+b*x)^2)^
(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))
^(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2707, 2710, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{2bd^2}-\frac{c}{bd\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] -(c/(b*d*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])) + (Sqrt[d*Csc[a + b*x]
]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
/(2*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{3/2}}\,dx = -\frac{c}{bd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\int \sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,dx}{2d^2}$$

$$= -\frac{c}{bd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(\right.}{}$$

$$= -\frac{c}{bd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2\right.}{2d^2}$$

$$= -\frac{c}{bd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec}}{2bd^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.73, size = 80, normalized size = 0.86

$$-\frac{\left(1+\cos(2(a+bx))+\left(-\cot^2(a+bx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)(c\sec(a+bx))^{3/2}}{2bcd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] -1/2*((1 + Cos[2*(a + b*x)] + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2
, 3/4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Csc[a +
b*x]])
```

Maple [A]
time = 31.64, size = 188, normalized size = 2.02

method	result

default	$-\frac{\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a}{\sin(bx+a)}}\right.\right.}{2b(-1+\cos(bx+a))\left(\frac{d}{\sin(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos
(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)+cos(b*x+a)^
2*2^(1/2)-2^(1/2)*cos(b*x+a))*(c/cos(b*x+a))^(1/2)/(-1+cos(b*x+a))/(d/sin(b
*x+a))^(3/2)/sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(d^2*csc(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\sec{(a+bx)}}}{(d\csc{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(c*sec(a + b*x))/(d*csc(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{c}{\cos(a+b\,x)}}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(3/2),x)
[Out] int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(3/2), x)
```

3.237 $\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=322

$$-\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}\,bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\frac{3\text{ArcTan}\Big(1}{4\sqrt{2}\,b}$$

```
[Out] -1/2*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2)+3/8*arctan(-1+2^(1/2)*
tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/b/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/t
an(b*x+a)^(1/2)+3/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)
/b/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+3/16*ln(1-2^(1/2)*tan(
b*x+a)^(1/2)+tan(b*x+a))*(c*sec(b*x+a))^(1/2)/b/d^2*2^(1/2)/(d*csc(b*x+a))^
(1/2)/tan(b*x+a)^(1/2)-3/16*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(c*se
c(b*x+a))^(1/2)/b/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2707, 2709, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{3\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{2bd\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] -1/2*c/(b*d*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc
[a + b*x]]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*
Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*
x]]) + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a +
 b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log[
1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqr
t[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{c\sec(a+bx)}}{(d\csc(a+bx))^{5/2}}\,dx = -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}+\frac{3\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{4d^2}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}+\frac{\left(3\sqrt{c\sec(a+bx)}\right)\int \sqrt{\tan(a+bx)}}{4d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}+\frac{\left(3\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx\right.}{4bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a}}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}+\frac{\left(3\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,\right.}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(}}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}-\frac{\left(3\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,\right.}{4bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(}}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}+\frac{\left(3\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x}\right.}{8bd^2\sqrt{d\csc(a+bx)}\sqrt{}}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}+\frac{3\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(\right.}{8\sqrt{2}\,bd^2\sqrt{d\csc(a+bx)}\sqrt{}}$$

$$= -\frac{c}{2bd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c}}{4\sqrt{2}\,bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a}}$$

Mathematica [A]

time = 1.15, size = 157, normalized size = 0.49

$$-\frac{\left(4\cos^2(a+bx)+3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}+3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}\right)\sqrt{c\sec(a+bx)}\,\tan(a+bx)}{8bd^2\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sec[a + b*x]]/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] -1/8*((4*Cos[a + b*x]^2 + 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqr
t[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) + 3*Sqrt[2]*ArcTanh[(S
qrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)
^(3/4))*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x])/(b*d^2*Sqrt[d*Csc[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 34.27, size = 514, normalized size = 1.60

method	result
default	$-\frac{\left(3i\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{1-\cos(bx+a)+}{\sin(bx-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(3*I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)-3*I*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)-3*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)-3*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)+2*cos(b*x+a)^2*2^(1/2)-2*2^(1/2)*cos(b*x+a))*(c/cos(b*x+a))^(1/2
)/(-1+cos(b*x+a))/(d/sin(b*x+a))^(5/2)/sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(1/2)/(d*csc(b*x+a))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(1/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sec(b*x + a))/(d*csc(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{c}{\cos(a+b\,x)}}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(5/2),x)
[Out] int((c/cos(a + b*x))^(1/2)/(d/sin(a + b*x))^(5/2), x)
```

3.238 $\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}-\frac{16cd^3(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{21b}-\frac{2cd(d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}}{7b}$$

```
[Out] -16/21*c*d^3*(d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2)/b-2/7*c*d*(d*csc(b*x
+a))^(7/2)*(c*sec(b*x+a))^(1/2)/b+64/21*c*d^5*(c*sec(b*x+a))^(1/2)/b/(d*csc
(b*x+a))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2705, 2699}

$$\frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}-\frac{16cd^3\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{3/2}}{21b}-\frac{2cd\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (64*c*d^5*Sqrt[c*Sec[a + b*x]])/(21*b*Sqrt[d*Csc[a + b*x]]) - (16*c*d^3*(d*
Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(21*b) - (2*c*d*(d*Csc[a + b*x])^
(7/2)*Sqrt[c*Sec[a + b*x]])/(7*b)
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rubi steps

$$\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}}{7b}+\frac{1}{7}\left(8d^2\right)\int (d\csc(a+b$$
$$= -\frac{16cd^3(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{21b}-\frac{2cd(d\csc(a+bx))^{7/}}{7b}$$
$$= \frac{64cd^5\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}-\frac{16cd^3(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{21b}$$

Mathematica [A]
time = 0.31, size = 57, normalized size = 0.55

$$-\frac{2cd^5\left(-32+8\csc^2(a+bx)+3\csc^4(a+bx)\right)\sqrt{c\sec(a+bx)}}{21b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(3/2),x]
[Out] (-2*c*d^5*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[c*Sec[a + b*x]])
/(21*b*Sqrt[d*Csc[a + b*x]])
```

Maple [A]
time = 60.66, size = 64, normalized size = 0.62

method	result	size
default	$\frac{2\left(32\left(\cos^4(bx+a)\right)-56\left(\cos^2(bx+a)\right)+21\right)\cos(bx+a)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{9}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)}{21b}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/21/b*(32*cos(b*x+a)^4-56*cos(b*x+a)^2+21)*cos(b*x+a)*(d/sin(b*x+a))^(9/2)
*(c/cos(b*x+a))^(3/2)*sin(b*x+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [**A**]
time = 3.39, size = 85, normalized size = 0.82

$$-\frac{2\left(32\,cd^4\cos\left(bx+a\right)^4-56\,cd^4\cos\left(bx+a\right)^2+21\,cd^4\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{21\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*(32*c*d^4*cos(b*x + a)^4 - 56*c*d^4*cos(b*x + a)^2 + 21*c*d^4)*sqrt(c
/cos(b*x + a))*sqrt(d/sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [**B**]
time = 2.17, size = 110, normalized size = 1.06

$$-\frac{16\,c\,d^4\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(41\,\sin\left(a+b\,x\right)-29\,\sin\left(3\,a+3\,b\,x\right)+12\,\sin\left(5\,a+5\,b\,x\right)-2\,\sin\left(7\,a+7\,b\,x\right)\right)}{21\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(9/2),x)
```

```
[Out] -(16*c*d^4*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(41*sin(a + b*x) -
 29*sin(3*a + 3*b*x) + 12*sin(5*a + 5*b*x) - 2*sin(7*a + 7*b*x)))/(21*b*(15
*cos(2*a + 2*b*x) - 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.239 $\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=166

$$\frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}{5b}-\frac{\ }{5b\sqrt{\ }}$$

```
[Out] 24/5*c*d^5*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(3/2)-2/5*c*d*(d*csc(b*x+a
))^(5/2)*(c*sec(b*x+a))^(1/2)/b-12/5*c*d^3*(d*csc(b*x+a))^(1/2)*(c*sec(b*x+
a))^(1/2)/b+24/5*c^2*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Elli
pticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2
)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2705, 2706, 2710, 2652, 2719}

$$-\frac{24c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{5b}-\frac{2cd\sqrt{c\sec(a+bx)}(d\csc(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (24*c*d^5*Sqrt[c*Sec[a + b*x]])/(5*b*(d*Csc[a + b*x])^(3/2)) - (12*c*d^3*Sq
rt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(5*b) - (2*c*d*(d*Csc[a + b*x])^(5
/2)*Sqrt[c*Sec[a + b*x]])/(5*b) - (24*c^2*d^4*EllipticE[a - Pi/4 + b*x, 2])
/(5*b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
```

```
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}{5b}+\frac{1}{5}(6d^2)\int (d\csc(a+bx)\\ &= -\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b}\\ &= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\\ &= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\\ &= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\\ &= \frac{24cd^5\sqrt{c\sec(a+bx)}}{5b(d\csc(a+bx))^{3/2}}-\frac{12cd^3\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{5b}-\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.16, size = 114, normalized size = 0.69

$$-\frac{2cd^3\sqrt{d\csc(a+bx)}\left(\cot^2(a+bx)\left(6\cos(2(a+bx))+\csc^2(a+bx)\right)+12\cos^2(a+bx)\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}\tan^2(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d^3*Sqrt[d*Csc[a + b*x]]*(Cot[a + b*x]^2*(6*Cos[2*(a + b*x)] + Csc[a
+ b*x]^2) + 12*Cos[a + b*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/
2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(165) = 330$.
time = 59.42, size = 996, normalized size = 6.00

method	result	size
default	Expression too large to display	996

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*(24*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(
cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3-12*(-(
cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-s
in(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3+24*(-(cos(b*x+a)-1-s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin
(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-12*(-(cos(b*x+a)-1-sin(b*x+a))/sin
(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))*cos(b*x+a)^2-24*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*
cos(b*x+a)+12*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-24*(-
(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+12*(-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))-12*2^(1/2)*cos(b*x+a)^3+6*cos(b*x+a)^2*2^(1/2)+12*2^(1/2)*c
os(b*x+a)-5*2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(7/2)*(c/cos(b*x+a))^(3/2)*s
in(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{3/2}\,{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{7/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(7/2),x)
```

```
[Out] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(7/2), x)
```

3.240 $\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{3b}$$

```
[Out] -2/3*c*d*(d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(1/2)/b+8/3*c*d^3*(c*sec(b*x+a
))^(1/2)/b/(d*csc(b*x+a))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$,
Rules used = {2705, 2699}

$$\frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (8*c*d^3*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc[a
 + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]])/(3*b)
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{3b}+\frac{1}{3}\left(4d^2\right)\int \sqrt{d\csc(a+\\ &= \frac{8cd^3\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 45, normalized size = 0.65

$$-\frac{2cd^3(-4+\csc^2(a+bx))\sqrt{c\sec(a+bx)}}{3b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d^3*(-4 + Csc[a + b*x]^2)*Sqrt[c*Sec[a + b*x]])/(3*b*Sqrt[d*Csc[a + b
*x]])
```

Maple [A]
time = 58.42, size = 54, normalized size = 0.78

method	result	size
default	$-\frac{2\left(4\left(\cos^2(bx+a)\right)-3\right)\cos(bx+a)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{5}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)}{3b}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-3)*cos(b*x+a)*(d/sin(b*x+a))^(5/2)*(c/cos(b*x+a))^(3
/2)*sin(b*x+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2), x)
```

Fricas [A]
time = 2.27, size = 58, normalized size = 0.84

$$-\frac{2\left(4\,cd^2\cos\left(bx+a\right)^2-3\,cd^2\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{3\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*c*d^2*cos(b*x + a)^2 - 3*c*d^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x
 + a))/(b*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [**B**]
time = 0.76, size = 61, normalized size = 0.88

$$\frac{2\,c\,d^2\,\left(2\,\sin\left(a+b\,x\right)-\sin\left(3\,a+3\,b\,x\right)\right)\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{3\,b\,{\sin\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(5/2),x)
```

```
[Out] (2*c*d^2*(2*sin(a + b*x) - sin(3*a + 3*b*x))*(c/cos(a + b*x))^(1/2)*(d/sin(
a + b*x))^(1/2))/(3*b*sin(a + b*x)^2)
```

3.241 $\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2cd\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}{b}-\frac{4c^2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2b}$$

```
[Out] 4*c*d^3*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(3/2)-2*c*d*(d*csc(b*x+a))^(1
/2)*(c*sec(b*x+a))^(1/2)/b+4*c^2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*
Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(
b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2705, 2706, 2710, 2652, 2719}

$$-\frac{4c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2cd\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (4*c*d^3*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c*d*Sqrt[d*C
sc[a + b*x]]*Sqrt[c*Sec[a + b*x]])/b - (4*c^2*d^2*EllipticE[a - Pi/4 + b*x,
 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
```

```
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}\,dx = -\frac{2cd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}{b} + \left(2d^2\right)\int \frac{(c\sec(a+bx)}{\sqrt{d\csc(a+b}}$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}{b} - ($$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}{b} -$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}{b} -$$

$$= \frac{4cd^3\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2cd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}{b} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.55, size = 99, normalized size = 0.79

$$-\frac{2cd\sqrt{d\csc(a+bx)}\left(\cos(2(a+bx))\cot^2(a+bx)+2\cos^2(a+bx)\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}\,\tan^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2),x]
```

[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]]*(Cos[2*(a + b*x)]*Cot[a + b*x]^2 + 2*Cos[a + b*x]^2*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]]*Tan[a + b*x]^2)/b

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(136) = 272$.
time = 58.21, size = 498, normalized size = 3.98

method	result
default	$-\frac{\left(-4\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(-4*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-4*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cos(b*x+a)-2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/cos(b*x+a))^(3/2)*sin(b*x+a)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{c}{\cos\left(a+b\,x\right)}\right)^{3/2}\,\left(\frac{d}{\sin\left(a+b\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2),x)
```

```
[Out] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2), x)
```

3.242 $\int \sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}\,dx$

Optimal. Leaf size=31

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

```
[Out] 2*c*d*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2699}

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}\,dx = \frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

Mathematica [A]
time = 0.07, size = 31, normalized size = 1.00

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2),x]
```

[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*Sqrt[d*Csc[a + b*x]])

Maple [A]
time = 59.82, size = 42, normalized size = 1.35

method	result	size
default	$\frac{2\sin(bx+a)\cos(bx+a)\sqrt{\frac{d}{\sin(bx+a)}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}}{b}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/b*sin(b*x+a)*cos(b*x+a)*(d/sin(b*x+a))^(1/2)*(c/cos(b*x+a))^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)

Fricas [A]
time = 2.55, size = 36, normalized size = 1.16

$$\frac{2\,c\sqrt{\frac{c}{\cos\left(bx+a\right)}}\,\sqrt{\frac{d}{\sin\left(bx+a\right)}}\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="fricas")

[Out] 2*c*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x + a)/b

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2), x)
```

Mupad [**B**]
time = 0.30, size = 36, normalized size = 1.16

$$\frac{2\,c\,\sin\left(a+b\,x\right)\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(1/2),x)
[Out] (2*c*sin(a + b*x)*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2))/b
```

3.243 $\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal. Leaf size=89

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2*c*d*(c*sec(b*x+a))^(1/2)/b/(d*csc(b*x+a))^(3/2)+2*c^2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b*
x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2706, 2710, 2652, 2719}

$$\frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}-\frac{2c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (2*c*d*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2)) - (2*c^2*EllipticE[
a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c\sec(a+bx))^{3/2}}{\sqrt{d\csc(a+bx)}}\,dx &= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - (2c^2)\int \frac{1}{\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}\,dx \\
&= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{(2c^2)\int \sqrt{c\cos(a+bx)}\,\sqrt{d\sin(a+bx)}\,dx}{\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{d\sin(a}} \\
&= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{(2c^2)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}} \\
&= \frac{2cd\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}} - \frac{2c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.41, size = 66, normalized size = 0.74

$$-\frac{2cd\left(-1+\sqrt[4]{-\cot^2(a+bx)}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{b(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (-2*c*d*(-1 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc
[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*(d*Csc[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. 2(104) = 208.
time = 60.53, size = 497, normalized size = 5.58

method	result

default	$\left(2\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \text{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(b}{\sin(bx+a)}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)*((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(b*x+a)+2^(
1/2))*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/(d/sin(b*x+a))^(1/2)/sin(b*x+a)*2^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sec\left(a+bx\right)\right)^{\frac{3}{2}}}{\sqrt{d\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Integral((c*sec(a + b*x))**(3/2)/sqrt(d*csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/sqrt(d*csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{3/2}}{\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(1/2),x)
```

```
[Out] int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(1/2), x)
```

3.244 $\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=327

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}+\frac{c^2\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}-\frac{c^2\mathrm{ArcTan}\left(1+\right.}{}$$

```
[Out] 2*c*(c*sec(b*x+a))^(1/2)/b/d/(d*csc(b*x+a))^(1/2)-1/2*c^2*arctan(-1+2^(1/2)
*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/d^2*2^(1/2)/(c*s
ec(b*x+a))^(1/2)-1/2*c^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^
(1/2)*tan(b*x+a)^(1/2)/b/d^2*2^(1/2)/(c*sec(b*x+a))^(1/2)+1/4*c^2*ln(1-2^(1
/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/d^
2*2^(1/2)/(c*sec(b*x+a))^(1/2)-1/4*c^2*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*
x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/d^2*2^(1/2)/(c*sec(b*x+a))^(1
/2)
```

Rubi [A]
time = 0.15, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2704, 2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{c^2\sqrt{\tan(a+bx)}\,\mathrm{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}-\frac{c^2\sqrt{\tan(a+bx)}\,\mathrm{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}+\frac{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}-\frac{c^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}+\frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*Sqrt[d*Csc[a + b*x]]) + (c^2*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*
b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*S
qrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]
) + (c^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a +
b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) - (c^2*Sqr
t[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[T
an[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{c^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx}{d^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,d\ldots\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(2c^2\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \ldots\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\ldots}\right.}{bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,d\ldots\right.}{2bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} + \frac{c^2\sqrt{d\csc(a+bx)}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}+\tan(a+\ldots\right.}{2\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{2c\sqrt{c\sec(a+bx)}}{bd\sqrt{d\csc(a+bx)}} + \frac{c^2\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\,\sqrt{\tan(\ldots}}{\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}$$

Mathematica [A]

time = 1.43, size = 142, normalized size = 0.43

$$\frac{c\left(4+\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}\right)\sqrt{c\sec(a+bx)}}{2bd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (c*(4 + Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2
)^(1/4))]*(Cot[a + b*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2
)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(1/4))*Sqrt[c*Sec[a +
 b*x]])/(2*b*d*Sqrt[d*Csc[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 56.84, size = 646, normalized size = 1.98

method	result
default	$\frac{\left(i\sin(bx+a)\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(I*sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(
b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-I
*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+sin(b*x+a
)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)+sin(b*x+a)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*sin(b*x+a)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cos(b*x+a)-2*2^(1/2))*cos(b*x+a)*(c/co
s(b*x+a))^(3/2)/(-1+cos(b*x+a))/(d/sin(b*x+a))^(3/2)/sin(b*x+a)*2^(1/2)
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sec{(a+bx)})^{\frac{3}{2}}}{(d\csc{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(3/2),x)
[Out] Integral((c*sec(a + b*x))**(3/2)/(d*csc(a + b*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{3/2}}{{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(3/2),x)
[Out] int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(3/2), x)
```

3.245 $\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{bd^2\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2*c*(c*sec(b*x+a))^(1/2)/b/d/(d*csc(b*x+a))^(3/2)+3*c^2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/d^2/(d*cs
c(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2704, 2710, 2652, 2719}

$$\frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)) - (3*c^2*EllipticE[
a - Pi/4 + b*x, 2])/(b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[S
in[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
```

```
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c\sec(a+bx))^{3/2}}{(d\csc(a+bx))^{5/2}}\,dx &= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{(3c^2)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{d^2} \\
&= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{(3c^2)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{d^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}} \\
&= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{(3c^2)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}} \\
&= \frac{2c\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}} - \frac{3c^2E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.47, size = 69, normalized size = 0.73

$$-\frac{c\left(-2+3\sqrt[4]{-\cot^2(a+bx)}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{bd(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(3/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] -((c*(-2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[
a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*d*(d*Csc[a + b*x])^(3/2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(109) = 218.
time = 57.90, size = 512, normalized size = 5.45

method	result

default	$\frac{\left(6\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx-}{\sin(bx+a)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)
*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF
(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(
1/2)-3*2^(1/2)*cos(b*x+a)+2*2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(3/2)/sin(b*
x+a)^3/(d/sin(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*c*sec(b*x + a)/(d^3*csc(
b*x + a)^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(3/2)/(d*csc(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(3/2)/(d*csc(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(5/2),x)
```

```
[Out] int((c/cos(a + b*x))^(3/2)/(d/sin(a + b*x))^(5/2), x)
```

3.246 $\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b}-\frac{2cd(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}}{7b}+$$

```
[Out] -20/21*c*d^3*(d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2)/b-2/7*c*d*(d*csc(b*x
+a))^(7/2)*(c*sec(b*x+a))^(3/2)/b+40/21*c*d^5*(c*sec(b*x+a))^(3/2)/b/(d*csc
(b*x+a))^(1/2)-40/21*c^2*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*
EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(1
/2)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]

time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2705, 2706, 2710, 2653, 2720}

$$\frac{40c^2d^4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{21b}+\frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{21b}-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (40*c*d^5*(c*Sec[a + b*x])^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]]) - (20*c*d^3*(
d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(21*b) - (2*c*d*(d*Csc[a + b*
x])^(7/2)*(c*Sec[a + b*x])^(3/2))/(7*b) + (40*c^2*d^4*Sqrt[d*Csc[a + b*x]]*
EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{9/2}(c\sec(a+bx))^{5/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{3/2}}{7b}+\frac{1}{7}\left(10d^2\right)\int (d\csc(a+\\
&= -\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{21b}-\frac{2cd(d\csc(a+bx))}{}\\
&= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))}{21b}\\
&= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))}{21b}\\
&= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))}{21b}\\
&= \frac{40cd^5(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}-\frac{20cd^3(d\csc(a+bx))^{3/2}(c\sec(a+bx))}{21b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.51, size = 92, normalized size = 0.55

$$-\frac{2cd^5\left(-7+\cot^2(a+bx)\left(13+3\csc^2(a+bx)\right)+20\left(-\cot^2(a+bx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)(c\sec(a+bx))^{3/2}}{21b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d^5*(-7 + Cot[a + b*x]^2*(13 + 3*Csc[a + b*x]^2) + 20*(-Cot[a + b*x]^
2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*(c*Sec[a + b*x])
^(3/2))/(21*b*Sqrt[d*Csc[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(165) = 330$.
time = 55.39, size = 563, normalized size = 3.39

method	result
default	$\frac{\left(-40\left(\cos^4(bx+a)\right)\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/b*(-40*cos(b*x+a)^4*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))-40*cos(b*x+a)^3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))+40*cos(b*x+a)^2*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
40*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+20*2^
(1/2)*cos(b*x+a)^4-30*cos(b*x+a)^2*2^(1/2)+7*2^(1/2))*cos(b*x+a)*(d/sin(b*x
+a))^(9/2)*(c/cos(b*x+a))^(5/2)*sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.88, size = 223, normalized size = 1.34

$$-\frac{2\left(10\left(i\,c^2d^4\cos\left(bx+a\right)^3-i\,c^2d^4\cos\left(bx+a\right)\right)\sqrt{-4i\,cd}\,\mathrm{ellipticF}(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1)\sin\left(bx+a\right)+10\left(-i\,c^2d^4\cos\left(bx+a\right)^3+i\,c^2d^4\cos\left(bx+a\right)\right)\sqrt{4i\,cd}\,\mathrm{ellipticF}(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1)\sin\left(bx+a\right)+\left(20\,c^2d^4\cos\left(bx+a\right)^4-30\,c^2d^4\cos\left(bx+a\right)^2+7\,c^2d^4\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}\right)}{21\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/21*(10*(I*c^2*d^4*cos(b*x + a)^3 - I*c^2*d^4*cos(b*x + a))*sqrt(-4*I*c*d
)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1)*sin(b*x + a) + 10*(-I*c^2*d^
4*cos(b*x + a)^3 + I*c^2*d^4*cos(b*x + a))*sqrt(4*I*c*d)*ellipticF(cos(b*x
+ a) - I*sin(b*x + a), -1)*sin(b*x + a) + (20*c^2*d^4*cos(b*x + a)^4 - 30*c
^2*d^4*cos(b*x + a)^2 + 7*c^2*d^4)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)
))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{5/2}\,{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{9/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(9/2),x)
```

```
[Out] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(9/2), x)
```

3.247 $\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=106

$$-\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}}+\frac{16cd^3\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{15b}-\frac{2cd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}{5b}$$

```
[Out] -2/5*c*d*(d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(3/2)/b+16/15*c*d^3*(c*sec(b*x
+a))^(3/2)*(d*csc(b*x+a))^(1/2)/b-64/15*c^3*d^3*(d*csc(b*x+a))^(1/2)/b/(c*s
ec(b*x+a))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2705, 2706, 2699}

$$-\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}}+\frac{16cd^3(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{15b}-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-64*c^3*d^3*Sqrt[d*Csc[a + b*x]])/(15*b*Sqrt[c*Sec[a + b*x]]) + (16*c*d^3*
Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(15*b) - (2*c*d*(d*Csc[a + b*x
])^(5/2)*(c*Sec[a + b*x])^(3/2))/(5*b)
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
```

```
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}{5b}+\frac{1}{5}\left(8d^2\right)\int (d\csc(a+\\ &= \frac{16cd^3\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{15b}-\frac{2cd(d\csc(a+bx))^{5/2}(\\ &= -\frac{64c^3d^3\sqrt{d\csc(a+bx)}}{15b\sqrt{c\sec(a+bx)}}+\frac{16cd^3\sqrt{d\csc(a+bx)}\,(c\sec(a+bx)}{15b}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 57, normalized size = 0.54

$$-\frac{2cd^3\left(-5+32\cos^2(a+bx)+3\cot^2(a+bx)\right)\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d^3*(-5 + 32*Cos[a + b*x]^2 + 3*Cot[a + b*x]^2)*Sqrt[d*Csc[a + b*x]]*
(c*Sec[a + b*x])^(3/2))/(15*b)
```

Maple [A]
time = 55.84, size = 64, normalized size = 0.60

method	result	size
default	$\frac{2\left(32\left(\cos^4(bx+a)\right)-40\left(\cos^2(bx+a)\right)+5\right)\cos(bx+a)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{7}{2}}\left(\frac{c}{\cos(bx+a)}\right)^{\frac{5}{2}}\sin(bx+a)}{15b}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/b*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+5)*cos(b*x+a)*(d/sin(b*x+a))^(7/2)*
(c/cos(b*x+a))^(5/2)*sin(b*x+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [A]
time = 3.17, size = 89, normalized size = 0.84

$$-\frac{2 \left(32 c^{2} d^{3} \cos \left(bx+a\right)^{4}-40 c^{2} d^{3} \cos \left(bx+a\right)^{2}+5 c^{2} d^{3}\right) \sqrt{\frac{c}{\cos \left(bx+a\right)}} \sqrt{\frac{d}{\sin \left(bx+a\right)}}}{15 \left(b \cos \left(bx+a\right)^{3}-b \cos \left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(32*c^2*d^3*cos(b*x + a)^4 - 40*c^2*d^3*cos(b*x + a)^2 + 5*c^2*d^3)*s
qrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [B]
time = 2.32, size = 112, normalized size = 1.06

$$\frac{16\,c^2\,d^3\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(5\,\cos\left(a+b\,x\right)-3\,\cos\left(3\,a+3\,b\,x\right)-4\,\cos\left(5\,a+5\,b\,x\right)+2\,\cos\left(7\,a+7\,b\,x\right)\right)}{15\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+2\,\cos\left(4\,a+4\,b\,x\right)-\cos\left(6\,a+6\,b\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(7/2),x)
```

```
[Out] (16*c^2*d^3*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(5*cos(a + b*x) -
 3*cos(3*a + 3*b*x) - 4*cos(5*a + 5*b*x) + 2*cos(7*a + 7*b*x)))/(15*b*(cos(
2*a + 2*b*x) + 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) - 2))
```

3.248 $\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b}+\frac{4c^2d^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{3b}$$

```
[Out] -2/3*c*d*(d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(3/2)/b+4/3*c*d^3*(c*sec(b*x+a
))^(3/2)/b/(d*csc(b*x+a))^(1/2)-4/3*c^2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin
(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))^(1/2)*(c
*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]

time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2705, 2706, 2710, 2653, 2720}

$$\frac{4c^2d^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{3b}+\frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (4*c*d^3*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) - (2*c*d*(d*Csc
[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))/(3*b) + (4*c^2*d^2*Sqrt[d*Csc[a +
b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*
x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
```

```
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}\,dx &= -\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b}+\left(2d^2\right)\int \sqrt{d\csc(a+\\ &= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b}\\ &= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b}\\ &= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b}\\ &= \frac{4cd^3(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}-\frac{2cd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.71, size = 87, normalized size = 0.66

$$-\frac{2c^3d(d\csc(a+bx))^{3/2}\left(-1+\cot^2(a+bx)+2(-\cot^2(a+bx))^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)\tan^2(a+bx)}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c^3*d*(d*Csc[a + b*x])^(3/2)*(-1 + Cot[a + b*x]^2 + 2*(-Cot[a + b*x]^2)
^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Tan[a + b*x]^2)/(3
*b*Sqrt[c*Sec[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(136) = 272$.
time = 56.53, size = 306, normalized size = 2.34

method	result
default	$-\frac{\left(-4\left(\cos^2(bx+a)\right)\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(-4*cos(b*x+a)^2*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1
/2))-4*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2
*cos(b*x+a)^2*2^(1/2)-2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(5/2)*(c/cos(b*x+a
))^(5/2)*sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.62, size = 158, normalized size = 1.21

$$-\frac{2\left(i\sqrt{-4i\,cd}\,c^2d^2\cos(bx+a)\,\mathrm{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)\sin(bx+a)-i\sqrt{4i\,cd}\,c^2d^2\cos(bx+a)\,\mathrm{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)\sin(bx+a)+\left(2c^2d^2\cos(bx+a)^2-c^2d^2\right)\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\right)}{3b\cos(bx+a)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(I*sqrt(-4*I*c*d)*c^2*d^2*cos(b*x + a)*ellipticF(cos(b*x + a) + I*sin(
b*x + a), -1)*sin(b*x + a) - I*sqrt(4*I*c*d)*c^2*d^2*cos(b*x + a)*ellipticF
(cos(b*x + a) - I*sin(b*x + a), -1)*sin(b*x + a) + (2*c^2*d^2*cos(b*x + a)^
```

```
2 - c^2*d^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a)))/(b*cos(b*x + a)*sin
(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)*(c*sec(b*x+a))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{c}{\cos\left(a+b\,x\right)}\right)^{5/2}\,\left(\frac{d}{\sin\left(a+b\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2),x)
[Out] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2), x)
```

3.249 $\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=69

$$-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}+\frac{2cd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*c*d*(c*sec(b*x+a))^(3/2)*(d*csc(b*x+a))^(1/2)/b-8/3*c^3*d*(d*csc(b*x+a)
)^(1/2)/b/(c*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2706, 2699}

$$\frac{2cd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}{3b}-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-8*c^3*d*Sqrt[d*Csc[a + b*x]])/(3*b*Sqrt[c*Sec[a + b*x]]) + (2*c*d*Sqrt[d*
Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2))/(3*b)
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int (d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}\,dx&=\frac{2cd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{3b}+\frac{1}{3}\left(4c^2\right)\int (d\csc(a+bx))^{3}\\&=-\frac{8c^3d\sqrt{d\csc(a+bx)}}{3b\sqrt{c\sec(a+bx)}}+\frac{2cd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 45, normalized size = 0.65

$$-\frac{2cd(1+2\cos(2(a+bx)))\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2),x]
[Out] (-2*c*d*(1 + 2*Cos[2*(a + b*x)])*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2
))/(3*b)
```

Maple [A]
time = 55.29, size = 54, normalized size = 0.78

method	result	size
default	$-\frac{2\left(4\left(\cos^{2}\left(bx+a\right)\right)-1\right)\cos\left(bx+a\right)\left(\frac{d}{\sin\left(bx+a\right)}\right)^{\frac{3}{2}}\left(\frac{c}{\cos\left(bx+a\right)}\right)^{\frac{5}{2}}\sin\left(bx+a\right)}{3b}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b*(4*cos(b*x+a)^2-1)*cos(b*x+a)*(d/sin(b*x+a))^(3/2)*(c/cos(b*x+a))^(5
/2)*sin(b*x+a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2), x)
```

Fricas [A]
time = 3.11, size = 58, normalized size = 0.84

$$-\frac{2\left(4\,c^{2}d\cos\left(bx+a\right)^{2}-c^{2}d\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*c^2*d*cos(b*x + a)^2 - c^2*d)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x +
 a))/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2), x)
```

Mupad [B]
time = 0.83, size = 64, normalized size = 0.93

$$-\frac{4\,c^2\,d\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(2\,\cos\left(a+b\,x\right)+\cos\left(3\,a+3\,b\,x\right)\right)}{3\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(3/2),x)
```

```
[Out] -(4*c^2*d*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(2*cos(a + b*x) + c
os(3*a + 3*b*x)))/(3*b*(cos(2*a + 2*b*x) + 1))
```

3.250 $\int \sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}\,dx$

Optimal. Leaf size=93

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}+\frac{2c^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}{3b}$$

```
[Out] 2/3*c*d*(c*sec(b*x+a))^(3/2)/b/(d*csc(b*x+a))^(1/2)-2/3*c^2*(sin(a+1/4*Pi+b
*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(
b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2706, 2710, 2653, 2720}

$$\frac{2c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{3b}+\frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]]) + (2*c^2*Sqrt[d*C
sc[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
```

```
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}\,dx &= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}+\frac{1}{3}\left(2c^2\right)\int \sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\\ &= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}+\frac{1}{3}\Big(2c^2\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+bx)}\,\sqrt{c}\\ &= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}+\frac{1}{3}\Big(2c^2\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{s}\\ &= \frac{2cd(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}+\frac{2c^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c s}}{3b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.65, size = 68, normalized size = 0.73

$$-\frac{2cd\left(-1+\left(-\cot^2(a+bx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)(c\sec(a+bx))^{3/2}}{3b\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d*(-1 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[
a + b*x]^2])*(c*Sec[a + b*x])^(3/2))/(3*b*Sqrt[d*Csc[a + b*x]])
```

Maple [A]
time = 55.88, size = 188, normalized size = 2.02

method	result
default	$-\frac{\left(2\sin(bx+a)\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx}{\mathrm{si}}}\right.\right.}{3b(-1+\cos(bx+a))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(2*sin(b*x+a)*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-
2^(1/2)*cos(b*x+a)+2^(1/2))*cos(b*x+a)*(d/sin(b*x+a))^(1/2)*(c/cos(b*x+a))^
(5/2)*sin(b*x+a)/(-1+cos(b*x+a))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.01, size = 117, normalized size = 1.26

$$\frac{-i\sqrt{-4i\,cd}\,c^2\cos(bx+a)\,\mathrm{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)+i\sqrt{4i\,cd}\,c^2\cos(bx+a)\,\mathrm{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)+2\,c^2\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sin(bx+a)}{3\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(-4*I*c*d)*c^2*cos(b*x + a)*ellipticF(cos(b*x + a) + I*sin(b*x
+ a), -1) + I*sqrt(4*I*c*d)*c^2*cos(b*x + a)*ellipticF(cos(b*x + a) - I*sin
(b*x + a), -1) + 2*c^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x +
a))/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)*(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)*(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{c}{\cos\left(a+b\,x\right)}\right)^{5/2}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2),x)
```

```
[Out] int((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2), x)
```

3.251 $\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx$

Optimal. Leaf size=33

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

```
[Out] 2/3*c*d*(c*sec(b*x+a))^(3/2)/b/(d*csc(b*x+a))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2699\}$

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{5/2}}{\sqrt{d\csc(a+bx)}}\,dx = \frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

Mathematica [A]
time = 0.12, size = 33, normalized size = 1.00

$$\frac{2cd(c\sec(a+bx))^{3/2}}{3b(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/Sqrt[d*Csc[a + b*x]],x]
```

```
[Out] (2*c*d*(c*Sec[a + b*x])^(3/2))/(3*b*(d*Csc[a + b*x])^(3/2))
```

Maple [A]
time = 58.52, size = 42, normalized size = 1.27

method	result	size
default	$\frac{2\cos(bx+a)\left(\frac{c}{\cos(bx+a)}\right)^{\frac{5}{2}}\sin(bx+a)}{3b\sqrt{\frac{d}{\sin(bx+a)}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/b*cos(b*x+a)*(c/cos(b*x+a))^(5/2)*sin(b*x+a)/(d/sin(b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sec(b*x + a))^(5/2)/sqrt(d*csc(b*x + a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.
time = 2.51, size = 58, normalized size = 1.76

$$-\frac{2\left(c^2\cos\left(bx+a\right)^2-c^2\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{3\,bd\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2/3*(c^2*cos(b*x + a)^2 - c^2)*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))/(
b*d*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/sqrt(d*csc(b*x + a)), x)
```

Mupad [**B**]
time = 0.86, size = 66, normalized size = 2.00

$$\frac{c^2\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(\cos\left(a+b\,x\right)-\cos\left(3\,a+3\,b\,x\right)\right)}{3\,b\,d\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(1/2),x)
```

```
[Out] (c^2*(c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)*(cos(a + b*x) - cos(3*a
+ 3*b*x)))/(3*b*d*(cos(2*a + 2*b*x) + 1))
```

3.252 $\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}{3bd^2}$$

```
[Out] 2/3*c*(c*sec(b*x+a))^(3/2)/b/d/(d*csc(b*x+a))^(1/2)+1/3*c^2*(sin(a+1/4*Pi+b
*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(
b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2704, 2710, 2653, 2720}

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]]) - (c^2*Sqrt[d*Csc
[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{3/2}}\,dx = \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{c^2\int\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,dx}{3d^2}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{d\,\mathrm{s}\ldots}\right.}{3d^2}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{\left(c^2\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}\right)\int\ldots}{3d^2}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}} - \frac{c^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\mathrm{si}\ldots}}{3bd^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.58, size = 70, normalized size = 0.71

$$\frac{c\left(2+\left(-\cot^2(a+bx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)(c\sec(a+bx))^{3/2}}{3bd\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(3/2),x]
```

```
[Out] (c*(2 + (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*
x]^2])*(c*Sec[a + b*x])^(3/2))/(3*b*d*Sqrt[d*Csc[a + b*x]])
```

Maple [A]
time = 56.98, size = 192, normalized size = 1.96

method	result

default	$\frac{\left(\cos(bx+a)\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+\cdots}{}}\right.\right.}{3b(-1+\cos(bx+a))\left(\frac{d}{\sin(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/3/b*(cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)-2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(5/2)/(-1+cos(b*x+a))/(d/sin(b*x+a))^(3/2)/sin(b*x+a)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(3/2), x)

Fricas [C] Result contains complex when optimal does not.
time = 1.09, size = 120, normalized size = 1.22

$$\frac{i\sqrt{-4i\,cd}\,c^2\cos(bx+a)\,\mathrm{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)-i\sqrt{4i\,cd}\,c^2\cos(bx+a)\,\mathrm{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)+4\,c^2\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\sin(bx+a)}{6\,bd^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="fricas")

[Out] 1/6*(I*sqrt(-4*I*c*d)*c^2*cos(b*x + a)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) - I*sqrt(4*I*c*d)*c^2*cos(b*x + a)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) + 4*c^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*sin(b*x + a))/(b*d^2*cos(b*x + a))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(3/2),x)

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{5/2}}{{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(3/2),x)
[Out] int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(3/2), x)
```

3.253 $\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}+\frac{c^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}-\frac{c^2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+\right.}}{\sqrt{2}\,bd^2\sqrt{d\csc(a+bx)}}$$

```
[Out] 2/3*c*(c*sec(b*x+a))^(3/2)/b/d/(d*csc(b*x+a))^(3/2)-1/2*c^2*arctan(-1+2^(1/
2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/b/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2
)/tan(b*x+a)^(1/2)-1/2*c^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a)
)^(1/2)/b/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-1/4*c^2*ln(1-2^
(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(c*sec(b*x+a))^(1/2)/b/d^2*2^(1/2)/(d*cs
c(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/4*c^2*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(
b*x+a))*(c*sec(b*x+a))^(1/2)/b/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^
(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2704, 2709, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{c^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c^2\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c^2\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{c^2\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,bd^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] (2*c*(c*Sec[a + b*x])^(3/2))/(3*b*d*(d*Csc[a + b*x])^(3/2)) + (c^2*ArcTan[1
 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*
Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (c^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*
x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]]) - (c^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Se
c[a + b*x]])/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (c
^2*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])
/(2*Sqrt[2]*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2704

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Dist[b^2*((m + 1)/(a^2*(n - 1))), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{(c\sec(a+bx))^{5/2}}{(d\csc(a+bx))^{5/2}}\,dx = \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{c^2\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\int \sqrt{\tan(a+bx)}\,dx}{d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(a+bx)\right)}{bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(2c^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right.}{bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} + \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right.}{bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{\left(c^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(a\right.}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} - \frac{c^2\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a}}{2\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}$$

$$= \frac{2c(c\sec(a+bx))^{3/2}}{3bd(d\csc(a+bx))^{3/2}} + \frac{c^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bd^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}} - \frac{c^2}{}$$

Mathematica [A]

time = 1.31, size = 143, normalized size = 0.43

$$\frac{c\left(4+3\sqrt{2}\,\text{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}+3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}\right)(c\sec(a+bx))^{3/2}}{6bd(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sec[a + b*x])^(5/2)/(d*Csc[a + b*x])^(5/2),x]
```

```
[Out] (c*(4 + 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]
^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*
x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(3/4))*(c*Sec[a +
 b*x])^(3/2))/(6*b*d*(d*Csc[a + b*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 58.86, size = 544, normalized size = 1.65

method	result
default	$\frac{\left(-3i\cos(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\text{EllipticPi}\left(\sqrt{-\frac{\cos(bx+a)-}{\sin(}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b*(-3*I*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(
b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*E
llipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))+3*I*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-
3*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*cos(
b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos
(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*2^(1/2)*co
s(b*x+a)-2*2^(1/2))*cos(b*x+a)*(c/cos(b*x+a))^(5/2)/(-1+cos(b*x+a))/(d/sin(
b*x+a))^(5/2)/sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))**(5/2)/(d*csc(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(b*x + a))^(5/2)/(d*csc(b*x + a))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(5/2),x)
```

```
[Out] int((c/cos(a + b*x))^(5/2)/(d/sin(a + b*x))^(5/2), x)
```

3.254 $\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=69

$$-\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

```
[Out] -8/21*c*d^3*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(3/2)-2/7*c*d*(d*csc(b*x+
a))^(7/2)/b/(c*sec(b*x+a))^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2705, 2699}

$$-\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-8*c*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d
*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(3/2))
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\csc(a+bx))^{9/2}}{\sqrt{c\sec(a+bx)}}\,dx &= -\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}+\frac{1}{7}\left(4d^2\right)\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx\\ &= -\frac{8cd^3(d\csc(a+bx))^{3/2}}{21b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 45, normalized size = 0.65

$$\frac{2cd(-5+2\cos(2(a+bx)))(d\csc(a+bx))^{7/2}}{21b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (2*c*d*(-5 + 2*Cos[2*(a + b*x)])*(d*Csc[a + b*x])^(7/2))/(21*b*(c*Sec[a + b
*x])^(3/2))
```

Maple [A]
time = 36.54, size = 54, normalized size = 0.78

method	result	size
default	$\frac{2\left(4\left(\cos^{2}(bx+a)\right)-7\right)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{9}{2}}\cos(bx+a)\sin(bx+a)}{21b\sqrt{\frac{c}{\cos(bx+a)}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/b*(4*cos(b*x+a)^2-7)*(d/sin(b*x+a))^(9/2)*cos(b*x+a)*sin(b*x+a)/(c/cos
(b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [A]
time = 3.30, size = 79, normalized size = 1.14

$$-\frac{2\left(4\,d^{4}\cos\left(bx+a\right)^{4}-7\,d^{4}\cos\left(bx+a\right)^{2}\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{21\left(bc\cos\left(bx+a\right)^{2}-bc\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(4*d^4*cos(b*x + a)^4 - 7*d^4*cos(b*x + a)^2)*sqrt(c/cos(b*x + a))*sq
rt(d/sin(b*x + a))/((b*c*cos(b*x + a)^2 - b*c)*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [**B**]
time = 1.83, size = 99, normalized size = 1.43

$$\frac{8\,d^4\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(11\,\sin\left(2\,a+2\,b\,x\right)-7\,\sin\left(4\,a+4\,b\,x\right)+\sin\left(6\,a+6\,b\,x\right)\right)}{21\,b\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(1/2),x)
```

```
[Out] (8*d^4*(d/sin(a + b*x))^(1/2)*(11*sin(2*a + 2*b*x) - 7*sin(4*a + 4*b*x) + s
in(6*a + 6*b*x)))/(21*b*(c/cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) - 6*cos
(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.255 $\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}-\frac{4d^4E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2/5*c*d*(d*csc(b*x+a))^(5/2)/b/(c*sec(b*x+a))^(3/2)-4/5*c*d^3*(d*csc(b*x+a
))^(1/2)/b/(c*sec(b*x+a))^(3/2)+4/5*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*s
ec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2705, 2710, 2652, 2719}

$$-\frac{4d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-4*c*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*(c*Sec[a + b*x])^(3/2)) - (2*c*d*(d*Cs
c[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(3/2)) - (4*d^4*EllipticE[a - Pi/4
 + b*x, 2])/(5*b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2
*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\csc(a+bx))^{7/2}}{\sqrt{c\sec(a+bx)}}\,dx &= -\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}+\frac{1}{5}\left(2d^2\right)\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx\\
&= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}-\frac{1}{5}\left(4d^4\right)\int \frac{1}{\sqrt{d\csc(a+bx)}\,\sqrt{c\ldots}}\\
&= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}-\frac{\left(4d^4\right)\int \sqrt{c\cos(a\ldots}}{5\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+\ldots}}\\
&= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}-\frac{\left(4d^4\right)\int \sqrt{\sin(2a+\ldots}}{5\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+\ldots}}\\
&= -\frac{4cd^3\sqrt{d\csc(a+bx)}}{5b(c\sec(a+bx))^{3/2}}-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{3/2}}-\frac{4d^4E\left(a-\frac{\pi}{4}+\ldots\right.}{5b\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+\ldots}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.13, size = 104, normalized size = 0.81

$$-\frac{2d^2(d\csc(a+bx))^{3/2}\left(-((-2+\cos(2(a+bx)))\cot^3(a+bx))+\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\sin(2(a+bx))\right)\tan^2(a+bx)}{5b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*d^2*(d*Csc[a + b*x])^(3/2)*(-((-2 + Cos[2*(a + b*x)])*Cot[a + b*x]^3) +
 (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]*
Sin[2*(a + b*x)])*Tan[a + b*x]^2)/(5*b*Sqrt[c*Sec[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 991 vs. 2(133) = 266.
time = 34.55, size = 992, normalized size = 7.75

method	result	size
default	Expression too large to display	992

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*(4*cos(b*x+a)^3*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(
b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*E
llipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b
*x+a)^3*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos
(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*cos(b*x+a)^2*(-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b
*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*(-(cos(b*x+a)-1-sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+
a))^(1/2),1/2*2^(1/2))-4*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))+2*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a
)^3*2^(1/2)-4*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*(-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)+2*2^(1/2)*cos(b*x+a))*(
d/sin(b*x+a))^(7/2)*sin(b*x+a)/(c/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}}{\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(1/2),x)
```

```
[Out] int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(1/2), x)
```

3.256 $\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

```
[Out] -2/3*c*d*(d*csc(b*x+a))^(3/2)/b/(c*sec(b*x+a))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2699\}$

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{5/2}}{\sqrt{c\sec(a+bx)}}\,dx = -\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

Mathematica [A]
time = 0.13, size = 33, normalized size = 1.00

$$-\frac{2cd(d\csc(a+bx))^{3/2}}{3b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(3/2))/(3*b*(c*Sec[a + b*x])^(3/2))
```

Maple [A]
time = 34.09, size = 42, normalized size = 1.27

method	result	size
default	$-\frac{2 \cos \left(b x +a \right) \sin \left(b x +a \right) \left(\frac{d}{\sin \left(b x +a \right)}\right)^{\frac{5}{2}}}{3 b \sqrt{\frac{c}{\cos \left(b x +a \right)}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b*cos(b*x+a)*sin(b*x+a)*(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(5/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [A]
time = 3.14, size = 51, normalized size = 1.55

$$-\frac{2 \, d^{2} \sqrt{\frac{c}{\cos\left(b x + a\right)}} \sqrt{\frac{d}{\sin\left(b x + a\right)}} \cos\left(b x + a\right)^{2}}{3 \, b c \sin\left(b x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2/3*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^2/(b*c*sin(
b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [B]
time = 0.80, size = 49, normalized size = 1.48

$$-\frac{d^2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{3\,b\,{\sin\left(a+b\,x\right)}^2\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(1/2),x)
```

```
[Out] -(d^2*sin(2*a + 2*b*x)*(d/sin(a + b*x))^(1/2))/(3*b*sin(a + b*x)^2*(c/cos(a
 + b*x))^(1/2))
```

3.257 $\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}-\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2*c*d*(d*csc(b*x+a))^(1/2)/b/(c*sec(b*x+a))^(3/2)+2*d^2*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/(d*csc(b
*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2705, 2710, 2652, 2719}

$$-\frac{2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*c*d*Sqrt[d*Csc[a + b*x]])/(b*(c*Sec[a + b*x])^(3/2)) - (2*d^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[
2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx &= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - (2d^2)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx \\
&= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{(2d^2)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}} \\
&= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{(2d^2)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}} \\
&= -\frac{2cd\sqrt{d\csc(a+bx)}}{b(c\sec(a+bx))^{3/2}} - \frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.50, size = 80, normalized size = 0.90

$$-\frac{2d^2\left(\cot^2(a+bx)+\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\tan(a+bx)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] (-2*d^2*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1
/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[
a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(104) = 208.
time = 34.23, size = 502, normalized size = 5.64

method	result

default	$\frac{\left(2\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \text{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)*(
-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1
-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*(-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))-(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-
1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
F((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(b*
x+a))*(d/sin(b*x+a))^(3/2)*sin(b*x+a)/(c/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\csc{(a+bx)})^{\frac{3}{2}}}{\sqrt{c\sec{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral((d*csc(a + b*x))**(3/2)/sqrt(c*sec(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/sqrt(c*sec(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{d}{\sin(a+b\,x)}\right)}^{3/2}}{\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(1/2),x)
```

```
[Out] int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(1/2), x)
```

3.258 $\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=270

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}\,b\sqrt{c\sec(a+bx)}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{}}{\sqrt{2}\,b\sqrt{c\sec(a}}$$

```
[Out] 1/2*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/
2)/b*2^(1/2)/(c*sec(b*x+a))^(1/2)+1/2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d
*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b*2^(1/2)/(c*sec(b*x+a))^(1/2)-1/4*ln(1
-2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)
/b*2^(1/2)/(c*sec(b*x+a))^(1/2)+1/4*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a
))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b*2^(1/2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,b\sqrt{c\sec(a+bx)}}+\frac{\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,b\sqrt{c\sec(a+bx)}}-\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,b\sqrt{c\sec(a+bx)}}+\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,b\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/Sqrt[c*Sec[a + b*x]],x]
```

```
[Out] -((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(Sqrt[2]*b*Sqrt[c*Sec[a + b*x]])) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a
+ b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*Sqrt[c*Sec[a +
 b*x]]) - (Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a
+ b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Cs
c[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(2*Sqrt[2]*b*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

`IntegerQ[n]`

Rubi steps

$$\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx = \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,dx}{\sqrt{c\sec(a+bx)}}$$
$$= \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx, x, \tan(a+bx)\right)}{b\sqrt{c\sec(a+bx)}}$$
$$= \frac{\left(2\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c\sec(a+bx)}}$$
$$= \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(a+bx)}\right)}{b\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\,c\right.}{}$$
$$= \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(a+bx)}\right)}{2b\sqrt{c\sec(a+bx)}} +$$
$$= -\frac{\sqrt{d\csc(a+bx)}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{\tan(a+bx)}}{2\sqrt{2}\,b\sqrt{c\sec(a+bx)}} +$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}}{\sqrt{2}\,b\sqrt{c\sec(a+bx)}} + \frac{\tan^{-1}\left(1+\right.}{}$$

Mathematica [A]
time = 0.84, size = 123, normalized size = 0.46

$$-\frac{\left(\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\right)\cot(a+bx)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,b\cot^2(a+bx)^{3/4}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/Sqrt[c*Sec[a + b*x]],x]

[Out] -(((ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))] -
ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])])*Cot[a
 + b*x]*Sqrt[d*Csc[a + b*x]])/(Sqrt[2]*b*(Cot[a + b*x]^2)^(3/4)*Sqrt[c*Sec[
a + b*x]]))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 33.56, size = 316, normalized size = 1.17

method	result
default	$-\frac{\sqrt{\frac{d}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)-}{\sin(bx-}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(d/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2
*2^(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2
*I,1/2*2^(1/2))-2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2)))*sin(b*x+a)^2/(c/cos(b*x+a))^(1/2)/cos(b*x+a)/(-1+cos(b*
x+a))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\csc{(a+bx)}}}{\sqrt{c\sec{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(a + b*x))/sqrt(c*sec(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/sqrt(c*sec(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(1/2),x)
```

```
[Out] int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(1/2), x)
```

3.259 $\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=53

$$\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),
2^(1/2))/b/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2710, 2652, 2719}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*S
qrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx = \frac{\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\sin(a+bx)}}$$
$$= \frac{\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$
$$= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.25, size = 66, normalized size = 1.25

$$\frac{\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\tan(a+bx)}{b\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] ((-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2]*
Tan[a + b*x])/(b*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(73) = 146.
time = 32.53, size = 509, normalized size = 9.60

method	result
default	$-\frac{\left(2\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
```

```
)/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2
)-2^(1/2)*cos(b*x+a))/(d/sin(b*x+a))^(1/2)/(c/cos(b*x+a))^(1/2)/sin(b*x+a)/
cos(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d*csc(b*x + a)*sec(b*
x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sec{\left(a+bx \right)}}\sqrt{d\csc{\left(a+bx \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*sec(a + b*x))*sqrt(d*csc(a + b*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{c}{\cos(a+b\,x)}}\,\sqrt{\frac{d}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)),x)
[Out] int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(1/2)), x)
```

3.260 $\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=322

$$-\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}-\frac{\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{4\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}+$$

```
[Out] -1/2*c/b/d/(c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2)+1/8*arctan(-1+2^(1/2)*
tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/d^2*2^(1/2)/(c*se
c(b*x+a))^(1/2)+1/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)
*tan(b*x+a)^(1/2)/b/d^2*2^(1/2)/(c*sec(b*x+a))^(1/2)-1/16*ln(1-2^(1/2)*tan(
b*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/d^2*2^(1/2
)/(c*sec(b*x+a))^(1/2)+1/16*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*cs
c(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/d^2*2^(1/2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2707, 2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{\tan(a+bx)}\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{4\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}+\frac{\sqrt{\tan(a+bx)}\operatorname{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{4\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}-\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}+\frac{\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bd^2\sqrt{c\sec(a+bx)}}-\frac{c}{2bd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -1/2*c/(b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (ArcTan[1 - Sqrt
[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]
*b*d^2*Sqrt[c*Sec[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt
[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
 - (Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]
*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a
 + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a + b*
x]])/(8*Sqrt[2]*b*d^2*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}\,dx = -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{4d^2}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan\right.}{4d^2\sqrt{}}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan\right.}{}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan\right.}{}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan\right.}{}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{\tan\right.}{}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} - \frac{\sqrt{d\csc(a+bx)}\,\log\left(1\right.}{}$$

$$= -\frac{c}{2bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} - \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(\right.}{4\sqrt{}}$$

Mathematica [A]

time = 1.56, size = 156, normalized size = 0.48

$$-\frac{\left(4\cos^2(a+bx)+\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}\right)\sqrt{d\csc(a+bx)}\,\tan(a+bx)}{8bd^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -1/8*((4*Cos[a + b*x]^2 + Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[
2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[
2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(1/
4))*Sqrt[d*Csc[a + b*x]]*Tan[a + b*x])/(b*d^2*Sqrt[c*Sec[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 29.89, size = 658, normalized size = 2.04

method	result
default	$-\frac{\left(-i\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+s}{\sin(bx+}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(-I*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*
x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))
+I*sin(b*x+a)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)+sin(b*x
+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+sin(b*x+a)*Ellipt
icPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-2*sin(b*x+a)*((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)^3*2^(1/2)-2*cos(b*x+a)^2*2^(1/2))
/(-1+cos(b*x+a))/sin(b*x+a)/(d/sin(b*x+a))^(3/2)/(c/cos(b*x+a))^(1/2)/cos(b
*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sec{(a+bx)}}\,(d\csc{(a+bx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*sec(a + b*x))*(d*csc(a + b*x))**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*sqrt(c*sec(b*x + a))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(3/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(3/2)), x)
```

3.261 $\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2bd^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/3*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2)-1/2*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/d^2/(d*c
sc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2707, 2710, 2652, 2719}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{3bd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -1/3*c/(b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a -
Pi/4 + b*x, 2]/(2*b*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[
2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
```

```
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}}\,dx = -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\,\sqrt{c}}}{2d^2}$$

$$= -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{c\,c}}{2d^2\sqrt{c\cos(a+bx)}\,\sqrt{}}$$

$$= -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{\sin}}{2d^2\sqrt{d\csc(a+bx)}\,\sqrt{}}$$

$$= -\frac{c}{3bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{E(a}{2bd^2\sqrt{d\csc(a+bx)}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.40, size = 84, normalized size = 0.88

$$-\frac{\left(1+\cos(2(a+bx))-3\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\tan(a+bx)}{6bd^2\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*Sqrt[c*Sec[a + b*x]]),x]
```

```
[Out] -1/6*((1 + Cos[2*(a + b*x)] - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-
1/2, 1/4, 1/2, Csc[a + b*x]^2])*Tan[a + b*x])/(b*d^2*Sqrt[d*Csc[a + b*x]]*S
qrt[c*Sec[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(106) = 212.
time = 33.32, size = 523, normalized size = 5.51

method	result

default	$\frac{\left(2\sqrt{2}\ (\cos^4(bx+a))+3\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*(2*2^(1/2)*cos(b*x+a)^4+3*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))-6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin
(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))-5*cos(b*x+a)^2*2^(1/2)+3*2^(1/2)*cos(b*x+a))/sin(b*x+a)^3/(d/sin(b*
x+a))^(5/2)/(c/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c*d^3*csc(b*x + a)^3*se
c(b*x + a)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*sqrt(c*sec(b*x + a))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(1/2)*(d/sin(a + b*x))^(5/2)), x)
```

3.262 $\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}}+\frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

```
[Out] 2/45*d^3*(d*csc(b*x+a))^(5/2)/b/c/(c*sec(b*x+a))^(1/2)-2/9*d*(d*csc(b*x+a))
^(9/2)/b/c/(c*sec(b*x+a))^(1/2)+8/45*d^5*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*
x+a))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2703, 2705, 2699}

$$\frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}}+\frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(11/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (8*d^5*Sqrt[d*Csc[a + b*x]])/(45*b*c*Sqrt[c*Sec[a + b*x]]) + (2*d^3*(d*Csc[
a + b*x])^(5/2))/(45*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d*(d*Csc[a + b*x])^(9/2
))/(9*b*c*Sqrt[c*Sec[a + b*x]])
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
```

```
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\csc(a+bx))^{11/2}}{(c\sec(a+bx))^{3/2}}\,dx &= -\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}-\frac{d^2\int (d\csc(a+bx))^{7/2}\sqrt{c\sec(a+bx)}\,dx}{9c^2}\\ &= \frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}-\frac{(4d^4)\int (d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}\,dx}{45c^2}\\ &= \frac{8d^5\sqrt{d\csc(a+bx)}}{45bc\sqrt{c\sec(a+bx)}}+\frac{2d^3(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{9/2}}{9bc\sqrt{c\sec(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 57, normalized size = 0.52

$$\frac{2d^3(-7+2\cos(2(a+bx)))\cot^2(a+bx)(d\csc(a+bx))^{5/2}}{45bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(11/2)/(c*Sec[a + b*x])^(3/2),x]
[Out] (2*d^3*(-7 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2*(d*Csc[a + b*x])^(5/2))/(45
*b*c*Sqrt[c*Sec[a + b*x]])
```

Maple [A]
time = 34.36, size = 54, normalized size = 0.49

method	result	size
default	$\frac{2\left(4\left(\cos^2(bx+a)\right)-9\right)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{11}{2}}\cos(bx+a)\sin(bx+a)}{45b\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45/b*(4*cos(b*x+a)^2-9)*(d/sin(b*x+a))^(11/2)*cos(b*x+a)*sin(b*x+a)/(c/co
s(b*x+a))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [**A**]
time = 4.24, size = 88, normalized size = 0.80

$$\frac{2\left(4\,d^5\cos\left(bx+a\right)^5-9\,d^5\cos\left(bx+a\right)^3\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{45\left(bc^2\cos\left(bx+a\right)^4-2\,bc^2\cos\left(bx+a\right)^2+bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*d^5*cos(b*x + a)^5 - 9*d^5*cos(b*x + a)^3)*sqrt(c/cos(b*x + a))*sqr
t(d/sin(b*x + a))/(b*c^2*cos(b*x + a)^4 - 2*b*c^2*cos(b*x + a)^2 + b*c^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(11/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(11/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(11/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [**B**]
time = 3.69, size = 125, normalized size = 1.14

$$\frac{8\,d^5\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(9\,\cos\left(2\,a+2\,b\,x\right)+14\,\cos\left(4\,a+4\,b\,x\right)-9\,\cos\left(6\,a+6\,b\,x\right)+\cos\left(8\,a+8\,b\,x\right)-15\right)}{45\,b\,c\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\left(28\,\cos\left(4\,a+4\,b\,x\right)-56\,\cos\left(2\,a+2\,b\,x\right)-8\,\cos\left(6\,a+6\,b\,x\right)+\cos\left(8\,a+8\,b\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(11/2)/(c/cos(a + b*x))^(3/2),x)
```

```
[Out] (8*d^5*(d/sin(a + b*x))^(1/2)*(9*cos(2*a + 2*b*x) + 14*cos(4*a + 4*b*x) - 9
*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) - 15))/(45*b*c*(c/cos(a + b*x))^(1/2)*
(28*cos(4*a + 4*b*x) - 56*cos(2*a + 2*b*x) - 8*cos(6*a + 6*b*x) + cos(8*a +
 8*b*x) + 35))
```

3.263 $\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}-\frac{2d^4\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a}}{21bc^2}$$

```
[Out] 2/21*d^3*(d*csc(b*x+a))^(3/2)/b/c/(c*sec(b*x+a))^(1/2)-2/7*d*(d*csc(b*x+a))
^(7/2)/b/c/(c*sec(b*x+a))^(1/2)+2/21*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+
1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))^(1/2)*(c*se
c(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/c^2
```

Rubi [A]

time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2703, 2705, 2710, 2653, 2720}

$$-\frac{2d^4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{21bc^2}+\frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*d^3*(d*Csc[a + b*x])^(3/2))/(21*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d*(d*Csc[
a + b*x])^(7/2))/(7*b*c*Sqrt[c*Sec[a + b*x]]) - (2*d^4*Sqrt[d*Csc[a + b*x]]
*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/
(21*b*c^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{d^2\int (d\csc(a+bx))^{5/2}\sqrt{c\sec(a+bx)}\,dx}{7c^2}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{(2d^4)\int \sqrt{d\csc(a+bx)}\sqrt{c\sec(a-}{21c^2}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{\left(2d^4\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+b}{}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{\left(2d^4\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+b}{}$$

$$= \frac{2d^3(d\csc(a+bx))^{3/2}}{21bc\sqrt{c\sec(a+bx)}} - \frac{2d(d\csc(a+bx))^{7/2}}{7bc\sqrt{c\sec(a+bx)}} - \frac{2d^4\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.52, size = 119, normalized size = 0.88

$$-\frac{d^3\cos(2(a+bx))(d\csc(a+bx))^{3/2}\left((5+\cos(2(a+bx)))\csc^4(a+bx)-2(-\cot^2(a+bx))^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\sec^2(a+bx)\right)}{21bc\left(-2+\csc^2(a+bx)\right)\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] -1/21*(d^3*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*((5 + Cos[2*(a + b*x)])*
Csc[a + b*x]^4 - 2*(-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 Csc[a + b*x]^2]*Sec[a + b*x]^2))/(b*c*(-2 + Csc[a + b*x]^2)*Sqrt[c*Sec[a +
 b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(140) = 280.
time = 34.14, size = 550, normalized size = 4.07

method	result
default	$\frac{\left(2(\cos^3(bx+a))\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{\mathrm{co}}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/b*(2*cos(b*x+a)^3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))+2*cos(b*x+a)^2*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-
2*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*sin(
b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^3*2^(1/2)-2*
2^(1/2)*cos(b*x+a))*(d/sin(b*x+a))^(9/2)*sin(b*x+a)/(c/cos(b*x+a))^(3/2)/co
s(b*x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.50, size = 181, normalized size = 1.34

$$\frac{\left(i\,d^4\cos\left(bx+a\right)^2-i\,d^4\right)\sqrt{-4i\,cd}\;\mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)\sin\left(bx+a\right)+\left(-i\,d^4\cos\left(bx+a\right)^2+i\,d^4\right)\sqrt{4i\,cd}\;\mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)\sin\left(bx+a\right)+2\left(d^4\cos\left(bx+a\right)^3+2\,d^4\cos\left(bx+a\right)\right)\sqrt{\frac{c}{\cos\left(bx+a\right)}}\sqrt{\frac{d}{\sin\left(bx+a\right)}}}{21\left(bc^2\cos\left(bx+a\right)^2-bc^2\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*((I*d^4*cos(b*x + a)^2 - I*d^4)*sqrt(-4*I*c*d)*ellipticF(cos(b*x + a)
+ I*sin(b*x + a), -1)*sin(b*x + a) + (-I*d^4*cos(b*x + a)^2 + I*d^4)*sqrt(4
*I*c*d)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1)*sin(b*x + a) + 2*(d^4*
cos(b*x + a)^3 + 2*d^4*cos(b*x + a))*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x +
a)))/((b*c^2*cos(b*x + a)^2 - b*c^2)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{d}{\sin\left(a+b\,x\right)}\right)}^{9/2}}{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(3/2),x)
```

```
[Out] int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(3/2), x)
```

3.264 $\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

```
[Out] -2/5*c*d*(d*csc(b*x+a))^(5/2)/b/(c*sec(b*x+a))^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2699}

$$-\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(5/2))/(5*b*(c*Sec[a + b*x])^(5/2))
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{3/2}}\,dx = -\frac{2cd(d\csc(a+bx))^{5/2}}{5b(c\sec(a+bx))^{5/2}}$$

Mathematica [A]
time = 0.15, size = 45, normalized size = 1.36

$$-\frac{2d^3\cot^2(a+bx)\sqrt{d\csc(a+bx)}}{5bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(3/2),x]
```

[Out] (-2*d^3*Cot[a + b*x]^2*Sqrt[d*Csc[a + b*x]])/(5*b*c*Sqrt[c*Sec[a + b*x]])

Maple [A]
time = 34.29, size = 42, normalized size = 1.27

method	result	size
default	$-\frac{2\sin(bx+a)\left(\frac{d}{\sin(bx+a)}\right)^{\frac{7}{2}}\cos(bx+a)}{5b\left(\frac{c}{\cos(bx+a)}\right)^{\frac{3}{2}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/5/b*sin(b*x+a)*(d/sin(b*x+a))^(7/2)*cos(b*x+a)/(c/cos(b*x+a))^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.
time = 2.56, size = 59, normalized size = 1.79

$$\frac{2\,d^3\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)^3}{5\left(bc^2\cos(bx+a)^2-bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")

[Out] 2/5*d^3*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^3/(b*c^2*cos(b*x + a)^2 - b*c^2)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [B]
time = 1.29, size = 70, normalized size = 2.12

$$\frac{2\,d^3\,\left(\cos\left(4\,a+4\,b\,x\right)-1\right)\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{5\,b\,c\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\left(\cos\left(4\,a+4\,b\,x\right)-4\,\cos\left(2\,a+2\,b\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(3/2),x)
```

```
[Out] (2*d^3*(cos(4*a + 4*b*x) - 1)*(d/sin(a + b*x))^(1/2))/(5*b*c*(c/cos(a + b*x
))^(1/2)*(cos(4*a + 4*b*x) - 4*cos(2*a + 2*b*x) + 3))
```

3.265 $\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}-\frac{d^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}{3bc^2}$$

```
[Out] -2/3*d*(d*csc(b*x+a))^(3/2)/b/c/(c*sec(b*x+a))^(1/2)+1/3*d^2*(sin(a+1/4*Pi+
b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc
(b*x+a))^(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/c^2
```

Rubi [A]

time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2703, 2710, 2653, 2720}

$$-\frac{d^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{3bc^2}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*Sqrt[c*Sec[a + b*x]]) - (d^2*Sqrt[d*Cs
c[a + b*x]]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
+ 2*b*x]])/(3*b*c^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{3/2}}\,dx &= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{d^2\int\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\,dx}{3c^2}\\ &= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{}\right.}{3c^{}}\\ &= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}\right)}{3c^2}\\ &= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc\sqrt{c\sec(a+bx)}} - \frac{d^2\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{}}{3bc^2}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.89, size = 105, normalized size = 1.07

$$-\frac{d\cos(2(a+bx))(d\csc(a+bx))^{3/2}\left(2\cot^2(a+bx)-(-\cot^2(a+bx))^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)\sec^3(a+bx)}{3b\left(-2+\csc^2(a+bx)\right)(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] -1/3*(d*Cos[2*(a + b*x)]*(d*Csc[a + b*x])^(3/2)*(2*Cot[a + b*x]^2 - (-Cot[a
 + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, Csc[a + b*x]^2])*Sec[a +
b*x]^3)/(b*(-2 + Csc[a + b*x]^2)*(c*Sec[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(109) = 218.
time = 32.90, size = 290, normalized size = 2.96

method	result
default	$-\frac{\left(\cos(bx+a)\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{\cos(b}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(
cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a))
*(d/sin(b*x+a))^(5/2)*sin(b*x+a)/(c/cos(b*x+a))^(3/2)/cos(b*x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.82, size = 120, normalized size = 1.22

$$\frac{i\sqrt{-4i\,cd}\,d^2\mathrm{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)\sin(bx+a)-i\sqrt{4i\,cd}\,d^2\mathrm{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)\sin(bx+a)-4\,d^2\sqrt{\frac{c}{\cos(bx+a)}}\sqrt{\frac{d}{\sin(bx+a)}}\cos(bx+a)}{6\,bc^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(I*sqrt(-4*I*c*d)*d^2*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1)*sin(
b*x + a) - I*sqrt(4*I*c*d)*d^2*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1)
*sin(b*x + a) - 4*d^2*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a
))/(b*c^2*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{d}{\sin(a+b\,x)}\right)}^{5/2}}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(3/2),x)
```

```
[Out] int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(3/2), x)
```

3.266 $\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=327

$$-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}+\frac{d^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}-\frac{d^2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+\right.}{\sqrt{2}bc^2\sqrt{d\csc(a+bx)}}$$

```
[Out] -2*d*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*x+a))^(1/2)-1/2*d^2*arctan(-1+2^(1/2
)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/b/c^2*2^(1/2)/(d*csc(b*x+a))^(1/2)
/tan(b*x+a)^(1/2)-1/2*d^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))
^(1/2)/b/c^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)-1/4*d^2*ln(1-2^(
1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(c*sec(b*x+a))^(1/2)/b/c^2*2^(1/2)/(d*csc
(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/4*d^2*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b
*x+a))*(c*sec(b*x+a))^(1/2)/b/c^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(
1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2703, 2709, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{d^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{d^2\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{d^2\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d^2\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*Sqrt[c*Sec[a + b*x]]) + (d^2*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a
 + b*x]]*Sqrt[Tan[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*
Sqrt[c*Sec[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]
]) - (d^2*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a +
 b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) + (d^2*Lo
g[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(2*S
qrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{3/2}}\,dx &= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}-\frac{d^2\int\frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{c^2}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}-\frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\int\sqrt{\tan(a+bx)}\,dx}{c^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}-\frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int\frac{\sqrt{x}}{1+x^2}\,dx,x,\tan(a+bx)\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}-\frac{\left(2d^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int\frac{x^2}{1+x^4}\,dx,x,\sqrt{\tan(a+bx)}\right.}{bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}+\frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int\frac{1-x^2}{1+x^4}\,dx,x,\sqrt{\tan(a+bx)}\right.}{bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}-\frac{\left(d^2\sqrt{c\sec(a+bx)}\right)\mathrm{Subst}\left(\int\frac{1}{1-\sqrt{2}\,x+x^2}\,dx,x,\sqrt{\tan(a\right.}{2bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}-\frac{d^2\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)\sqrt{c\sec(a}{2\sqrt{2}\,bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}\\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc\sqrt{c\sec(a+bx)}}+\frac{d^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{\sqrt{2}\,bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}-\frac{d^2}{}
\end{aligned}$$

Mathematica [A]

time = 0.83, size = 160, normalized size = 0.49

$$-\frac{d\left(4\cot^2(a+bx)-\sqrt{2}\,\text{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}\right)\sqrt{d\csc(a+bx)}\,\tan^2(a+bx)}{2bc\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] -1/2*(d*(4*Cot[a + b*x]^2 - Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqr
t[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) - Sqrt[2]*ArcTanh[(Sqr
t[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(
3/4))*Sqrt[d*Csc[a + b*x]]*Tan[a + b*x]^2)/(b*c*Sqrt[c*Sec[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.56, size = 975, normalized size = 2.98

method	result	size
default	Expression too large to display	975

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(I*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*
x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-I*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticP
i((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*(-
(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1
-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*(-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(
b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2),1/2-1/2*I,1/2*2^(1/2))-cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*
x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)-(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+
```

```
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*2^(1/2)*cos(b*x
+a))*(d/sin(b*x+a))^(3/2)*sin(b*x+a)/cos(b*x+a)^2/(c/cos(b*x+a))^(3/2)*2^(1
/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(3/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\csc(a+bx))^{\frac{3}{2}}}{(c\sec(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
[Out] Integral((d*csc(a + b*x))**(3/2)/(c*sec(a + b*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(3/2),x)
[Out] int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(3/2), x)
```

3.267 $\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{d}{bc\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}+\frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}{2bc^2}$$

```
[Out] d/b/c/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)-1/2*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))^
(1/2)*(c*sec(b*x+a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2708, 2710, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}{2bc^2}+\frac{d}{bc\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] d/(b*c*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*E
llipticF[a - Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(2
*b*c^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
```

```
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx = \frac{d}{bc\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\int \sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,dx}{2c^2}$$

$$= \frac{d}{bc\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{c\cos(a+bx)}\,\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(}\right.}{}$$

$$= \frac{d}{bc\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\left(\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a}\right.}{2c^2}$$

$$= \frac{d}{bc\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}} + \frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sec(}}{2bc^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.70, size = 84, normalized size = 0.91

$$\frac{d\left(1+\cos(2(a+bx))-\left(-\cot^2(a+bx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)\right)\sec^3(a+bx)}{2b\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(3/2),x]
```

```
[Out] (d*(1 + Cos[2*(a + b*x)] - (-Cot[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/2, 3
/4, 3/2, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(2*b*Sqrt[d*Csc[a + b*x]]*(c*Sec[
a + b*x])^(3/2))
```

Maple [A]
time = 35.59, size = 195, normalized size = 2.12

method	result

default	$\frac{\left(-\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{2b(-1+\cos(bx+a))\cos(bx+a)^2\left(\frac{c}{\cos(bx+a)}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(-sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^
2*2^(1/2)-2^(1/2)*cos(b*x+a))*(d/sin(b*x+a))^(1/2)*sin(b*x+a)/(-1+cos(b*x+a
))/cos(b*x+a)^2/(c/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*sec(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(d*csc(a + b*x))/(c*sec(a + b*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(3/2),x)
[Out] int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(3/2), x)
```

3.268 $\int \frac{1}{\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=322

$$\frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}\,bc^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}+\frac{\text{ArcTan}\left(1+\sqrt{2}\right.}{4\sqrt{2}\,bc^2\sqrt{\ }}$$

```
[Out] 1/2*d/b/c/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(1/2)+1/8*arctan(-1+2^(1/2)*t
an(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/b/c^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/ta
n(b*x+a)^(1/2)+1/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/
b/c^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+1/16*ln(1-2^(1/2)*tan(b
*x+a)^(1/2)+tan(b*x+a))*(c*sec(b*x+a))^(1/2)/b/c^2*2^(1/2)/(d*csc(b*x+a))^(
1/2)/tan(b*x+a)^(1/2)-1/16*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(c*sec
(b*x+a))^(1/2)/b/c^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2708, 2709, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}\,bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{4\sqrt{2}\,bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bc^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{d}{2bc\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] d/(2*b*c*(d*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (ArcTan[1 - Sqrt[2]
*Sqrt[Tan[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a +
b*x]]*Sqrt[Tan[a + b*x]]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*
Sec[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) +
(Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(
8*Sqrt[2]*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (Log[1 + Sqrt[2]
*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(8*Sqrt[2]*b*c^2*
Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}\,dx = \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\int \frac{\sqrt{c\sec(a+bx)}}{\sqrt{d\csc(a+bx)}}\,dx}{4c^2}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\int \sqrt{\tan(}}{4c^2\sqrt{d\csc(a+bx)}\sqrt{\mathrm{ta}}}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\Big(\int}{4bc^2\sqrt{d\csc(a+b}}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\Big(\int}{2bc^2\sqrt{d\csc(a+}}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\Big(\int}{4bc^2\sqrt{d\csc(a+}}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\sqrt{c\sec(a+bx)}\,\mathrm{Subst}\Big(\int}{8bc^2\sqrt{d\csc(a}}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} + \frac{\log\Big(1-\sqrt{2}\sqrt{\tan(a+b}}{8\sqrt{2}\,bc^2\sqrt{d\csc}}$$

$$= \frac{d}{2bc(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}} - \frac{\tan^{-1}\Big(1-\sqrt{2}\sqrt{\tan(a}}{4\sqrt{2}\,bc^2\sqrt{d\csc(a+}}$$

Mathematica [A]

time = 1.27, size = 157, normalized size = 0.49

$$\frac{d\left(4\cos^2(a+bx)-\sqrt{2}\,\text{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}-\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}\right)\sec^3(a+bx)}{8b(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] (d*(4*Cos[a + b*x]^2 - Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*
(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4) - Sqrt[2]*ArcTanh[(Sqrt[2]*
(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(3/4))
*Sec[a + b*x]^3)/(8*b*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 34.42, size = 526, normalized size = 1.63

method	result
default	$\frac{\left(i\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\text{EllipticPi}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-I*((cos(b*
x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2
*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*2^(1/2)*cos(b*x+a))*sin(b*x+a)/(-1+cos(b
*x+a))/(d/sin(b*x+a))^(1/2)/(c/cos(b*x+a))^(3/2)/cos(b*x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sec{(a+bx)})^{\frac{3}{2}}\sqrt{d\csc{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Integral(1/((c*sec(a + b*x))**(3/2)*sqrt(d*csc(a + b*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{3/2}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(1/2)), x)
```

3.269 $\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{c}{3bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}}+\frac{1}{6bcd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}+\frac{\sqrt{d\csc(a+bx)}\,F\left(a-\frac{\pi}{4}-\right.$$

```
[Out] -1/3*c/b/d/(c*sec(b*x+a))^(5/2)/(d*csc(b*x+a))^(1/2)+1/6/b/c/d/(d*csc(b*x+a
))^(1/2)/(c*sec(b*x+a))^(1/2)-1/12*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi
+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*(d*csc(b*x+a))^(1/2)*(c*sec(b*x+
a))^(1/2)*sin(2*b*x+2*a)^(1/2)/b/c^2/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2707, 2708, 2710, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}{12bc^2d^2}-\frac{c}{3bd(c\sec(a+bx))^{5/2}\sqrt{d\csc(a+bx)}}+\frac{1}{6bcd\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] -1/3*c/(b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)) + 1/(6*b*c*d*Sqrt[
d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]) + (Sqrt[d*Csc[a + b*x]]*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*c^2*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
```

```
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}\,dx = -\frac{c}{3bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}} + \frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{3/2}}\,dx}{6d^2}$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\,\sqrt{}}$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\,\sqrt{}}$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\,\sqrt{}}$$

$$= -\frac{c}{3bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}} + \frac{1}{6bcd\sqrt{d\csc(a+bx)}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.57, size = 89, normalized size = 0.66

$$\frac{-2\cos(2(a+bx)) + \frac{\csc^2(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\csc^2(a+bx)\right)}{\sqrt[4]{-\cot^2(a+bx)}}}{12bcd\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] (-2*Cos[2*(a + b*x)] + (Csc[a + b*x]^2*Hypergeometric2F1[1/2, 3/4, 3/2, Csc
[a + b*x]^2])/(-Cot[a + b*x]^2)^(1/4))/(12*b*c*d*Sqrt[d*Csc[a + b*x]]*Sqrt[
c*Sec[a + b*x]])
```

Maple [A]
time = 35.34, size = 220, normalized size = 1.63

method	result
default	$-\frac{\left(\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(}{\sin(bx+a)}}\right.\right.}{12b(-1+\cos(bx+a))\left(\frac{d}{\sin(bx+a)}\right)^{\frac{3}{2}}\left(\frac{c}{\cos(bx+a)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)
^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)+2^(1/2)*cos(b*x+a))/
(-1+cos(b*x+a))/(d/sin(b*x+a))^(3/2)/(c/cos(b*x+a))^(3/2)/sin(b*x+a)/cos(b*
x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^2*d^2*csc(b*x + a)^2*
sec(b*x + a)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(3/2)), x)
```

3.270 $\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=371

$$-\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}+\frac{3}{16bcd(d\csc(a+bx))^{3/2}\sqrt{c\sec(a+bx)}}-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{\text{ta}}\right.}{32\sqrt{2}\,bc^2d^2\sqrt{d\csc}}$$

```
[Out] -1/4*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2)+3/16/b/c/d/(d*csc(b*x+
a))^(3/2)/(c*sec(b*x+a))^(1/2)+3/64*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(c*
sec(b*x+a))^(1/2)/b/c^2/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+3
/64*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(c*sec(b*x+a))^(1/2)/b/c^2/d^2*2^(1/
2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)+3/128*ln(1-2^(1/2)*tan(b*x+a)^(1/2
)+tan(b*x+a))*(c*sec(b*x+a))^(1/2)/b/c^2/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/t
an(b*x+a)^(1/2)-3/128*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(c*sec(b*x+
a))^(1/2)/b/c^2/d^2*2^(1/2)/(d*csc(b*x+a))^(1/2)/tan(b*x+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2707, 2708, 2709, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}\,bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{c\sec(a+bx)}}{32\sqrt{2}\,bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{3\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{64\sqrt{2}\,bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{3\sqrt{c\sec(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{64\sqrt{2}\,bc^2d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{4bd(c\sec(a+bx))^{5/2}(d\csc(a+bx))^{3/2}}+\frac{3}{16bcd\sqrt{c\sec(a+bx)}\,(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] -1/4*c/(b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)) + 3/(16*b*c*d*(d
*Csc[a + b*x])^(3/2)*Sqrt[c*Sec[a + b*x]]) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan
[a + b*x]]]*Sqrt[c*Sec[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]
]*Sqrt[Tan[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[c*Se
c[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
 + (3*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x
]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]]) - (3*Log
[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[c*Sec[a + b*x]])/(64*S
qrt[2]*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{3/2}}\,dx = -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}\,(\ldots}}{8d^2}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

$$= -\frac{c}{4bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}} + \frac{3}{16bcd(d\csc(a+bx))^{3\ldots}}$$

Mathematica [A]

time = 1.63, size = 168, normalized size = 0.45

$$-\frac{\left(2(\cos(2(a+bx))+\cos(4(a+bx)))+3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}+3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\cot^2(a+bx)^{3/4}\right)\sec^3(a+bx)}{64bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(3/2)),x]
```

```
[Out] -1/64*((2*(Cos[2*(a + b*x)] + Cos[4*(a + b*x)]) + 3*Sqrt[2]*ArcTan[(-1 + Sq
rt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(3/4
) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*
```

```
x]^2])]*(Cot[a + b*x]^2)^(3/4))*Sec[a + b*x]^3)/(b*d*(d*Csc[a + b*x])^(3/2)
*(c*Sec[a + b*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 32.35, size = 548, normalized size = 1.48

method	result
default	$\frac{\left(3i\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{1-\cos(bx+a)+}{\sin(bx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(3*I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2+1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)-3*I*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)-8*cos(b*x+a)^4*2^(1/2)+3*(
(cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2
^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)+3*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)+8*cos(b*x+a)^3*2^(1/2)+6*cos(b*x+a)^2*2^(
1/2)-6*2^(1/2)*cos(b*x+a))/(-1+cos(b*x+a))/(d/sin(b*x+a))^(5/2)/(c/cos(b*x+
a))^(3/2)/sin(b*x+a)/cos(b*x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{3/2}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(5/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(3/2)*(d/sin(a + b*x))^(5/2)), x)
```

3.271 $\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

```
[Out] -2/7*c*d*(d*csc(b*x+a))^(7/2)/b/(c*sec(b*x+a))^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2699\}$

$$-\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*c*d*(d*Csc[a + b*x])^(7/2))/(7*b*(c*Sec[a + b*x])^(7/2))
```

Rule 2699

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n - 1
)/(f*(n - 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 2, 0] &&
NeQ[n, 1]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{9/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2cd(d\csc(a+bx))^{7/2}}{7b(c\sec(a+bx))^{7/2}}$$

Mathematica [A]
time = 0.17, size = 45, normalized size = 1.36

$$-\frac{2d^4\cot^3(a+bx)\sqrt{d\csc(a+bx)}}{7bc^2\sqrt{c\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(9/2)/(c*Sec[a + b*x])^(5/2),x]
```

[Out] (-2*d^4*Cot[a + b*x]^3*Sqrt[d*Csc[a + b*x]])/(7*b*c^2*Sqrt[c*Sec[a + b*x]])

Maple [A]
time = 34.99, size = 42, normalized size = 1.27

method	result	size
default	$-\frac{2 \sin \left(b x +a \right) \cos \left(b x +a \right) \left(\frac{d}{\sin \left(b x +a \right)}\right)^{\frac{9}{2}}}{7 b \left(\frac{c}{\cos \left(b x +a \right)}\right)^{\frac{5}{2}}}$	42

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/7/b*sin(b*x+a)*cos(b*x+a)*(d/sin(b*x+a))^(9/2)/(c/cos(b*x+a))^(5/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(27) = 54.
time = 5.01, size = 67, normalized size = 2.03

$$\frac{2 d^{4} \sqrt{\frac{c}{\cos \left(b x + a\right)}} \sqrt{\frac{d}{\sin \left(b x + a\right)}} \cos \left(b x + a\right)^{4}}{7 \left(b c^{3} \cos \left(b x + a\right)^{2} - b c^{3}\right) \sin \left(b x + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")

[Out] 2/7*d^4*sqrt(c/cos(b*x + a))*sqrt(d/sin(b*x + a))*cos(b*x + a)^4/((b*c^3*co
s(b*x + a)^2 - b*c^3)*sin(b*x + a))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(9/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(9/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(9/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [B]
time = 1.76, size = 93, normalized size = 2.82

$$\frac{2\,d^4\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}\,\left(3\,\sin\left(2\,a+2\,b\,x\right)-\sin\left(6\,a+6\,b\,x\right)\right)}{7\,b\,c^2\,\sqrt{\frac{c}{\cos\left(a+b\,x\right)}}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(9/2)/(c/cos(a + b*x))^(5/2),x)
```

```
[Out] (2*d^4*(d/sin(a + b*x))^(1/2)*(3*sin(2*a + 2*b*x) - sin(6*a + 6*b*x)))/(7*b
*c^2*(c/cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) - 6*cos(4*a + 4*b*x) + cos
(6*a + 6*b*x) - 10))
```

3.272 $\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}+\frac{6d^4E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2/5*d*(d*csc(b*x+a))^(5/2)/b/c/(c*sec(b*x+a))^(3/2)+6/5*d^3*(d*csc(b*x+a))
^(1/2)/b/c/(c*sec(b*x+a))^(3/2)-6/5*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/c^2/(d*csc(b*x+a))^(1/2)/
(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2703, 2705, 2710, 2652, 2719}

$$\frac{6d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}+\frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}}-\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (6*d^3*Sqrt[d*Csc[a + b*x]])/(5*b*c*(c*Sec[a + b*x])^(3/2)) - (2*d*(d*Csc[a
 + b*x])^(5/2))/(5*b*c*(c*Sec[a + b*x])^(3/2)) + (6*d^4*EllipticE[a - Pi/4
+ b*x, 2])/(5*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2705

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
```

```
 - 1)/(f*(m - 1))), x] + Dist[a^2*((m + n - 2)/(m - 1)), Int[(a*Csc[e + f*x
])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m
, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{7/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \frac{(d\csc(a+bx))^{3/2}}{\sqrt{c\sec(a+bx)}}\,dx}{5c^2}$$

$$= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{(6d^4)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{5c^2}$$

$$= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{(6d^4)\int \sqrt{c\cos(a+bx)}\ldots}{5c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\ldots}$$

$$= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{(6d^4)\int \sqrt{\sin(2a+2\ldots}}{5c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+b\ldots}}$$

$$= \frac{6d^3\sqrt{d\csc(a+bx)}}{5bc(c\sec(a+bx))^{3/2}} - \frac{2d(d\csc(a+bx))^{5/2}}{5bc(c\sec(a+bx))^{3/2}} + \frac{6d^4E\left(a-\frac{\pi}{4}+b\ldots\right.}{5bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.86, size = 101, normalized size = 0.75

$$\frac{d^5\left((1-3\cos(2(a+bx)))\cot^2(a+bx)\csc^2(a+bx)+6\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{5bc^3(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(7/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (d^5*((1 - 3*Cos[2*(a + b*x)])*Cot[a + b*x]^2*Csc[a + b*x]^2 + 6*(-Cot[a +
b*x]^2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec
[a + b*x]])/(5*b*c^3*(d*Csc[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(140) = 280$.
time = 33.46, size = 993, normalized size = 7.36

method	result	size
default	Expression too large to display	993

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*(6*cos(b*x+a)^3*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b
*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*El
lipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*
x+a)^3*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)^2*(-(cos(b
*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)^2*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2),1/2*2^(1/2))-6*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1
/2))+3*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)
^3*2^(1/2)-6*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((
-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*(-(cos(b*x+a)-1
-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*2^(1/2)+3*2^(1/2)*cos(b*x+a))*(d
/sin(b*x+a))^(7/2)*sin(b*x+a)/cos(b*x+a)^3/(c/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(b*x + a))^(7/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(5/2),x)
```

```
[Out] int((d/sin(a + b*x))^(7/2)/(c/cos(a + b*x))^(5/2), x)
```

3.273 $\int \frac{(d \csc(a+bx))^{5/2}}{(c \sec(a+bx))^{5/2}} \, dx$

Optimal. Leaf size=329

$$-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}+\frac{d^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}}{\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}-\frac{d^2\text{ArcTan}\left(1+\right.}{}$$

```
[Out] -2/3*d*(d*csc(b*x+a))^(3/2)/b/c/(c*sec(b*x+a))^(3/2)-1/2*d^2*arctan(-1+2^(1
/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2*2^(1/2)/(
c*sec(b*x+a))^(1/2)-1/2*d^2*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a
))^(1/2)*tan(b*x+a)^(1/2)/b/c^2*2^(1/2)/(c*sec(b*x+a))^(1/2)+1/4*d^2*ln(1-2
^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b
/c^2*2^(1/2)/(c*sec(b*x+a))^(1/2)-1/4*d^2*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan
(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2*2^(1/2)/(c*sec(b*x+a))
^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2703, 2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{d^2\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}-\frac{d^2\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}+\frac{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}-\frac{d^2\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{2\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}-\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*(d*Csc[a + b*x])^(3/2))/(3*b*c*(c*Sec[a + b*x])^(3/2)) + (d^2*ArcTan[
1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(S
qrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (d^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b
*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(Sqrt[2]*b*c^2*Sqrt[c*Sec[a
+ b*x]]) + (d^2*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + T
an[a + b*x]]*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) - (
d^2*Sqrt[d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]
*Sqrt[Tan[a + b*x]])/(2*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{(d\csc(a+bx))^{5/2}}{(c\sec(a+bx))^{5/2}}\,dx = -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{d^2\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{c^2}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\int \frac{1}{\sqrt{\tan(a+bx)}}\,d}{c^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(2d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x\right.}{bc^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} - \frac{\left(d^2\sqrt{d\csc(a+bx)}\sqrt{\tan(a+bx)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x}\right.}{2bc^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} + \frac{d^2\sqrt{d\csc(a+bx)}\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a\right.}{2\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

$$= -\frac{2d(d\csc(a+bx))^{3/2}}{3bc(c\sec(a+bx))^{3/2}} + \frac{d^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\sqrt{\text{ta}}}{\sqrt{2}bc^2\sqrt{c\sec(a+bx)}}$$

Mathematica [A]

time = 1.67, size = 154, normalized size = 0.47

$$-\frac{d^3\left(4\cot^2(a+bx)-3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}+3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}\right)\sqrt{c\sec(a+bx)}}{6bc^3\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(5/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] -1/6*(d^3*(4*Cot[a + b*x]^2 - 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/
(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) + 3*Sqrt[2]*ArcTan
h[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x
]^2)^(1/4))*Sqrt[c*Sec[a + b*x]])/(b*c^3*Sqrt[d*Csc[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 37.47, size = 1259, normalized size = 3.83

method	result	size
default	Expression too large to display	1259

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/b*(3*I*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))-3*I*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))+3*I*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a)
)^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1
/2*I,1/2*2^(1/2))-3*I*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I
,1/2*2^(1/2))+3*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-
1/2*I,1/2*2^(1/2))+3*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(
b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))-6*cos(b*x+a)*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))+3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2
```

```
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1
/2*2^(1/2))+3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(
1/2))-6*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a
)^2*2^(1/2))*(d/sin(b*x+a))^(5/2)*sin(b*x+a)/cos(b*x+a)^3/(c/cos(b*x+a))^(5
/2)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(5/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(5/2),x)
[Out] int((d/sin(a + b*x))^(5/2)/(c/cos(a + b*x))^(5/2), x)
```

3.274 $\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}-\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2*d*(d*csc(b*x+a))^(1/2)/b/c/(c*sec(b*x+a))^(3/2)+3*d^2*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/c^2/(d*c
sc(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2703, 2710, 2652, 2719}

$$-\frac{3d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bc^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*Sqrt[d*Csc[a + b*x]])/(b*c*(c*Sec[a + b*x])^(3/2)) - (3*d^2*EllipticE
[a - Pi/4 + b*x, 2])/(b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[
Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2703

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Dist[a^2*((n + 1)/(b^2*(m - 1))), Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
```

```
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\csc(a+bx))^{3/2}}{(c\sec(a+bx))^{5/2}}\,dx &= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \frac{1}{\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}}\,dx}{c^2} \\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \sqrt{c\cos(a+bx)}\sqrt{d\sin(a+bx)}\,dx}{c^2\sqrt{c\cos(a+bx)}\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{d\,s\cdots}} \\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{(3d^2)\int \sqrt{\sin(2a+2bx)}\,dx}{c^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}} \\
&= -\frac{2d\sqrt{d\csc(a+bx)}}{bc(c\sec(a+bx))^{3/2}} - \frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{bc^2\sqrt{d\csc(a+bx)}\sqrt{c\sec(a+bx)}\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.69, size = 80, normalized size = 0.85

$$
-\frac{d^3\left(2\cot^2(a+bx)+3\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{bc^3(d\csc(a+bx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[a + b*x])^(3/2)/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] -((d^3*(2*Cot[a + b*x]^2 + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2
, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(b*c^3*(d*Csc[a + b*x])^
(3/2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(109) = 218.
time = 32.24, size = 515, normalized size = 5.48

method	result

default	$\frac{\left(6\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \text{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(6*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+
a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*(-(cos(b*x+a)-1-sin(b*x+a
))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))-3*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b
*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*El
lipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+
a)^2*2^(1/2)-3*2^(1/2)*cos(b*x+a))*(d/sin(b*x+a))^(3/2)*sin(b*x+a)/cos(b*x+
a)^3/(c/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))*d*csc(b*x + a)/(c^3*sec(
b*x + a)^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*csc(b*x + a))^(3/2)/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\sin(a+b\,x)}\right)^{3/2}}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(5/2),x)
[Out] int((d/sin(a + b*x))^(3/2)/(c/cos(a + b*x))^(5/2), x)
```

3.275 $\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=322

$$\frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}-\frac{3\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}}{4\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}+$$

```
[Out] 1/2*d/b/c/(c*sec(b*x+a))^(3/2)/(d*csc(b*x+a))^(1/2)+3/8*arctan(-1+2^(1/2)*t
an(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2*2^(1/2)/(c*sec
(b*x+a))^(1/2)+3/8*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*
tan(b*x+a)^(1/2)/b/c^2*2^(1/2)/(c*sec(b*x+a))^(1/2)-3/16*ln(1-2^(1/2)*tan(b
*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2*2^(1/2)
/(c*sec(b*x+a))^(1/2)+3/16*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*csc
(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2*2^(1/2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2708, 2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{4\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}+\frac{3\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{4\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}-\frac{3\sqrt{\tan(a+bx)}\,\sqrt{d\csc(a+bx)}\,\log\left(\tan(a+bx)-\sqrt{2}\,\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}+\frac{3\sqrt{\tan(a+bx)}\,\sqrt{d\csc(a+bx)}\,\log\left(\tan(a+bx)+\sqrt{2}\,\sqrt{\tan(a+bx)}+1\right)}{8\sqrt{2}\,bc^2\sqrt{c\sec(a+bx)}}+\frac{d}{2bc(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] d/(2*b*c*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*
b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqr
t[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(4*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]
) - (3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*
x]]*Sqrt[Tan[a + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[d*
Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a
 + b*x]])/(8*Sqrt[2]*b*c^2*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx = \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{3\int \frac{\sqrt{d\csc(a+bx)}}{\sqrt{c\sec(a+bx)}}\,dx}{4c^2}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\int \frac{}{\sqrt{t}}}{4c^2\sqrt{c\sec(a+bx)}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\text{Subst}}{4bc^2\sqrt{c\sec(a}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\text{Subst}}{2bc^2\sqrt{c\sec(a+}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\text{Subst}}{4bc^2\sqrt{c\sec(a+}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} + \frac{\left(3\sqrt{d\csc(a+bx)}\,\sqrt{\tan(a+bx)}\right)\text{Subst}}{8bc^2\sqrt{c\sec(}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} - \frac{3\sqrt{d\csc(a+bx)}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(a+}\right.}{8\sqrt{2}\,bc^2\sqrt{c\sec}}$$

$$= \frac{d}{2bc\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(a+bx)}\right)\sqrt{d\csc}}{4\sqrt{2}\,bc^2\sqrt{c\sec(a+b}}$$

Mathematica [A]

time = 1.55, size = 152, normalized size = 0.47

$$\frac{d\left(4\cos^2(a+bx)-3\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}+3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}\right)\sqrt{c\sec(a+bx)}}{8bc^3\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[a + b*x]]/(c*Sec[a + b*x])^(5/2),x]
```

```
[Out] (d*(4*Cos[a + b*x]^2 - 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2
]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^(1/4) + 3*Sqrt[2]*ArcTanh[(Sqrt
[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(1
/4))*Sqrt[c*Sec[a + b*x]])/(8*b*c^3*Sqrt[d*Csc[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 34.88, size = 668, normalized size = 2.07

method	result
default	$\frac{\left(3i\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(bx+a)-1-}{\sin(bx}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(3*I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/
2)-3*I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-3
*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((
-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*sin(b
*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*sin(b*x+a)*
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)^3*2^(1/2)-2*cos(b
*x+a)^2*2^(1/2))*(d/sin(b*x+a))^(1/2)*sin(b*x+a)/(-1+cos(b*x+a))/cos(b*x+a)
^3/(c/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(b*x + a))/(c*sec(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}{{\left(\frac{c}{\cos\left(a+b\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(5/2),x)
[Out] int((d/sin(a + b*x))^(1/2)/(c/cos(a + b*x))^(5/2), x)
```

3.276 $\int \frac{1}{\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2bc^2\sqrt{d\csc(a+bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/3*d/b/c/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(3/2)-1/2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/c^2/(d*cs
c(b*x+a))^(1/2)/(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2708, 2710, 2652, 2719}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bc^2\sqrt{\sin(2a+2bx)}\,\sqrt{c\sec(a+bx)}\,\sqrt{d\csc(a+bx)}}+\frac{d}{3bc(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] d/(3*b*c*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + EllipticE[a - Pi/
4 + b*x, 2]/(2*b*c^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a
 + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
```

```
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{5/2}}\,dx &= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{d\csc(a+bx)}\,\sqrt{c\sec\cdots}}}{2c^2} \\ &= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{c\cos\cdots}}{2c^2\sqrt{c\cos(a+bx)}\,\sqrt{d\cdots}} \\ &= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{\int \sqrt{\sin(2\cdots}}{2c^2\sqrt{d\csc(a+bx)}\,\sqrt{c\,s\cdots}} \\ &= \frac{d}{3bc(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}} + \frac{E(a-\cdots}{2bc^2\sqrt{d\csc(a+bx)}\,\sqrt{c\cdots}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.39, size = 79, normalized size = 0.83

$$\frac{d\left(1+\cos(2(a+bx))+3\sqrt[4]{-\cot^2(a+bx)}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{6bc^3(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] (d*(1 + Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[-1/2
, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(6*b*c^3*(d*Csc[a + b*x]
)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(106) = 212.
time = 35.93, size = 530, normalized size = 5.58

method	result

default	$-\frac{\left(2\sqrt{2}\,(\cos^4(bx+a))+6\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\text{EllipticE}\left(\sqrt{-\frac{\mathrm{c}}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*(2*cos(b*x+a)^4*2^(1/2)+6*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))-3*cos(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*(
(cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*(-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*2^(1/2)*cos(b*x+a))/(d/sin(b*x+a))^
(1/2)/(c/cos(b*x+a))^(5/2)/cos(b*x+a)^3/sin(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d*csc(b*x + a)*sec(
b*x + a)^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(1/2)/(c*sec(b*x+a))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(1/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*csc(b*x + a))*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{5/2}\,\sqrt{\frac{d}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2)),x)
[Out] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(1/2)), x)
```

3.277 $\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=371

$$-\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}}+\frac{1}{16bcd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{3/2}}-\frac{3\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan\right.}{32\sqrt{}}$$

```
[Out] -1/4*c/b/d/(c*sec(b*x+a))^(7/2)/(d*csc(b*x+a))^(1/2)+1/16/b/c/d/(c*sec(b*x+
a))^(3/2)/(d*csc(b*x+a))^(1/2)+3/64*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(d*
csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2/d^2*2^(1/2)/(c*sec(b*x+a))^(1/2)+3
/64*arctan(1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2
)/b/c^2/d^2*2^(1/2)/(c*sec(b*x+a))^(1/2)-3/128*ln(1-2^(1/2)*tan(b*x+a)^(1/2
)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2/d^2*2^(1/2)/(c*se
c(b*x+a))^(1/2)+3/128*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+
a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2/d^2*2^(1/2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2707, 2708, 2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{32\sqrt{2}\,bc^2d^2\sqrt{c\sec(a+bx)}}+\frac{3\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{32\sqrt{2}\,bc^2d^2\sqrt{c\sec(a+bx)}}-\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{64\sqrt{2}\,bc^2d^2\sqrt{c\sec(a+bx)}}+\frac{3\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{64\sqrt{2}\,bc^2d^2\sqrt{c\sec(a+bx)}}-\frac{c}{4bd(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}+\frac{1}{16bcd(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -1/4*c/(b*d*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 1/(16*b*c*d*Sqrt
[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a
 + b*x]]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sq
rt[c*Sec[a + b*x]]) + (3*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[
a + b*x]]*Sqrt[Tan[a + b*x]])/(32*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]]) -
 (3*Sqrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]
*Sqrt[Tan[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]]) + (3*Sqrt[
d*Csc[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan
[a + b*x]])/(64*Sqrt[2]*b*c^2*d^2*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{\int \frac{\sqrt{d\csc(a+bx)}}{(c\sec(a+bx))^{5/2}}\,dx}{8d^2}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

$$= -\frac{c}{4bd\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}} + \frac{1}{16bcd\sqrt{d\csc(a+bx)}}$$

Mathematica [A]
time = 1.47, size = 165, normalized size = 0.44

$$-\frac{\left(4+6\cos(2(a+bx))+2\cos(4(a+bx))+3\sqrt{2}\,\text{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-3\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}\right)\sqrt{c\sec(a+bx)}}{64bc^3d\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -1/64*((4 + 6*Cos[2*(a + b*x)] + 2*Cos[4*(a + b*x)] + 3*Sqrt[2]*ArcTan[(-1
+ Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))]*(Cot[a + b*x]^2)^
```

```
(1/4) - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]^2)^(1/4))/(1 + Sqrt[Cot[a
+ b*x]^2])]*(Cot[a + b*x]^2)^(1/4))*Sqrt[c*Sec[a + b*x]])/(b*c^3*d*Sqrt[d*C
sc[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 34.92, size = 696, normalized size = 1.88

method	result
default	$\left(3i\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(bx+a)-1}{\sin(bx}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(3*I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)-3*I*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-
8*2^(1/2)*cos(b*x+a)^5-3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-3*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/
2*2^(1/2))+6*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+8*cos(
b*x+a)^4*2^(1/2)+2*cos(b*x+a)^3*2^(1/2)-2*cos(b*x+a)^2*2^(1/2))/(-1+cos(b*x
+a))/(d/sin(b*x+a))^(3/2)/(c/cos(b*x+a))^(5/2)/cos(b*x+a)^3/sin(b*x+a)*2^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(3/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(3/2)*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{c}{\cos(a+b\,x)}\right)}^{5/2}\,{\left(\frac{d}{\sin(a+b\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(3/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(3/2)), x)
```

3.278 $\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}}+\frac{1}{10bcd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{3/2}}+\frac{3E}{20bc^2d^2\sqrt{d\csc(a+bx}}$$

```
[Out] -1/5*c/b/d/(d*csc(b*x+a))^(3/2)/(c*sec(b*x+a))^(7/2)+1/10/b/c/d/(d*csc(b*x+
a))^(3/2)/(c*sec(b*x+a))^(3/2)-3/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*P
i+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))/b/c^2/d^2/(d*csc(b*x+a))^(1/2)/
(c*sec(b*x+a))^(1/2)/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2707, 2708, 2710, 2652, 2719}

$$\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bc^2d^2\sqrt{\sin(2a+2bx)}\sqrt{c\sec(a+bx)}\sqrt{d\csc(a+bx)}}-\frac{c}{5bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{3/2}}+\frac{1}{10bcd(c\sec(a+bx))^{3/2}(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -1/5*c/(b*d*(d*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(7/2)) + 1/(10*b*c*d*(d
*Csc[a + b*x])^(3/2)*(c*Sec[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x,
 2])/(20*b*c^2*d^2*Sqrt[d*Csc[a + b*x]]*Sqrt[c*Sec[a + b*x]]*Sqrt[Sin[2*a +
 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
```

```
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2710

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*(b*Sec[e + f*x])^n*(a*Sin[e + f*x]
)^m*(b*Cos[e + f*x])^n, Int[1/((a*Sin[e + f*x])^m*(b*Cos[e + f*x])^n), x],
x] /; FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[m - 1/2] && IntegerQ[n - 1/2
]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{3\int \frac{1}{\sqrt{d\csc(a+bx)}\,(\cdots}}{10d^2 \cdots}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3\cdots}}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3\cdots}}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3\cdots}}$$

$$= -\frac{c}{5bd(d\csc(a+bx))^{3/2}(c\sec(a+bx))^{7/2}} + \frac{1}{10bcd(d\csc(a+bx))^{3\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.68, size = 90, normalized size = 0.67

$$\frac{\left(-2\cos^2(a+bx)\cos(2(a+bx)) + 3\sqrt[4]{-\cot^2(a+bx)}\;{}_2F_1\left(-\tfrac{1}{2},\tfrac{1}{4};\tfrac{1}{2};\csc^2(a+bx)\right)\right)\sqrt{c\sec(a+bx)}}{20bc^3d(d\csc(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] ((-2*Cos[a + b*x]^2*Cos[2*(a + b*x)] + 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeome
tric2F1[-1/2, 1/4, 1/2, Csc[a + b*x]^2])*Sqrt[c*Sec[a + b*x]])/(20*b*c^3*d*
(d*Csc[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(140) = 280$.
time = 36.21, size = 536, normalized size = 3.97

method	result
default	$\frac{\left(4\sqrt{2}\,(\cos^{6}(bx+a))-6\sqrt{2}\,(\cos^{4}(bx+a))+3\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\mathrm{co}}{\sin(b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40/b*(4*2^(1/2)*cos(b*x+a)^6-6*cos(b*x+a)^4*2^(1/2)+3*cos(b*x+a)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))+3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipti
cF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-6*((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*2^(1/2)+3*2^(1/2)*cos(b*x+a))/s
in(b*x+a)^3/cos(b*x+a)^3/(d/sin(b*x+a))^(5/2)/(c/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*csc(b*x + a))*sqrt(c*sec(b*x + a))/(c^3*d^3*csc(b*x + a)^3*
sec(b*x + a)^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(5/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(5/2)*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(5/2)), x)
```

3.279 $\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=406

$$-\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}}-\frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\,(c\sec(a+bx))^{7/2}}+\frac{5}{192bcd^3\sqrt{d\csc(a+bx)}}$$

```
[Out] -1/6*c/b/d/(d*csc(b*x+a))^(5/2)/(c*sec(b*x+a))^(7/2)-5/48*c/b/d^3/(c*sec(b*
x+a))^(7/2)/(d*csc(b*x+a))^(1/2)+5/192/b/c/d^3/(c*sec(b*x+a))^(3/2)/(d*csc(
b*x+a))^(1/2)+5/256*arctan(-1+2^(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2
)*tan(b*x+a)^(1/2)/b/c^2/d^4*2^(1/2)/(c*sec(b*x+a))^(1/2)+5/256*arctan(1+2^
(1/2)*tan(b*x+a)^(1/2))*(d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2/d^4*2^(
1/2)/(c*sec(b*x+a))^(1/2)-5/512*ln(1-2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(
d*csc(b*x+a))^(1/2)*tan(b*x+a)^(1/2)/b/c^2/d^4*2^(1/2)/(c*sec(b*x+a))^(1/2)
+5/512*ln(1+2^(1/2)*tan(b*x+a)^(1/2)+tan(b*x+a))*(d*csc(b*x+a))^(1/2)*tan(b
*x+a)^(1/2)/b/c^2/d^4*2^(1/2)/(c*sec(b*x+a))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2707, 2708, 2709, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)\sqrt{d\csc(a+bx)}}{128\sqrt{2}\,bc^2d^4\sqrt{c\sec(a+bx)}}+\frac{5\sqrt{\tan(a+bx)}\,\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(a+bx)}+1\right)\sqrt{d\csc(a+bx)}}{128\sqrt{2}\,bc^2d^4\sqrt{c\sec(a+bx)}}-\frac{5\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)-\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{256\sqrt{2}\,bc^2d^4\sqrt{c\sec(a+bx)}}+\frac{5\sqrt{\tan(a+bx)}\sqrt{d\csc(a+bx)}\log\left(\tan(a+bx)+\sqrt{2}\sqrt{\tan(a+bx)}+1\right)}{256\sqrt{2}\,bc^2d^4\sqrt{c\sec(a+bx)}}-\frac{5c}{48bd^3(c\sec(a+bx))^{7/2}\sqrt{d\csc(a+bx)}}+\frac{5}{192bcd^3(c\sec(a+bx))^{3/2}\sqrt{d\csc(a+bx)}}-\frac{c}{6bd(c\sec(a+bx))^{7/2}(d\csc(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -1/6*c/(b*d*(d*Csc[a + b*x])^(5/2)*(c*Sec[a + b*x])^(7/2)) - (5*c)/(48*b*d^
3*Sqrt[d*Csc[a + b*x]]*(c*Sec[a + b*x])^(7/2)) + 5/(192*b*c*d^3*Sqrt[d*Csc[
a + b*x]]*(c*Sec[a + b*x])^(3/2)) - (5*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]
]]*Sqrt[d*Csc[a + b*x]]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*S
ec[a + b*x]]) + (5*ArcTan[1 + Sqrt[2]*Sqrt[Tan[a + b*x]]]*Sqrt[d*Csc[a + b*
x]]*Sqrt[Tan[a + b*x]])/(128*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) - (5*S
qrt[d*Csc[a + b*x]]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt
[Tan[a + b*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]]) + (5*Sqrt[d*Cs
c[a + b*x]]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]*Sqrt[Tan[a +
 b*x]])/(256*Sqrt[2]*b*c^2*d^4*Sqrt[c*Sec[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2707

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
```

```
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2709

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^m*((b*Sec[e + f*x])^n/Tan[e + f*x]^n
), Int[Tan[e + f*x]^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ
[n] && EqQ[m + n, 0]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d\csc(a+bx))^{7/2}(c\sec(a+bx))^{5/2}}\,dx = -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} + \frac{5\int \frac{1}{(d\csc(a+bx))^{3/2}(c\sec(\cdots}}{12d^2}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

$$= -\frac{c}{6bd(d\csc(a+bx))^{5/2}(c\sec(a+bx))^{7/2}} - \frac{5c}{48bd^3\sqrt{d\csc(a+bx)}\cdots}$$

Mathematica [A]
time = 1.97, size = 175, normalized size = 0.43

$$-\frac{\left(28+34\cos(2(a+bx))+2\cos(4(a+bx))-4\cos(6(a+bx))+15\sqrt{2}\,\mathrm{ArcTan}\left(\frac{-1+\sqrt{\cot^2(a+bx)}}{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}-15\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\cot^2(a+bx)}}{1+\sqrt{\cot^2(a+bx)}}\right)\sqrt[4]{\cot^2(a+bx)}\right)\sqrt{c\sec(a+bx)}}{768bc^3d^3\sqrt{d\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Csc[a + b*x])^(7/2)*(c*Sec[a + b*x])^(5/2)),x]
```

```
[Out] -1/768*((28 + 34*Cos[2*(a + b*x)] + 2*Cos[4*(a + b*x)] - 4*Cos[6*(a + b*x)]
 + 15*Sqrt[2]*ArcTan[(-1 + Sqrt[Cot[a + b*x]^2])/(Sqrt[2]*(Cot[a + b*x]^2)^
(1/4))]*(Cot[a + b*x]^2)^(1/4) - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*(Cot[a + b*x]^
2)^(1/4))/(1 + Sqrt[Cot[a + b*x]^2])]*(Cot[a + b*x]^2)^(1/4))*Sqrt[c*Sec[a
+ b*x]])/(b*c^3*d^3*Sqrt[d*Csc[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 35.20, size = 722, normalized size = 1.78

method	result
default	$\frac{\left(64\sqrt{2}\,(\cos^7(bx+a))-64\sqrt{2}\,(\cos^6(bx+a))-104\sqrt{2}\,(\cos^5(bx+a))-15i\sin(bx+a)\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/768/b*(64*2^(1/2)*cos(b*x+a)^7-64*2^(1/2)*cos(b*x+a)^6-104*2^(1/2)*cos(b*
x+a)^5-15*I*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*E
llipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/
2))+15*I*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))
+104*cos(b*x+a)^4*2^(1/2)-15*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-15*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2
*I,1/2*2^(1/2))+30*sin(b*x+a)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^
(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+
10*cos(b*x+a)^3*2^(1/2)-10*cos(b*x+a)^2*2^(1/2))/(-1+cos(b*x+a))/(d/sin(b*x
+a))^(7/2)/(c/cos(b*x+a))^(5/2)/cos(b*x+a)^3/sin(b*x+a)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))**(7/2)/(c*sec(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(b*x+a))^(7/2)/(c*sec(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*csc(b*x + a))^(7/2)*(c*sec(b*x + a))^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(\frac{c}{\cos(a+b\,x)}\right)^{5/2}\,\left(\frac{d}{\sin(a+b\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(7/2)),x)
```

```
[Out] int(1/((c/cos(a + b*x))^(5/2)*(d/sin(a + b*x))^(7/2)), x)
```

3.280 $\int \csc^n(e+fx)\sec^m(e+fx)\,dx$

Optimal. Leaf size=81

$$\frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec^{1+m}(e+fx)}{f(1-n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*m)*csc(f*x+e)^(-1+n)*hypergeom([1/2-1/2*n, 1/2+1/2*
m],[3/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)^(1+m)/f/(1-n)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2711, 2657}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)\csc^{n-1}(e+fx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^n*Sec[e + f*x]^m,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*(1 - n
))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int \csc^n(e+fx)\sec^m(e+fx)\,dx = \left(\cos^{1+m}(e+fx)\csc^{-1+n}(e+fx)\sec^{1+m}(e+fx)\sin^{-1+n}(e+fx)\right)\int \cos$$

$$= \frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec^{1+m}}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.32, size = 278, normalized size = 3.43

$$-\frac{(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};m,1-m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\csc^{-1+n}(e+fx)\sec^m(e+fx)}{f(-1+n)\left((-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};m,1-m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((-1+m+n)F_1\left(\frac{3}{2}-\frac{n}{2};m,2-m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+mF_1\left(\frac{3}{2}-\frac{n}{2};1+m,1-m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^n*Sec[e + f*x]^m,x]
```

```
[Out] -(((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Csc[e + f*x]^(-1 + n)*Sec[e + f*x]^m)/(f*(-1 + n)*((
-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[3/2 - n/2, m, 2 - m - n, 5/2
- n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2 - n/2, 1 +
 m, 1 - m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \left(\csc^n\left(fx+e\right)\right)\left(\sec^m\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

```
[Out] int(csc(f*x+e)^n*sec(f*x+e)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="fricas")
[Out] integral(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{n}\left(e + fx\right)\sec^{m}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**n*sec(f*x+e)**m,x)
[Out] Integral(csc(e + f*x)**n*sec(e + f*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*sec(f*x+e)^m,x, algorithm="giac")
[Out] integrate(csc(f*x + e)^n*sec(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{1}{\sin\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^m*(1/sin(e + f*x))^n,x)
[Out] int((1/cos(e + f*x))^m*(1/sin(e + f*x))^n, x)
```

3.281 $\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx$

Optimal. Leaf size=86

$$\frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(a\sec(e+fx))^{1+m}}{af(1-n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*m)*csc(f*x+e)^(-1+n)*hypergeom([1/2-1/2*n, 1/2+1/2*
m],[3/2-1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^(1+m)/a/f/(1-n)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2711, 2657}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}\csc^{n-1}(e+fx)(a\sec(e+fx))^{m+1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m)/2)*Csc[e + f*x]^(-1 + n)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))/(a*f*
(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int \csc^n(e+fx)(a\sec(e+fx))^m\,dx = \frac{((a\cos(e+fx))^{1+m}\csc^{-1+n}(e+fx)(a\sec(e+fx))^{1+m}\sin^{-1+n}(e+ \ldots}{a^2}$$
$$= \frac{\cos^2(e+fx)^{\frac{1+m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(a \ldots}{af(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.94, size = 280, normalized size = 3.26

$$-\frac{(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};m,1-m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\csc^{-1+n}(e+fx)(a\sec(e+fx))^m}{f(-1+n)\left((-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};m,1-m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((-1+m+n)F_1\left(\frac{3}{2}-\frac{n}{2};m,2-m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+mF_1\left(\frac{3}{2}-\frac{n}{2};1+m,1-m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] -(((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Csc[e + f*x]^(-1 + n)*(a*Sec[e + f*x])^m)/(f*(-1 + n
)*((-3 + n)*AppellF1[1/2 - n/2, m, 1 - m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[3/2 - n/2, m, 2 - m - n,
5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2 - n/2,
 1 + m, 1 - m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(\csc^n(fx+e)\right)(a\sec(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

```
[Out] int(csc(f*x+e)^n*(a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(e+fx))^m \csc^n(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**n*(a*sec(f*x+e))**m,x)
[Out] Integral((a*sec(e + f*x))**m*csc(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^n*(a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e))^m*csc(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{a}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{1}{\sin\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(e + f*x))^m*(1/sin(e + f*x))^n,x)
[Out] int((a/cos(e + f*x))^m*(1/sin(e + f*x))^n, x)
```

3.282 $\int (b\csc(e+fx))^n \sec^m(e+fx)\, dx$

Optimal. Leaf size=84

$$\frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\, {}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec^{1+m}(e+fx)}{f(1-n)}$$

```
[Out] b*(cos(f*x+e)^2)^(1/2+1/2*m)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/2-1/2*n, 1/
2+1/2*m],[3/2-1/2*n],sin(f*x+e)^2)*sec(f*x+e)^(1+m)/f/(1-n)
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2711, 2657}

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}\sec^{m+1}(e+fx)(b\csc(e+fx))^{n-1}\, {}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^m,x]
```

```
[Out] (b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x]^(1 + m))/(f*
(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n \sec^m(e+fx)\,dx = \left(b^2\cos^{1+m}(e+fx)(b\csc(e+fx))^{-1+n}\sec^{1+m}(e+fx)(b\sin(e+fx))^{-1}\right.$$

$$= \frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.76, size = 281, normalized size = 3.35

$$-\frac{b(-3+n)F_1\left(\frac{1-n}{2};m,1-m-n;\frac{3-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(b\csc(e+fx))^{-1+n}\sec^m(e+fx)}{f(-1+n)\left((-3+n)F_1\left(\frac{1-n}{2};m,1-m-n;\frac{3-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((-1+m+n)F_1\left(\frac{3-n}{2};m,2-m-n;\frac{5-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+mF_1\left(\frac{3-n}{2};1+m,1-m-n;\frac{5-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^m,x]
```

```
[Out] -((b*(-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*(b*Csc[e + f*x])^(-1 + n)*Sec[e + f*x]^m)/(f*(-1 +
 n)*((-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[(3 - n)/2, m, 2 - m - n
, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[(3 - n)/
2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n (\sec^m(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc (e+fx))^n \sec^m (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**m,x)
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n*(1/cos(e + f*x))^m,x)
[Out] int((b/sin(e + f*x))^n*(1/cos(e + f*x))^m, x)
```

3.283 $\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(a\sec(e+fx))^{1+m}}{af(1-n)}$$

```
[Out] b*(cos(f*x+e)^2)^(1/2+1/2*m)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/2-1/2*n, 1/
2+1/2*m],[3/2-1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^(1+m)/a/f/(1-n)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2711, 2657\}$

$$\frac{b\cos^2(e+fx)^{\frac{m+1}{2}}(a\sec(e+fx))^{m+1}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{af(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] (b*(Cos[e + f*x]^2)^((1 + m)/2)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^(1 + m))
/(a*f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n(a\sec(e+fx))^m\,dx = \frac{(b^2(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{-1+n}(a\sec(e+fx))^{1+m}(b\sin(}{a^2}$$
$$= \frac{b\cos^2(e+fx)^{\frac{1+m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1+m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+\right.}{af(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.78, size = 283, normalized size = 3.18

$$-\frac{b(-3+n)F_1\left(\frac{1-n}{2};m,1-m-n;\frac{3-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(b\csc(e+fx))^{-1+n}(a\sec(e+fx))^m}{f(-1+n)\left((-3+n)F_1\left(\frac{1-n}{2};m,1-m-n;\frac{3-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((-1+m+n)F_1\left(\frac{3-n}{2};m,2-m-n;\frac{5-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+mF_1\left(\frac{3-n}{2};1+m,1-m-n;\frac{5-n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Csc[e + f*x])^n*(a*Sec[e + f*x])^m,x]
```

```
[Out] -((b*(-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*(b*Csc[e + f*x])^(-1 + n)*(a*Sec[e + f*x])^m)/(f*(
-1 + n)*((-3 + n)*AppellF1[(1 - n)/2, m, 1 - m - n, (3 - n)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m + n)*AppellF1[(3 - n)/2, m, 2 - m
 - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[(3 -
 n)/2, 1 + m, 1 - m - n, (5 - n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
])*Tan[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\left(a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

```
[Out] int((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (e+fx))^m (b\csc (e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(a*sec(f*x+e))**m,x)
[Out] Integral((a*sec(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^n*(a*sec(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{a}{\cos \left(e+f\,x\right)}\right)}^m\,{\left(\frac{b}{\sin \left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/cos(e + f*x))^m*(b/sin(e + f*x))^n,x)
[Out] int((a/cos(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

3.284 $\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{(b\csc(e+fx))^{5+n}\,{}_2F_1\left(3,\frac{5+n}{2};\frac{7+n}{2};\csc^2(e+fx)\right)}{b^5f(5+n)}$$

```
[Out] (b*csc(f*x+e))^(5+n)*hypergeom([3, 5/2+1/2*n],[7/2+1/2*n],csc(f*x+e)^2)/b^5
/f/(5+n)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2701, 371}

$$\frac{(b\csc(e+fx))^{n+5}\,{}_2F_1\left(3,\frac{n+5}{2};\frac{n+7}{2};\csc^2(e+fx)\right)}{b^5f(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^5,x]
```

```
[Out] ((b*Csc[e + f*x])^(5 + n)*Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, Csc[e
+ f*x]^2])/(b^5*f*(5 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^5(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{4+n}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\csc(e+fx)\right)}{b^5f}\\ &= \frac{(b\csc(e+fx))^{5+n}\,{}_2F_1\left(3,\frac{5+n}{2};\frac{7+n}{2};\csc^2(e+fx)\right)}{b^5f(5+n)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 1.06

$$-\frac{b(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(3,\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^5,x]
[Out] -((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3, (1 - n)/2, (3 - n)/2, S
in[e + f*x]^2])/(f*(-1 + n)))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n \left(\sec^5(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n \sec^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**5,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/cos(e + f*x)^5,x)
```

```
[Out] int((b/sin(e + f*x))^n/cos(e + f*x)^5, x)
```

3.285 $\int (b\csc(e+fx))^n \sec^3(e+fx)\,dx$

Optimal. Leaf size=49

$$-\frac{(b\csc(e+fx))^{3+n}\,{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};\csc^2(e+fx)\right)}{b^3f(3+n)}$$

```
[Out] -(b*csc(f*x+e))^(3+n)*hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],csc(f*x+e)^2)/b^
3/f/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2701, 371}

$$-\frac{(b\csc(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};\csc^2(e+fx)\right)}{b^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^3,x]
```

```
[Out] -(((b*Csc[e + f*x])^(3 + n)*Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Csc[
e + f*x]^2])/(b^3*f*(3 + n)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\sec^3(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{2+n}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\csc(e+fx)\right)}{b^3f}\\ &= -\frac{(b\csc(e+fx))^{3+n}\,{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};\csc^2(e+fx)\right)}{b^3f(3+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 1.04

$$-\frac{b(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(2,\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^3,x]
[Out] -((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[2, (1 - n)/2, (3 - n)/2, S
in[e + f*x]^2])/(f*(-1 + n)))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n \left(\sec^3(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n \sec^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**3,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{b}{\sin(e+f\,x)}\right)}^n}{{\cos\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/cos(e + f*x)^3,x)
```

```
[Out] int((b/sin(e + f*x))^n/cos(e + f*x)^3, x)
```

3.286 $\int (b\csc(e+fx))^n \sec(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{(b\csc(e+fx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\csc^2(e+fx)\right)}{bf(1+n)}$$

```
[Out] (b*csc(f*x+e))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],csc(f*x+e)^2)/b/f
/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2701, 371}

$$\frac{(b\csc(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\csc^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x],x]
```

```
[Out] ((b*Csc[e + f*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Csc[e
+ f*x]^2])/(b*f*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n \sec(e+fx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+\frac{x^2}{b^2}}\,dx, x, b\csc(e+fx)\right)}{bf}\\ &= \frac{(b\csc(e+fx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\csc^2(e+fx)\right)}{bf(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 1.06

$$-\frac{b(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(1,\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x],x]
[Out] -((b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1, (1 - n)/2, (3 - n)/2, S
in[e + f*x]^2])/(f*(-1 + n)))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e),x)
[Out] int((b*csc(f*x+e))^n*sec(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="fricas")
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(e+fx\right)\right)^{n}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e),x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/cos(e + f*x),x)
```

```
[Out] int((b/sin(e + f*x))^n/cos(e + f*x), x)
```

3.287 $\int \cos(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=24

$$\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] b*(b*csc(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2701, 30}

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(e+fx)(b\csc(e+fx))^n\,dx &= -\frac{b\text{Subst}\left(\int x^{-2+n}\,dx, x, b\csc(e+fx)\right)}{f}\\ &= \frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 0.96

$$-\frac{b(b\csc(e+fx))^{-1+n}}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(b*Csc[e + f*x])^n,x]
```

```
[Out] -((b*(b*Csc[e + f*x])^(-1 + n))/(f*(-1 + n)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(24) = 48$.
time = 2.55, size = 66, normalized size = 2.75

method	result	size
norman	$-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\mathrm{e}^{n\ln\left(\frac{b\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}}{f(-1+n)\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$	66
risch	Expression too large to display	1312

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(b*csc(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/(-1+n)*tan(1/2*f*x+1/2*e)*exp(n*ln(1/2*b*(1+tan(1/2*f*x+1/2*e)^2)/tan(
1/2*f*x+1/2*e)))/(1+tan(1/2*f*x+1/2*e)^2)
```

Maxima [A]
time = 0.29, size = 31, normalized size = 1.29

$$-\frac{b^n \sin(fx+e)^{-n} \sin(fx+e)}{f(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] -b^n*sin(f*x + e)^(-n)*sin(f*x + e)/(f*(n - 1))
```

Fricas [A]
time = 3.27, size = 31, normalized size = 1.29

$$-\frac{\left(\frac{b}{\sin(fx+e)}\right)^n \sin(fx+e)}{fn-f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -(b/sin(f*x + e))^n*sin(f*x + e)/(f*n - f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n \cos(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e), x)
```

Mupad [B]
time = 0.32, size = 28, normalized size = 1.17

$$-\frac{\sin\left(e+f\,x\right)\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n}{f\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(b/sin(e + f*x))^n,x)
```

```
[Out] -(sin(e + f*x)*(b/sin(e + f*x))^n)/(f*(n - 1))
```

3.288 $\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=52

$$-\frac{b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b^3*(b*csc(f*x+e))^(-3+n)/f/(3-n)+b*(b*csc(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2701, 14}

$$\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}-\frac{b^3(b\csc(e+fx))^{n-3}}{f(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(b*Csc[e + f*x])^n,x]
```

```
[Out] -((b^3*(b*Csc[e + f*x])^(-3 + n))/(f*(3 - n))) + (b*(b*Csc[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(e+fx)(b\csc(e+fx))^n\,dx &= -\frac{b^3\text{Subst}\left(\int x^{-4+n}\left(-1+\frac{x^2}{b^2}\right)\,dx,x,b\csc(e+fx)\right)}{f}\\ &= -\frac{b^3\text{Subst}\left(\int\left(-x^{-4+n}+\frac{x^{-2+n}}{b^2}\right)\,dx,x,b\csc(e+fx)\right)}{f}\\ &= -\frac{b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 45, normalized size = 0.87

$$-\frac{b(-5+n+(-1+n)\cos(2(e+fx)))(b\csc(e+fx))^{-1+n}}{2f(-3+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(b*Csc[e + f*x])^n,x]
```

```
[Out] -1/2*(b*(-5 + n + (-1 + n)*Cos[2*(e + f*x)])*(b*Csc[e + f*x])^(-1 + n))/(f*
(-3 + n)*(-1 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.34, size = 2446, normalized size = 47.04

method	result	size
risch	Expression too large to display	2446

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(b*csc(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I*exp(I*(f*x+e))^n*b^n*(exp(2*I*(f*x+e))-1)^(-n)*2^n/(f*n-3*f)*exp(1/2*
I*(-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*n+csgn(I*b/(exp(2*I*
(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2*I
*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n-csgn(I*e
xp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*
x+e)))*csgn(I*b)*Pi*n+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/
(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n+csgn(b/(exp(2*I*(f*x+e))-1)*exp
(I*(f*x+e)))^3*Pi*n-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*Pi*n+csgn
(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(exp(2*I*(f*x+e))-1))*Pi*n
+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*Pi*n-
csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(
exp(2*I*(f*x+e))-1))*Pi*n-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csg
n(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*n-csgn(b/(exp(2*I*(f*x+e))-1)*e
xp(I*(f*x+e)))^2*Pi*n+Pi*n+6*f*x+6*e))+1/8*I*exp(I*(f*x+e))^n*b^n*(exp(2*I*
(f*x+e))-1)^(-n)*2^n/(-3+n)/(-1+n)/f*(n-9)*exp(1/2*I*(-csgn(I*b/(exp(2*I*(f
*x+e))-1)*exp(I*(f*x+e)))^3*Pi*n+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))^2*csgn(I*b)*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(
exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n-csgn(I*exp(I*(f*x+e))/(exp(2*I*(
f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*Pi*n+cs
gn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp
(I*(f*x+e)))^2*Pi*n+csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*n-csgn
(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))-1))^2*csgn(I/(exp(2*I*(f*x+e))-1))*Pi*n+csgn(I*exp(I*(f*x+e))/(
exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*Pi*n-csgn(I*exp(I*(f*x+e))/(e
```

```
xp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))-1))*Pi*
n-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)
*exp(I*(f*x+e)))*Pi*n-csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n+Pi
*n+2*f*x+2*e))-1/8*I/(f*n-3*f)*2^n*(exp(2*I*(f*x+e))-1)^(-n)*b^n*exp(I*(f*x
+e))^n*exp(-1/2*I*(csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*n-csg
n(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*Pi*n-csgn(I*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^
2*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e
))-1)*exp(I*(f*x+e)))*csgn(I*b)*Pi*n-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f
*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n-csgn(b/(exp(2*I*
(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-
1))^3*Pi*n-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(exp(2*I*(f
*x+e))-1))*Pi*n-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*
(f*x+e)))*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*
x+e)))*csgn(I/(exp(2*I*(f*x+e))-1))*Pi*n+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(
I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*n+csgn(b/(exp(2*
I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n-Pi*n+6*f*x+6*e))-1/8*I*exp(I*(f*x+e))^
n*b^n*(exp(2*I*(f*x+e))-1)^(-n)*2^n/(-3+n)/(-1+n)/f*(n-9)*exp(-1/2*I*(csgn(
I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*n-csgn(I*b/(exp(2*I*(f*x+e))-
1)*exp(I*(f*x+e)))^2*csgn(I*b)*Pi*n-csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n+csgn(I*exp(I*(f*x
+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*cs
gn(I*b)*Pi*n-csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*
(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*n-csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))^3*Pi*n+csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*Pi*n-csgn(I*exp(I*
(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(exp(2*I*(f*x+e))-1))*Pi*n-csgn(I*e
xp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*Pi*n+csgn(I*ex
p(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(
f*x+e))-1))*Pi*n+csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(
2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*n+csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x
+e)))^2*Pi*n-Pi*n+2*f*x+2*e))
```

Maxima [A]
time = 0.30, size = 62, normalized size = 1.19

$$\frac{\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)^3}{n-3}-\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*sin(f*x + e)^(-n)*sin(f*x + e)^3/(n - 3) - b^n*sin(f*x + e)^(-n)*sin(f
*x + e)/(n - 1))/f
```

Fricas [A]

time = 2.92, size = 52, normalized size = 1.00

$$-\frac{\left((n-1)\cos\left(fx+e\right)^{2}-2\right)\left(\frac{b}{\sin(fx+e)}\right)^{n}\sin\left(fx+e\right)}{fn^{2}-4\,fn+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -((n - 1)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^n*sin(f*x + e)/(f*n^2 - 4*f*
n + 3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(b*csc(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^3, x)
```

Mupad [**B**]
time = 0.63, size = 66, normalized size = 1.27

$$\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^{n}\left(9\,\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)-n\,\sin\left(e+f\,x\right)-n\,\sin\left(3\,e+3\,f\,x\right)\right)}{4\,f\,\left(n^{2}-4\,n+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(b/sin(e + f*x))^n,x)
```

```
[Out] ((b/sin(e + f*x))^n*(9*sin(e + f*x) + sin(3*e + 3*f*x) - n*sin(e + f*x) - n
*sin(3*e + 3*f*x)))/(4*f*(n^2 - 4*n + 3))
```

3.289 $\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=78

$$\frac{b^5(b\csc(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] b^5*(b*csc(f*x+e))^(-5+n)/f/(5-n)-2*b^3*(b*csc(f*x+e))^(-3+n)/f/(3-n)+b*(b*
csc(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A]

time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2701, 276}

$$\frac{b^5(b\csc(e+fx))^{n-5}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{n-3}}{f(3-n)}+\frac{b(b\csc(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b^5*(b*Csc[e + f*x])^(-5 + n))/(f*(5 - n)) - (2*b^3*(b*Csc[e + f*x])^(-3 +
 n))/(f*(3 - n)) + (b*(b*Csc[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^5(e+fx)(b\csc(e+fx))^n\,dx &= -\frac{b^5\text{Subst}\left(\int x^{-6+n}\left(-1+\frac{x^2}{b^2}\right)^2\,dx,x,b\csc(e+fx)\right)}{f}\\ &= -\frac{b^5\text{Subst}\left(\int\left(x^{-6+n}-\frac{2x^{-4+n}}{b^2}+\frac{x^{-2+n}}{b^4}\right)dx,x,b\csc(e+fx)\right)}{f}\\ &= \frac{b^5(b\csc(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\csc(e+fx))^{-3+n}}{f(3-n)}+\frac{b(b\csc(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 81, normalized size = 1.04

$$-\frac{(b\csc(e+fx))^n\left(3-4n+n^2-2(5-6n+n^2)\csc^2(e+fx)+(15-8n+n^2)\csc^4(e+fx)\right)\sin^5(e+fx)}{f(-5+n)(-3+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(b*Csc[e + f*x])^n,x]
```

```
[Out] -(((b*Csc[e + f*x])^n*(3 - 4*n + n^2 - 2*(5 - 6*n + n^2)*Csc[e + f*x]^2 + (
15 - 8*n + n^2)*Csc[e + f*x]^4)*Sin[e + f*x]^5)/(f*(-5 + n)*(-3 + n)*(-1 +
n)))
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int\left(\cos^5(fx+e)\right)(b\csc(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^5*(b*csc(f*x+e))^n,x)
```

Maxima [A]
time = 0.30, size = 92, normalized size = 1.18

$$-\frac{\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)^5}{n-5}-\frac{2\,b^n\sin(fx+e)^{-n}\sin(fx+e)^3}{n-3}+\frac{b^n\sin(fx+e)^{-n}\sin(fx+e)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] -(b^n*sin(f*x + e)^(-n)*sin(f*x + e)^5/(n - 5) - 2*b^n*sin(f*x + e)^(-n)*si
n(f*x + e)^3/(n - 3) + b^n*sin(f*x + e)^(-n)*sin(f*x + e)/(n - 1))/f
```

Fricas [A]
time = 3.36, size = 77, normalized size = 0.99

$$-\frac{\left((n^2-4\,n+3)\cos(fx+e)^4-4\,(n-1)\cos(fx+e)^2+8\right)\left(\frac{b}{\sin(fx+e)}\right)^n\sin(fx+e)}{fn^3-9\,fn^2+23\,fn-15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] -((n^2 - 4*n + 3)*cos(f*x + e)^4 - 4*(n - 1)*cos(f*x + e)^2 + 8)*(b/sin(f*x
 + e))^n*sin(f*x + e)/(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(b*csc(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^5, x)
```

Mupad [**B**]
time = 1.41, size = 134, normalized size = 1.72

$$-\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n\,\left(150\,\sin\left(e+f\,x\right)+25\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)+3\,n^2\,\sin\left(3\,e+3\,f\,x\right)+n^2\,\sin\left(5\,e+5\,f\,x\right)-24\,n\,\sin\left(e+f\,x\right)-28\,n\,\sin\left(3\,e+3\,f\,x\right)-4\,n\,\sin\left(5\,e+5\,f\,x\right)+2\,n^2\,\sin\left(e+f\,x\right)\right)}{16\,f\,\left(n^3-9\,n^2+23\,n-15\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(b/sin(e + f*x))^n,x)
```

```
[Out] -((b/sin(e + f*x))^n*(150*sin(e + f*x) + 25*sin(3*e + 3*f*x) + 3*sin(5*e +
5*f*x) + 3*n^2*sin(3*e + 3*f*x) + n^2*sin(5*e + 5*f*x) - 24*n*sin(e + f*x)
- 28*n*sin(3*e + 3*f*x) - 4*n*sin(5*e + 5*f*x) + 2*n^2*sin(e + f*x)))/(16*f
*(23*n - 9*n^2 + n^3 - 15))
```

3.290 $\int (b\csc(e+fx))^n \sec^6(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{7}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}$$

```
[Out] b*(b*csc(f*x+e))^(-1+n)*hypergeom([7/2, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2711, 2657}

$$\frac{b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{7}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^6,x]
```

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[7/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n \sec^6(e+fx)\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \sec^6(e+fx)(b\sin(e+fx$$

$$= \frac{b\sqrt{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{7}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(}{f(1-n)}$$

Mathematica [A]
time = 0.65, size = 77, normalized size = 1.07

$$\frac{(b\csc(e+fx))^n\,{}_2F_1\left(-2-\frac{n}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(e+fx)\right)\sec^2(e+fx)^{-n/2}\tan(e+fx)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^6,x]
```

```
[Out] ((b*Csc[e + f*x])^n*Hypergeometric2F1[-2 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan[
e + f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n\left(\sec^6(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/cos(e + f*x)^6,x)
```

```
[Out] int((b/sin(e + f*x))^n/cos(e + f*x)^6, x)
```

3.291 $\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}$$

```
[Out] b*(b*csc(f*x+e))^(-1+n)*hypergeom([5/2, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2711, 2657}

$$\frac{b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^4,x]
```

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \sec^4(e+fx)(b\sin(e+f$$

$$= \frac{b\sqrt{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\text{se}}{f(1-n)}$$

Mathematica [A]
time = 0.56, size = 77, normalized size = 1.07

$$\frac{(b\csc(e+fx))^n\,{}_2F_1\left(-1-\frac{n}{2},\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(e+fx)\right)\sec^2(e+fx)^{-n/2}\tan(e+fx)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^4,x]
```

```
[Out] ((b*Csc[e + f*x])^n*Hypergeometric2F1[-1 - n/2, 1/2 - n/2, 3/2 - n/2, -Tan[
e + f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n \left(\sec^4(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n \sec^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**4,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/cos(e + f*x)^4,x)
```

```
[Out] int((b/sin(e + f*x))^n/cos(e + f*x)^4, x)
```

3.292 $\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(e+fx)}{f(1-n)}$$

```
[Out] b*(b*csc(f*x+e))^(-1+n)*hypergeom([3/2, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^2
)*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2711, 2657}

$$\frac{b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sec[e + f*x]^2,x]
```

```
[Out] (b*Sqrt[Cos[e + f*x]^2]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/2, (1
 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sec[e + f*x])/(f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n \sec^2(e+fx)\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \sec^2(e+fx)(b\sin(e+fx$$

$$= \frac{b\sqrt{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sec(}{f(1-n)}$$

Mathematica [A]
time = 0.49, size = 75, normalized size = 1.04

$$\frac{(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2}-\frac{n}{2},-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2(e+fx)\right)\sec^2(e+fx)^{-n/2}\tan(e+fx)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sec[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^n*Hypergeometric2F1[1/2 - n/2, -1/2*n, 3/2 - n/2, -Tan[e
+ f*x]^2]*Tan[e + f*x])/(f*(1 - n)*(Sec[e + f*x]^2)^(n/2))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n\left(\sec^2(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

```
[Out] int((b*csc(f*x+e))^n*sec(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n\sec^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*sec(f*x+e)**2,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sec(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*sec(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*sec(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n}{{\cos\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/cos(e + f*x)^2,x)
```

```
[Out] int((b/sin(e + f*x))^n/cos(e + f*x)^2, x)
```

3.293 $\int (b\csc(e+fx))^n\,dx$

Optimal. Leaf size=72

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] b*cos(f*x+e)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],s
in(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3857, 2722}

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/2, (1 - n)/2,
 (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^n\,dx &= (b\csc(e+fx))^n\left(\frac{\sin(e+fx)}{b}\right)^n\int\left(\frac{\sin(e+fx)}{b}\right)^{-n}dx\\ &= \frac{\cos(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sin(e+fx)}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.90

$$-\frac{\cos(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3}{2};\cos^2(e+fx)\right)\sin(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-1+n)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, 3/2, C
os[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 + n)/2))/f)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n,x)
```

```
[Out] int((b*csc(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n,x)
```

```
[Out] int((b/sin(e + f*x))^n, x)
```

3.294 $\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=72

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] b*cos(f*x+e)*(b*csc(f*x+e))^(-1+n)*hypergeom([-1/2, 1/2-1/2*n],[3/2-1/2*n],
sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2711, 2657}

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)(b\csc(e+fx))^n\,dx &= \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \cos^2(e+fx)(b\sin(e+f\\ &= \frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 165 vs. 2(72) = 144.
time = 0.50, size = 165, normalized size = 2.29

$$-\frac{2(b\csc(e+fx))^n\left({}_2F_1\left(1-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\,{}_2F_1\left(2-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+4\,{}_2F_1\left(3-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}\tan\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(b*Csc[e + f*x])^n*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan
[(e + f*x)/2]^2] - 4*Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e
 + f*x)/2]^2] + 4*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2])/(f*(-1 + n)*(Sec[(e + f*x)/2]^2)^n)
```

Maple [**F**]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(b\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(b*csc(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n\cos^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^2\left(\frac{b}{\sin(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(b/sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(b/sin(e + f*x))^n, x)
```

3.295 $\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=72

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

```
[Out] b*cos(f*x+e)*(b*csc(f*x+e))^(-1+n)*hypergeom([-3/2, 1/2-1/2*n],[3/2-1/2*n],
sin(f*x+e)^2)/f/(1-n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2711, 2657}

$$\frac{b\cos(e+fx)(b\csc(e+fx))^{n-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-3/2, (1 - n)/2
, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 - n)*Sqrt[Cos[e + f*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\begin{aligned}\int \cos^4(e+fx)(b\csc(e+fx))^n\,dx &= \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \cos^4(e+fx)(b\sin(e+\\ &= \frac{b\cos(e+fx)(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 246 vs. 2(72) = 144.
time = 0.63, size = 246, normalized size = 3.42

$$-\frac{2(b\csc(e+fx))^n\left({}_2F_1\left(1-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-8\left({}_2F_1\left(2-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-3\,{}_2F_1\left(3-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+4\,{}_2F_1\left(4-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\,{}_2F_1\left(5-n,\frac{1}{2}-\frac{n}{2};\frac{3}{2}-\frac{n}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}\tan\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(b*Csc[e + f*x])^n*(Hypergeometric2F1[1 - n, 1/2 - n/2, 3/2 - n/2, -Tan
[(e + f*x)/2]^2] - 8*(Hypergeometric2F1[2 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(
e + f*x)/2]^2] - 3*Hypergeometric2F1[3 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e +
 f*x)/2]^2] + 4*Hypergeometric2F1[4 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*
x)/2]^2] - 2*Hypergeometric2F1[5 - n, 1/2 - n/2, 3/2 - n/2, -Tan[(e + f*x)/
2]^2]))*Tan[(e + f*x)/2])/(f*(-1 + n)*(Sec[(e + f*x)/2]^2)^n)
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(fx+e\right)\right)\left(b\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(b*csc(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n \cos^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(b*csc(f*x+e))**n,x)
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*csc(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^4\,\left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(b/sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^4*(b/sin(e + f*x))^n, x)
```

3.296 $\int (b\csc(e+fx))^n (c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=81

$$\frac{b\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(c\sec(e+fx))^{5/2}}{cf(1-n)}$$

```
[Out] b*(cos(f*x+e)^2)^(5/4)*(b*csc(f*x+e))^(-1+n)*hypergeom([5/4, 1/2-1/2*n],[3/
2-1/2*n],sin(f*x+e)^2)*(c*sec(f*x+e))^(5/2)/c/f/(1-n)
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2711, 2657}

$$\frac{b\cos^2(e+fx)^{5/4}(c\sec(e+fx))^{5/2}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[5/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(5/2))/(c*f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n(c\sec(e+fx))^{3/2}\,dx = \frac{\left(b^2(c\cos(e+fx))^{5/2}(b\csc(e+fx))^{-1+n}(c\sec(e+fx))^{5/2}(b\sin(e+\right.}{c^2}$$

$$= \frac{b\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Mathematica [A]
time = 11.93, size = 92, normalized size = 1.14

$$\frac{2\cot(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sec^2(e+fx)\right)(c\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{\frac{1+n}{2}}}{f(3+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*(c*Sec[e + f*x])^(3/2),x]
[Out] (2*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4
, (7 + 2*n)/4, Sec[e + f*x]^2]*(c*Sec[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^((1
 + n)/2))/(f*(3 + 2*n))
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\left(c\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
[Out] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sec(f*x + e))^(3/2)*(b*csc(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n*c*sec(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sec(f*x + e))^(3/2)*(b*csc(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{c}{\cos\left(e+f\,x\right)}\right)^{3/2}\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^(3/2)*(b/sin(e + f*x))^n,x)
```

```
[Out] int((c/cos(e + f*x))^(3/2)*(b/sin(e + f*x))^n, x)
```

3.297 $\int (b\csc(e+fx))^n \sqrt{c\sec(e+fx)}\, dx$

Optimal. Leaf size=81

$$\frac{b\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)(c\sec(e+fx))^{3/2}}{cf(1-n)}$$

```
[Out] b*(cos(f*x+e)^2)^(3/4)*(b*csc(f*x+e))^(-1+n)*hypergeom([3/4, 1/2-1/2*n],[3/
2-1/2*n],sin(f*x+e)^2)*(c*sec(f*x+e))^(3/2)/c/f/(1-n)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2711, 2657}

$$\frac{b\cos^2(e+fx)^{3/4}(c\sec(e+fx))^{3/2}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (b*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[3/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*(c*Sec[e + f*x])^(3/2))/(c*f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int (b\csc(e+fx))^n \sqrt{c\sec(e+fx)}\, dx = \frac{\left(b^2(c\cos(e+fx))^{3/2}(b\csc(e+fx))^{-1+n}(c\sec(e+fx))^{3/2}(b\sin(e-\right.}{c^2}$$

$$= \frac{b\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Mathematica [A]
time = 11.71, size = 90, normalized size = 1.11

$$\frac{2\cot(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sec^2(e+fx)\right)\sqrt{c\sec(e+fx)}\,(-\tan^2(e+fx))^{\frac{1+n}{2}}}{f+2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4
, (5 + 2*n)/4, Sec[e + f*x]^2]*Sqrt[c*Sec[e + f*x]]*(-Tan[e + f*x]^2)^((1 +
 n)/2))/(f + 2*f*n)
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^n\sqrt{c\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n\sqrt{c\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*csc(e + f*x))**n*sqrt(c*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(c*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{c}{\cos\left(e+f\,x\right)}}\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/cos(e + f*x))^(1/2)*(b/sin(e + f*x))^n,x)
```

```
[Out] int((c/cos(e + f*x))^(1/2)*(b/sin(e + f*x))^n, x)
```

3.298 $\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx$

Optimal. Leaf size=81

$$\frac{b\sqrt[4]{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sqrt{c\sec(e+fx)}}{cf(1-n)}$$

```
[Out] b*(cos(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(-1+n)*hypergeom([1/4, 1/2-1/2*n],[3/
2-1/2*n],sin(f*x+e)^2)*(c*sec(f*x+e))^(1/2)/c/f/(1-n)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2711, 2657}

$$\frac{b\sqrt[4]{\cos^2(e+fx)}\,\sqrt{c\sec(e+fx)}\,(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n/Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (b*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[1/4,
(1 - n)/2, (3 - n)/2, Sin[e + f*x]^2]*Sqrt[c*Sec[e + f*x]])/(c*f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int \frac{(b\csc(e+fx))^n}{\sqrt{c\sec(e+fx)}}\,dx = \frac{\left(b^2\sqrt{c\cos(e+fx)}\,(b\csc(e+fx))^{-1+n}\sqrt{c\sec(e+fx)}\,(b\sin(e+fx))^{-1+n}\right)\int \surd}{c^2}$$

$$= \frac{b\sqrt[4]{\cos^2(e+fx)}\,(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)\sqrt{c\sec(e+fx)}}{cf(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 32.71, size = 326, normalized size = 4.02

$$-\frac{4(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-\frac{1}{2},\frac{3}{2}-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(b\csc(e+fx))^n\sin\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)\sqrt{c\sec(e+fx)}\left(2(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-\frac{1}{2},\frac{3}{2}-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)-F_1\left(\frac{3}{2}-\frac{n}{2};\frac{1}{2},\frac{3}{2}-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(-1+\cos(e+fx))+2(3-2n)F_1\left(\frac{3}{2}-\frac{n}{2};-\frac{1}{2},\frac{5}{2}-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Csc[e + f*x])^n/Sqrt[c*Sec[e + f*x]],x]
```

```
[Out] (-4*(-3 + n)*AppellF1[1/2 - n/2, -1/2, 3/2 - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Csc[e + f*x])^n*Sin[(e + f*x
)/2])/(f*(-1 + n)*Sqrt[c*Sec[e + f*x]]*(2*(-3 + n)*AppellF1[1/2 - n/2, -1/2
, 3/2 - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x
)/2]^2 - AppellF1[3/2 - n/2, 1/2, 3/2 - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(-1 + Cos[e + f*x]) + 2*(3 - 2*n)*AppellF1[3/2 - n/2, -
1/2, 5/2 - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e +
f*x)/2]^2))
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(b\csc(fx+e))^n}{\sqrt{c\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n/(c*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\csc{(e+fx)})^n}{\sqrt{c\sec{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n/(c*sec(f*x+e))**(1/2),x)
[Out] Integral((b*csc(e + f*x))**n/sqrt(c*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^n/sqrt(c*sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\sqrt{\frac{c}{\cos{(e+f\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(1/2),x)
[Out] int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(1/2), x)
```

3.299 $\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{b(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{c\sec(e+fx)}}$$

```
[Out] b*(b*csc(f*x+e))^(-1+n)*hypergeom([-1/4, 1/2-1/2*n],[3/2-1/2*n],sin(f*x+e)^
2)/c/f/(1-n)/(cos(f*x+e)^2)^(1/4)/(c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2711, 2657}

$$\frac{b(b\csc(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n/(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(b*Csc[e + f*x])^(-1 + n)*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2,
Sin[e + f*x]^2])/(c*f*(1 - n)*(Cos[e + f*x]^2)^(1/4)*Sqrt[c*Sec[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2711

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(
n + 1)*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1), Int[1/((a*Sin[e +
 f*x])^m*(b*Cos[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !S
implerQ[-m, -n]
```

Rubi steps

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{3/2}}\,dx = \frac{\left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int (c\cos(e+fx))^{3/2}(b\sin(e+fx))^{-n}\,dx}{c^2\sqrt{c\cos(e+fx)}\,\sqrt{c\sec(e+fx)}}$$
$$= \frac{b(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{cf(1-n)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{c\sec(e+fx)}}$$

Mathematica [A]
time = 21.13, size = 115, normalized size = 1.42

$$-\frac{2\cos(2(e+fx))\cot(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\sec^2(e+fx)\right)\sqrt{c\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{\frac{1+n}{2}}}{c^2f(-3+2n)\left(-2+\sec^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n/(c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[2*(e + f*x)]*Cot[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 +
 n)/2, (-3 + 2*n)/4, (1 + 2*n)/4, Sec[e + f*x]^2]*Sqrt[c*Sec[e + f*x]]*(-Ta
n[e + f*x]^2)^((1 + n)/2))/(c^2*f*(-3 + 2*n)*(-2 + Sec[e + f*x]^2))
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(b\csc(fx+e))^n}{(c\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

```
[Out] int((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sec(f*x + e))*(b*csc(f*x + e))^n/(c^2*sec(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\csc(e+fx))^n}{(c\sec(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n/(c*sec(f*x+e))**(3/2),x)
[Out] Integral((b*csc(e + f*x))**n/(c*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n/(c*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^n/(c*sec(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^n}{\left(\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(3/2),x)
[Out] int((b/sin(e + f*x))^n/(c/cos(e + f*x))^(3/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*     is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*     antiderivative*)
(* "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```